

Evaluation Method for Pressure Loss using Energy Dissipation

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ABSTRACT

In order to make more effective use of simulation, a new evaluation method on pressure loss has been developed. Using this method, the location and amount of pressure loss is clarified. As a result, enormous simulation data have been used effectively. The authors have confirmed the validity of the method about fundamental shapes.

INTRODUCTION

Due to the dramatic evolution of computer hardware and software, it is becoming possible to simulate very large models associated with product development, where it is important to examine the phenomenon based on the simulated results, and come up with an improvement idea.

However, in doing this, the enormous simulation data are usually not used effectively. Even in future, although accurate calculation system might be more easily available, the simulated results would become less useful without appropriate evaluation system. From now on, it is necessary not only to perform more accurate calculations, but also to develop better analysis tools.

To deal with this problem, we have developed a method to evaluate pressure loss from energy dissipation.

Thus, this paper describes in detail this evaluation method as well as results of pressure loss in duct flow.

THEORY OF EVALUATION METHOD

One of the most important factors to evaluate the duct flow performance is pressure loss. In general, the pressure loss just means the total energy loss in the whole duct domain, but in engineering, it is important to know where and how much the pressure loss occurs. To do this, it is necessary to examine each element of the pressure loss.

The time-averaged Navier-Stokes equations (RANS: Reynolds Averaged Navier-Stokes) for mean flow quantities of incompressible flow can be written as follows.

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j^2} - \frac{\partial \bar{u}_i' u_j'}{\partial x_j} \quad (1)$$

From the inner product of the RANS equations and $\bar{u}_i$, the transport equation of the kinetic energy can be obtained.

$$\frac{\partial K}{\partial t} + \bar{u}_i \frac{\partial K}{\partial x_i} = -\frac{1}{\rho} \bar{u}_i \frac{\partial \bar{p}}{\partial x_i} + \nu \bar{u}_i \frac{\partial^2 \bar{u}_i}{\partial x_j^2} - \bar{u}_i \frac{\partial \bar{u}_i' u_j'}{\partial x_j} \quad (2)$$

where K ($K = (1/2) \bar{u}_i' u_i'$) is the kinetic energy. Integrating this equation over the whole volume, and applying Gauss's divergence theorem, eq. (3) is obtained (See Appendix A).

$$\begin{aligned} \int_S \frac{1}{\rho} (\rho K + \bar{p}) \bar{u}_i n_i dS = \\ \int_S \nu \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \bar{u}_j n_i dS - \int_S \bar{u}_i' u_j' n_j dS \\ - \int_V \nu \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \frac{\partial \bar{u}_i}{\partial x_j} dV + \int_V \bar{u}_i' u_j' \frac{\partial \bar{u}_i}{\partial x_j} dV \end{aligned} \quad (3)$$

We consider an internal flow with an inlet, an outlet, and a wall (see Fig. 1), where $\bar{u}_i = 0$ at the wall boundary. Then, we assume that the inlet flow is equal to the outlet one, leading to eq. (4).

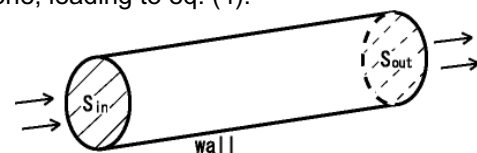


Figure 1 Internal Flow

$$\begin{aligned}
\int_S \frac{1}{\rho} (\rho K + \bar{p}) \bar{u}_i n_i dS &= \int_{S_{in}} \frac{P_t}{\rho} \bar{u}_i n_i dS + \int_{S_{out}} \frac{P_t}{\rho} \bar{u}_i n_i dS \\
&= \int_{S_{out}} \frac{P_t}{\rho} \bar{u}_i n_i dS - \int_{S_{in}} \frac{P_t}{\rho} \bar{u}_i (-n_i) dS \\
&= \frac{Q}{\rho} (\bar{P}_{t,out} - \bar{P}_{t,in}) = \frac{Q}{\rho} \Delta P_t
\end{aligned} \quad (4)$$

where P_t and Q are the total pressure and the flow rate, respectively.

From eq. (4), it turns out that the pressure loss on the left side of eq. (3) has been transformed into the difference in total pressure between the inlet and outlet.

By applying $\bar{u}_i = 0$ at the wall in the first and second terms on its right side, eq. (3) can be arranged into eq. (5).

$$\begin{aligned}
\frac{Q}{\rho} \Delta P_t &= \int_{S_{in}+S_{out}} \nu \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \bar{u}_j n_i dS \\
&\quad - \int_{S_{in}+S_{out}} \bar{u}_i' \bar{u}_j' \bar{u}_j n_i dS \\
&\quad - \int_V \nu \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \frac{\partial \bar{u}_i}{\partial x_j} dV + \int_V \bar{u}_i' \bar{u}_j' \frac{\partial \bar{u}_i}{\partial x_j} dV
\end{aligned} \quad (5)$$

The right side of eq. (5) represents the energy dissipation. Therefore, we can evaluate a local contribution of the pressure loss by integrating the energy dissipation in each domain.

The first term on the right side of eq. (5) shows the energy loss by molecular viscosity at the inlet and outlet, while the second term shows the energy loss by turbulent viscosity at the inlet and outlet. The third term is energy dissipation by molecular viscosity, and the fourth term energy dissipation by turbulent viscosity; i.e., the third term is the production of internal energy, and the fourth term the turbulence production.

TURBULENCE PRODUCTION MODEL

The fourth term of the right side of eq. (5) corresponds to turbulence production. When this term has been calculated by a linear k-e model, the accuracy of the term would be frequently deteriorated. The production of turbulence energy is written out as follows.

$$\begin{aligned}
P_k &= -\bar{u}_i' \bar{u}_k' \frac{\partial \bar{u}_i}{\partial x_k} \\
&= -\bar{u}' \bar{u}' \frac{\partial \bar{u}}{\partial x} - \bar{u}' \bar{v}' \frac{\partial \bar{u}}{\partial y} - \bar{u}' \bar{w}' \frac{\partial \bar{u}}{\partial z} \\
&\quad - \bar{v}' \bar{u}' \frac{\partial \bar{v}}{\partial x} - \bar{v}' \bar{v}' \frac{\partial \bar{v}}{\partial y} - \bar{v}' \bar{w}' \frac{\partial \bar{v}}{\partial z} \\
&\quad - \bar{w}' \bar{u}' \frac{\partial \bar{w}}{\partial x} - \bar{w}' \bar{v}' \frac{\partial \bar{w}}{\partial y} - \bar{w}' \bar{w}' \frac{\partial \bar{w}}{\partial z}
\end{aligned} \quad (6)$$

Following Boussinesq's approximation of eq. (7), the Reynolds stresses are represented by the eddy viscosity ν_T and the strain tensor $\bar{D}_{ik}$.

$$-\bar{u}_i' \bar{u}_k' = 2\nu_T \bar{D}_{ik} - \frac{2}{3} \delta_{ik} k, \quad (7)$$

where the strain tensor is defined as follows.

$$\bar{D}_{ik} = \frac{1}{2} \left(\frac{\partial \bar{u}_i}{\partial x_k} + \frac{\partial \bar{u}_k}{\partial x_i} \right) \quad (8)$$

Applying eq. (7) to eq. (6), eq. (9) is derived.

$$\begin{aligned}
P_k &= 2\nu_T \bar{D}_{ik} \bar{D}_{ik} = 2\nu_T \frac{1}{2} \left(\frac{\partial \bar{u}_i}{\partial x_k} + \frac{\partial \bar{u}_k}{\partial x_i} \right) \frac{1}{2} \left(\frac{\partial \bar{u}_i}{\partial x_k} + \frac{\partial \bar{u}_k}{\partial x_i} \right) \\
&= \frac{\nu_T}{2} \left(2 \frac{\partial \bar{u}}{\partial x} \right)^2 + \frac{\nu_T}{2} \left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right)^2 + \frac{\nu_T}{2} \left(\frac{\partial \bar{u}}{\partial z} + \frac{\partial \bar{w}}{\partial x} \right)^2 \\
&\quad + \frac{\nu_T}{2} \left(\frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{u}}{\partial y} \right)^2 + \frac{\nu_T}{2} \left(2 \frac{\partial \bar{v}}{\partial y} \right)^2 + \frac{\nu_T}{2} \left(\frac{\partial \bar{v}}{\partial z} + \frac{\partial \bar{w}}{\partial y} \right)^2 \\
&\quad + \frac{\nu_T}{2} \left(\frac{\partial \bar{w}}{\partial x} + \frac{\partial \bar{u}}{\partial z} \right)^2 + \frac{\nu_T}{2} \left(\frac{\partial \bar{w}}{\partial y} + \frac{\partial \bar{v}}{\partial z} \right)^2 + \frac{\nu_T}{2} \left(2 \frac{\partial \bar{w}}{\partial z} \right)^2
\end{aligned} \quad (9)$$

If we consider a two-dimensional flow and neglect shear components, eq. (9) becomes eq. (10).

$$P_k = -\bar{u}' \bar{u}' \frac{\partial \bar{u}}{\partial x} - \bar{v}' \bar{v}' \frac{\partial \bar{v}}{\partial y} \quad (10)$$

Applying the continuity equation to eq. (10), eq. (11) is obtained.

$$P_k = -(\bar{u}' \bar{u}' - \bar{v}' \bar{v}') \frac{\partial \bar{u}}{\partial x} \quad (11)$$

Using the linear k-e model, we have eq. (12).

$$P_k = \frac{\nu_T}{2} \left(2 \frac{\partial \bar{u}}{\partial x} \right)^2 + \frac{\nu_T}{2} \left(2 \frac{\partial \bar{v}}{\partial y} \right)^2 = 4\nu_T \left(\frac{\partial \bar{u}}{\partial x} \right)^2 \quad (12)$$

In eq. (11), turbulence energy production depends on the difference between $\bar{u}' \bar{u}'$ and $\bar{v}' \bar{v}'$, while in eq. (12), it consists of the sum of each component. This is a big problem for linear k-e turbulence models.

In order to solve this problem, we propose a modified k-e turbulence production model, as shown in eq. (13).

$$\begin{aligned}
P_k &= 2\nu_T \bar{D}_{ik} \bar{D}_{ik} - 2\nu_T \left(\frac{\partial \bar{u}_i}{\partial x_k} + \frac{\partial \bar{u}_k}{\partial x_i} \right) \frac{\partial \bar{u}_i}{\partial x_k} \delta_{ik} \\
&= \frac{\nu_T}{2} \left(2 \frac{\partial \bar{u}}{\partial x} \right)^2 - 2\nu_T \left(\frac{\partial \bar{u}}{\partial x} + \left| \frac{\partial \bar{u}}{\partial x} \right| \right) \frac{\partial \bar{u}}{\partial x} + \frac{\nu_T}{2} \left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right)^2 + \frac{\nu_T}{2} \left(\frac{\partial \bar{u}}{\partial z} + \frac{\partial \bar{w}}{\partial x} \right)^2 \\
&\quad + \frac{\nu_T}{2} \left(\frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{u}}{\partial y} \right)^2 + \frac{\nu_T}{2} \left(2 \frac{\partial \bar{v}}{\partial y} \right)^2 - 2\nu_T \left(\frac{\partial \bar{v}}{\partial y} + \left| \frac{\partial \bar{v}}{\partial y} \right| \right) \frac{\partial \bar{v}}{\partial y} + \frac{\nu_T}{2} \left(\frac{\partial \bar{v}}{\partial z} + \frac{\partial \bar{w}}{\partial y} \right)^2 \\
&\quad + \frac{\nu_T}{2} \left(\frac{\partial \bar{w}}{\partial x} + \frac{\partial \bar{u}}{\partial z} \right)^2 + \frac{\nu_T}{2} \left(\frac{\partial \bar{w}}{\partial y} + \frac{\partial \bar{v}}{\partial z} \right)^2 + \frac{\nu_T}{2} \left(2 \frac{\partial \bar{w}}{\partial z} \right)^2 - 2\nu_T \left(\frac{\partial \bar{w}}{\partial z} + \left| \frac{\partial \bar{w}}{\partial z} \right| \right) \frac{\partial \bar{w}}{\partial z}
\end{aligned} \quad (13)$$

This model can calculate the turbulence production adequately due to additional terms. In particular, this model can protect overestimation of turbulent energy in an accelerated flow.

APPLICATION TO HIGH-RE K-E MODEL

In engineering applications, high-Re k-e models have been usually used, because the grid number in the near wall can be reduced by using the wall function. However,

eq. (5) cannot directly apply to the wall function, because integration over the near wall region causes a large error due to a rapid change in velocity there.

To deal with this problem, a new application method of the wall function has been developed in this research. The first and second terms of eq. (5) have $\overline{u}_i = 0$ in the near wall region, which does not produce a significant effect. The preconditioning for correction is considered, following the law of wall. Assuming that the velocity change has a single direction, which is normal to the wall, the fourth term of eq. (5) is approximated by using the eddy viscosity ν_T as follows.

$$(3rd \text{ and } 4th \text{ term})_{\text{correction at wall}} = - \int_V (\nu + \nu_T) \left(\frac{\partial \overline{u}_s}{\partial x_n} \right)^2 dV \quad (14)$$

where s is the shearing direction and n the normal direction to the wall. The velocity gradient $\partial \overline{u}_s / \partial x_n$ can be calculated by the velocity profile for the wall function of eq. (15) (See Fig. 2).

$$\overline{u}^+ = \begin{cases} y^+ & 0 < y^+ < 4 \\ 4.19 - 5.75 \ln y^+ + 5.11 \ln(y^+)^2 & 4 < y^+ < 30 \\ -0.767 \ln(y^+)^3 & 4 < y^+ < 30 \\ 2.5 \ln(y^+) + 5.5 & 30 < y^+ \end{cases} \quad (15)$$

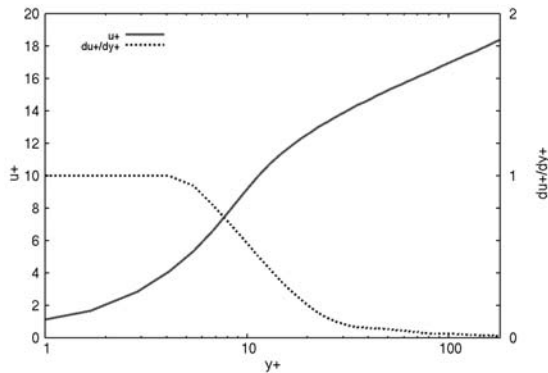


Figure 2 Relationship between $\overline{u}^+$, $d\overline{u}^+/dy^+$ and y^+

Using Cebecci & Smith's turbulence model, ν_t is obtained by eq. (16) (see Fig. 3).

$$\nu_t = \frac{\mu_t}{\rho} = \left\{ \kappa y \left[1 - \exp\left(-\frac{y^+}{A}\right) \right] \right\}^2 \frac{d\overline{u}}{dy} \quad (16)$$

where $\kappa = 0.4$ and $A = 26$.

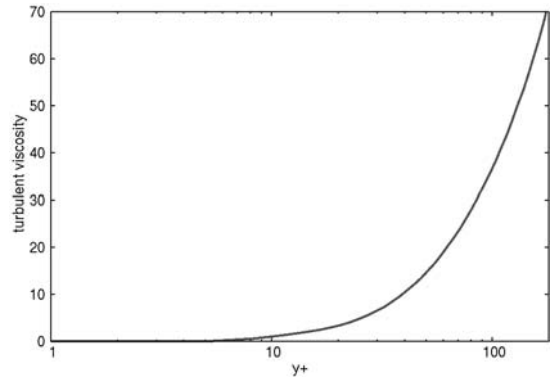


Figure 3 Relationship between Turbulent Viscosity and y^+

Thus, by applying these approximations for $\partial \overline{u}_s / \partial x_n$ and ν_t , integration in the near-wall region can be performed by using the functions in terms of wall distance. As a result, the energy dissipation there can be calculated with better accuracy.

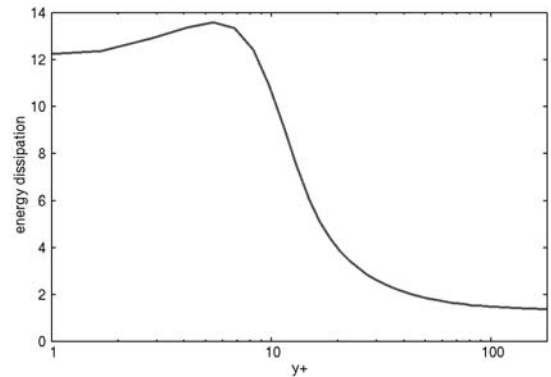


Figure 4 Energy Dissipation and y^+

To confirm its validity, the proposed method has been applied to the problem of three-dimensional model (see Fig. 5, Fig. 6), using the STAR-CD software.

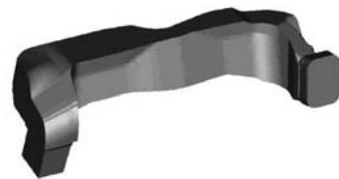


Figure 5 Product Model A

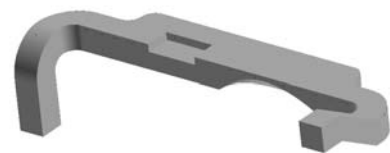


Figure 6 Product Model B