

Dynamic Behavior of Segmented Telescoping Structures in Automotive Systems

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ABSTRACT

Vehicles contain many examples of transversely loaded telescoping structures with overlapped sections, including Macpherson struts, hydraulic and pneumatic shock absorbers, closure actuators, and various powertrain elements such as splined shafts. This paper presents results of research into the mathematical modeling and dynamic response of such components. Specifically, the systems are modeled as Euler beams with overlapping sections of varying length. These systems are characterized according to their physical configurations and constraints. Solution methodologies are discussed. The paper concludes with parametric studies that are pertinent to vehicle design applications, and which yield insight into the interrelationships among modal characteristics, dynamic response, and optimization of segmented structures.

INTRODUCTION

Vibration is a common and important issue in many engineering applications, including spacecraft, aircraft, automotive, civil, and naval structures. Vibratory motion can be characterized in different ways, but the most frequent description is in terms of the system natural frequencies and corresponding deformation patterns (mode shapes). These natural properties need to be determined in order to know how the system will respond to excitation by external forces and initial conditions that may be either anticipated or unanticipated. Another common objective of vibration analyses is to provide information that allows a designer to adjust a system to isolate it from damaging or otherwise undesirable inputs. An automobile suspension is an example of such a system. Proper parametric design permits the vehicle to respond in a quasi-static fashion to low-frequency inputs, while transmitting little motion to the passenger compartment when subjected to high-frequency inputs from road perturbations. As a result, noise, vibration, and harshness (NVH) analyses are a fundamental part of modern vehicle design practice.

The subject of this paper is segmented, telescoping, beam-like structures. Such structures represent an important class of engineering systems with nonuniform geometric and physical parameters. They are characterized by multiple segments that can retract inside of, or relative to one another during operation. Examples of these systems present in vehicles include shock absorbers, pneumatic and hydraulic cylinders, steering actuators, and retractable antennas. This list is extensive, but by no means comprehensive.

Because vibration problems are so likely in multi-segmented telescoping beam structures, they should be subjected to modal analysis as stand-alone subsystems. The mathematical models for such analyses must reflect the complex interactions that occur among the segments during deployment and retraction. These interactions in turn produce a very complicated spatial and temporal redistribution of stiffness and inertia properties whose effect on performance and stability needs to be understood. From a mechanics perspective, the mathematical modeling of multi-segment telescoping systems differs substantially from that of corresponding single-segment systems with fixed configurations. Because inertia and stiffness distributions within segmented systems vary along with the segment configuration, they introduce analytical and computational challenges and that justify this study.

Due to the topic's relevance to automotive, civil, and mechanical engineering applications, the vibration of uniform and non-uniform beams has been studied extensively and is still receiving attention in the literature, particularly in relation to numerical issues encountered in computation of the modal frequencies. These problems are found in the analysis of both uniform and non-uniform (stepped, tapered, etc.) beams. Chronologically, the first publications on the transverse vibration of Euler-Bernoulli beams with a step change in cross-section date from as early as 1961. Taleb and Suppiger [1] and Levinson, [2], both derived the frequency equation for a simply supported stepped beam. In 1974 Klein [3] presented a general analysis of the free vibration of elastic beams

with non-uniform characteristics. Theoretical investigation of the free vibration of non-uniform beams was based on a new method that combined the advantages of finite element analysis and Rayleigh-Ritz analysis. At the same time Vaicaitis [4] investigated the free vibration of beams with random characteristics. In this work the approximate solution was obtained by using a two-variable perturbation expansion method. Bailey [5] presented a direct analytical solution to the vibration of non-uniform beams with and without discontinuities, and with various boundary conditions. Results were compared to the exact solution for cases where the exact solution was available. Two different methods were developed by Porat [6] for numerical computation of the natural frequencies associated with lateral vibration of a non-uniform beam with various boundary conditions. Only three natural frequencies were obtained. It appears that the methods presented were difficult to apply, and that convergence was not assured.

The research available on the vibration of beams with more than one step change in cross-section is not extensive. To study the effect of abrupt change of cross-section on the natural frequency of uniform beams, Sato [7] applied a combined type of transfer matrix method in which the finite element method is employed. Sato considered irregular stress distribution and local deformation, which occur near the section discontinuity. He pointed out that applying simple beam theory to such a beam results in overestimation of the stiffness and thus gives natural frequency predictions that are higher than the actual values. Chehil et al. [8] presented a procedure based on the Galerkin method to estimate the first few natural frequencies of a beam with varying sectional properties and included the effects due to rotary inertia. This approach was described as an alternative to more expensive procedures such as the finite element method. Jang and Bert [9] were among the first researchers to derive the frequency equations as fourth order determinants equated to zero for combinations of the classical clamped, pinned and free end supports. They described this methodology as an "exact solution." Stress concentrations at the junction of the two parts of the beam were neglected. Only the fundamental natural frequency of stepped beams was calculated in this paper. By extending the computations, higher mode frequencies for a stepped beam with two different cross-sections were found and presented by Jang and Bert in a later paper [10]. Results proved that increasing step sizes lowers the natural frequencies, thus making the structure more prone to low frequency vibration damage. However, in some particular cases of the boundary conditions, the natural frequencies were found to increase.

Naguleswaran [11, 12] extended the work of Jang and Bert and expressed the frequency equation as a second order determinant equated to zero. Here the roots of the frequency equation were determined by an iterative procedure based on linear interpolation. The transverse

vibration of a beam with up to three step changes in cross-section and four uniform beam sections was first considered by Naguleswaran [13].

Several papers on beam vibration mention that evaluation of the natural frequencies may encounter with numerical instability. Goncalves et al. [14] pointed out that the most common form of the expression for the mode shape function of a uniform Euler-Bernoulli beam may permit the evaluation of only the first twelve modes or so due to "numerical issues." These researchers derived alternative approximate expressions for the mode shape functions, which allowed numerically stable evaluation of the mode shapes functions. Although this approximation allows evaluation of all mode shape functions, some errors can occur in the calculations of the lower - and often most important - modes.

Abrate [15] showed that for some non-uniform rods and beams the equation of motion can be transformed into the equation of motion for a uniform rod or beam. Specifically, when the ends are completely fixed, the eigenvalues for non-uniform continua are the same as those of the corresponding uniform continua. To avoid numerical instability, the Rayleigh-Ritz approach for non-uniform beams was used to formulate the problem.

The work of Singh et al. [16] presents numerical methods for calculating the natural frequencies of non-uniform composite beams. The authors admit that the presence of transcendental functions introduce errors in determining the higher eigenvalues, which in turn lead to inaccuracies in physical parameter estimation. In another study [17], Singh and Ram extend the work presented in [16] to develop certain direct transcendental eigenvalue problems (DTEP) associated with non-uniform composite beams. They approximated the non-uniform composite beam with another composite beam having piecewise-constant material properties. The authors of [17] emphasized that when solving transcendental eigenvalue problems, it is necessary to evaluate the determinant of matrices containing transcendental elements, and that this can be difficult. To overcome this issue and other computational challenges, the authors proposed a numerical method that they call Newton's eigenvalue iteration method (NEIM). This method was originally developed by Yang [18] in 1983 for solving transcendental eigenvalue problems.

Dynamics and stability of a two-segment telescopic beam is the subject of the research by Amos and Qu [19] and Amos [20]. However, these authors appeared to have interpreted numerical instability encountered during the solution of the eigenproblem as physical phenomena. The closely spaced eigenvalues were considered a manifestation of system instability. Discontinuous distributions in inertia and stiffness properties were considered responsible for such phenomena as "modal agglomeration" and frequency grouping in a closely

spaced region. These anomalous results (closely spaced roots for the eigenvalue equation, discontinuous mode shapes) appeared at the higher natural frequencies. Sharp discontinuities apparent on the mode corresponding to these “agglomerated” natural frequencies were held to be consistent with this phenomenon. As will be discussed later, Amos’ interpretation is not consistent with the findings of the presented research in this work.

The research presented in this paper extends the current body of knowledge by developing a comprehensive, systematic modeling methodology for telescoping beams, which are transversely vibrating structures that find important practical applications, yet are little studied. With certain exceptions, the work focuses on two and three segment cantilevered telescoping beams (one fixed end and one free end). This focus is a result of the frequency of which this configuration is found in practical systems.

Mathematical models for segmented transversely vibrating telescoping beam system are established based on classical continuous beam theory, yielding an Euler-Bernoulli fourth-order partial differential equation for the continuum. This approximation encompasses several physical assumptions, including requirements that beam sections be slender and have a uniform cross-section along their axial direction, that transverse deformations remain small; that plane sections of the beam remain plane (i.e., no warping), and that the beam materials be both isotropic and homogeneous within a given section.

The segmented vibrating system equation of motion is formulated as a continuous system eigenproblem, which in turn is solved for an infinite set of the eigenvalues and eigenfunctions representing, respectively, the natural frequencies and mode shapes. The infinite dimensional distributed-parameter system may then be reduced to a finite-dimensional system using modal coordinates and expressed as a set of independent ordinary differential equations. As will be shown, numerical stability of the eigenproblem solution algorithm is an issue. The dynamic response of a two-segment cantilever beam structure in transverse vibration is investigated for numerical instability, and the manifestations of these problems are addressed in this work. However, stability enhancement is beyond the scope of the research. The paper concludes with parametric studies that illustrate how the modal parameters of telescoping beam configurations with three segments can be modified independently of the total system length.

PROBLEM FORMULATION

Figure 1 shows a representative telescoping beam system. Cantilever constraints are a primary focus of this research, so these conditions are depicted. The modeling procedure begins with a discretization process that divides the system into homogeneous but still continuous “sections.” In contrast to sections, which are often comprised of multiple physical components, the term “segment” as used in this paper will refer to a physically contiguous portion of the beam structure than cannot telescope upon itself. If two segments overlap during operation of the telescoping beam, the longitudinally coincident portion of the two segments will form a new section. Sections of this type are referred to as “overlap sections.” As the telescoping beam changes length during the processes of deployment or retraction, the relative size of overlap sections and non-overlap section changes, dramatically affecting the beam’s physical characteristics and dynamic response behavior.

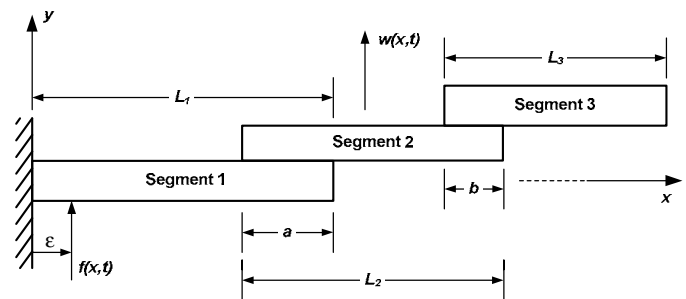


Figure 1. Three-segment cantilever beam with overlapping sections.

The equation of motion for a distributed parameter Euler-Bernoulli beam is:

$$E(x)I(x)\frac{\partial^4 w(x,t)}{\partial x^4} + \rho(x)A(x)\frac{\partial^2 w(x,t)}{\partial t^2} = f(x,t) \quad (1)$$

Euler-Bernoulli beam theory as embodied in Eq. 1 is applied to segmented structures where $I(x)$, and $A(x)$ vary at different axial locations. The parameters $\rho(x)$ and $E(x)$ may also vary, but are assumed constant in the systems examined here.

Consider a beam with two overlapped segments having constant E , I , ρ , and A over the segment length (Figure 2). Section discretization requires definition of three Euler-Bernoulli sections: one for the portion of segment 1 not overlapped, one for the portion of segment 2 not overlapped, and a special overlap section for the overlapped portions of segments 1 and 2. After section discretization is complete, a distinct, unique, Euler-Bernoulli partial differential equation of motion of the form in Eq. 1 is defined for each section.

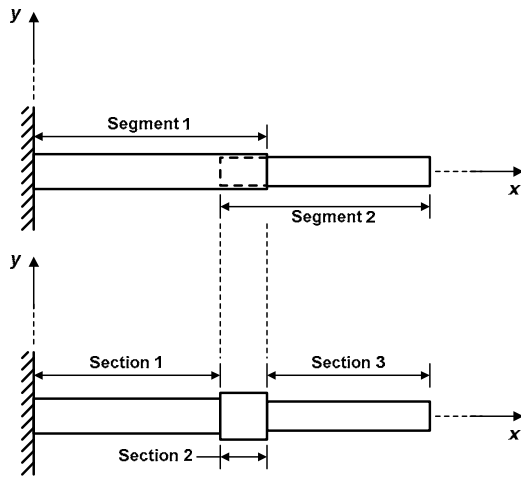


Figure 2. Euler-Bernoulli section discretization for a telescoping beam with two segment having constant sectional modulus (EI), and mass per unit length (pA).

The Euler-Bernoulli model provides a reasonable approximation for slender beams, e.g., beams with cross sectional dimensions less than one-tenth the beam length [21]. Each individual section generated during the discretization process should satisfy this criterion. Overlapped sections will have their shortest possible length when the beam is almost fully deployed, and the slenderness ratio criterion for these sections is most likely to be violated in such a configuration. However, operational constraints associated with vehicle applications will require that enough overlap remains to prevent excessive bending and wear at the interface, thus, it is unlikely that non-Euler behavior will develop in practical structures. Even if it does develop, the very short lengths involved means that there will be little effect on the overall system modal response. The longitudinal stresses developed in any Euler beam section will be linearly distributed, and the transverse shear deformations do not significantly affect the total elastic strain energy. The inertia due to rotation of the cross sections is small and is ignored in Euler theory.

For the cantilevered telescoping beams that are the focus of this research, boundary conditions for a fixed constraint must be applied to one of the beam's end sections, and boundary conditions for a free constraint must be applied to the other end section. These correspond to no-displacement and no-slope conditions for the fixed end:

$$w(x,t) = 0; \quad \frac{\partial w(0,t)}{\partial x} = 0, \quad (2)$$

and no-shear force and no bending moment conditions at the free end:

$$\frac{\partial^2 w(l,t)}{\partial x^2} = 0; \quad \frac{\partial^3 w(l,t)}{\partial x^3} = 0 \quad (3)$$

The Euler-Bernoulli is a well-tried and proven approach to modeling beams. The most restrictive assumption - a high aspect ratio - is quite typical for segmented structures. The process of Euler-Bernoulli section discretization introduced here adds no assumptions to those inherent to Euler-Bernoulli theory. Thus, the results from analyses based upon this approach should exhibit accuracy comparable to that of the traditional method. This approach to modeling telescoping beams will be called the distributed-parameter Euler-Bernoulli sectioning method (DEBSEM).

MODEL DEVELOPMENT AND SOLUTION

TELESCOPING BEAM SEGMENTATION - Consider again the three segment telescoping beam introduced in Figure 1. Upon application of the distributed-parameter Euler-Bernoulli sectioning method, the beam will be modeled as shown in Figure 3, with five sections, of which two, section 2 and section 4, are overlap sections. To complete the parameterization of the model requires specification of the segment lengths, L_1 , L_2 , and L_3 , the section flexural rigidities, $EI(x)$, and section densities. Also in Figure 3, $w(x,t)$ represents the transverse displacement of the beam from its equilibrium position, and $f(x,t)$ is any external transverse force that might be acting upon the beam.

An Euler-Bernoulli fourth order partial differential equation of motion (Eq. 1) is formulated for each beam section, and can be expressed for the complete five section beam as:

$$EI^{[i]} \frac{\partial^4 w^{[i]}(x,t)}{\partial x^4} + m^{[i]} \frac{\partial^2 w^{[i]}(x,t)}{\partial t^2} = f(x,t), \quad i \in [1,5] \quad (4)$$

In this equation $m^{[i]}$ is the mass per unit of length of the beam section, which is given by the product of mass density and the area of the corresponding section cross sectional area, e.g., $m^{[i]} = \rho A^{[i]}$. The bracketed subscripts, "[i]" associated with the constants provide a section reference and emphasize the fact that each beam section has its own partial differential equation of motion.

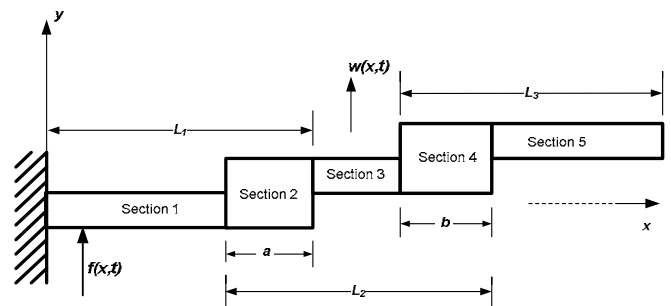


Figure 3. Division of a three-segment telescoping beam into distributed parameter Euler-Bernoulli sections.