

Superelement, Component Mode Synthesis, and Automated Multilevel Substructuring for Rapid Vehicle Development

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ABSTRACT

This paper presents the new techniques/methods being used for the rapid vehicle development and system level performance assessment. It consists of two parts: the first part presents the automated multilevel substructuring (AMLS) technique, which greatly reduces the computational demands of larger finite element models with millions of degrees of freedom (DOF) and extends the capabilities to higher frequencies and higher level of accuracy; the second part is on the superelement in conjunction with the Component Mode Synthesis (CMS) and also Automated Component Mode Synthesis (ACMS) techniques. In superelement, a full vehicle model is divided into components such as Body-in-white, Front cradle/chassis, Rear cradle/chassis, Exhaust, Engine, Transmission, Driveline, Front suspension, Rear suspension, Brake, Seats, Instrument panel, Steering system, tires, etc. with each piece represented by reduced stiffness, mass, and damping matrices. Superelement has the advantage that matrices can be passed from one organization to another without revealing proprietary details or concern. This virtual technique is very efficient, economical and versatile for the automotive and aerospace applications. The component mode synthesis is a form of superelement dynamic reduction wherein matrices are defined in terms of modal co-ordinates (corresponding to the superelement modes) and physical co-ordinates (corresponding to grids on the superelement boundaries). In the AMLS method, the structure is recursively divided into thousands of superelements/substructures and eigenvectors associated with these subdomains are used to represent the structure's response rather than the traditional global eigenvectors. Reduction of the finite element discretization is based on many small, local, inexpensive eigenvalue problems. The mobility at various interfaces between body and other components, and acoustic sensitivity response at driver's ear are presented for a five million DOF and 4.8 million DOF vehicles. A comparative study of CPU time and Elapsed (wall-clock) time for different methods and number of CPUs are presented. It is concluded that the methods such as AMLS/ACMS, and acoustic superelement in conjunction with CMS are very cost effective and 4 to 6 times faster than the use of conventional techniques.

INTRODUCTION

Automotive companies today face intense global competition, increasing product complexity, compressed product cycles, price and profit pressures, strict regulatory and liability environments, systems integration, high costs of testing and physical prototyping. In the fiercely competitive world of today's automotive industry, Computer Aided Engineering (CAE) is playing a more and more important role in shortening the design cycle time, minimizing costs and improving the product quality. A recent advancement in technological changes has made it possible to have affordable and powerful workstations with very large memory and disc space to run industrial finite element analysis with millions of DOF models.

Finite element method [1-4] has become an essential tool to support the virtual product development. The present author Mallikarjuna Bennur and his co-researchers published a series of papers on various higher-order theories in conjunction with superparametric element [5-10], and hybrid technique based on FEA and experimental test data [11,12]. At present, the main numerical prediction techniques for vibro-acoustic analysis are based on finite element, infinite element and boundary element methods [13 - 18]. In these deterministic methods, the continuum domains or their boundary surfaces are discretized into small elements. Since the field variables within each element are described in terms of simple, but approximating shape functions, a substantial amount of elements must be used in order to keep the approximation error within acceptable levels. In their use for dynamic problems, the element based methods exhibit numerical dispersion in that the wave number of the numerical solution differs from the wave number of the exact problem solution [19]. Since the dispersion error increases for increasing frequency, the mesh density and subsequent computational efforts, required for getting a reasonable level of accuracy, also increase with frequency.

In order to overcome the aforementioned practical limitations of long computation time, extensive work has been performed on Superelement or Substructuring and Component Mode Synthesis (CMS) techniques. Another

possible approach to reduce the computational efforts, involved with solving large dynamic FE models, is domain decomposition, in which the size of the original FE model is substantially reduced or where solving one large problem is replaced by solving many thousands of smaller subproblems. In the CMS approach, the structure is divided into components and each component is then analyzed independently for natural frequencies and mode shapes. The component mode shapes are then assembled to give displacement shapes of the original structure. These mode shapes are not eigenvectors of the original structure, but they are only used to transform the original degrees of freedom into generalized degrees of freedom. This may reduce significantly the size of the original system matrix. A survey of CMS methods can be studied in [20]. The difference in these methods lies in how the modes are combined over the boundary between the subdomains. A commonly used method is the Craig-Bampton approach [21, 22], where two sets of substructure modes are used to transform the subcomponent degrees of freedom: component normal modes, i.e. normal modes with all degrees of freedom of the interface held fixed, and constraint modes, i.e. deflections induced in the component structures by applying a unit displacement to one interface degree of freedom while all other interface degrees of freedom are held fixed. A significant model size reduction can be obtained with the Craig-Bampton CMS method. It should be noted, however, that, since there is one constraint mode needed for each interface degree of freedom, the cost of this method could still be prohibitive for a sufficiently fine mesh. In this respect, the Automated Multi-level Substructuring method (AMLS) has been proposed recently [23-27] which significantly reduces the computational effort as is illustrated in this paper for real-life automotive engineering applications.

THEORETICAL FORMULATION

The following sections will briefly describe the various techniques/methods employed in this project. Superelement analysis is a form of substructuring wherein a model is divided into pieces with each piece represented by a reduced stiffness, mass, and damping matrix. Each superelement is processed individually, and the solutions are then combined to solve the entire model. The final analysis (in which all of the individual superelement solutions are combined) involves much smaller matrices than would be required if the entire model were solved in a single solution. This technique has the advantage of reducing computer resource requirements, especially if changes are made to only one portion (superelement) of the model; in this case, only the affected superelement needs to be reanalyzed and the final analysis repeated. Superelement analysis presents procedural advantages as well, particularly when multiple engineering contractors are involved in an analysis. Each contractor can reduce his model to its boundary degrees-of-freedom, which is suitable for superelement analysis. The systems integrator then combines these reduced models into one model for a

final analysis. Superelement analysis has the advantage that matrices can be passed from one organization to another without revealing proprietary modeling details or concern about whether the same grid point and element numbers are used by every participant.

Component Mode Synthesis [20 - 22] is a form of superelement dynamic reduction wherein matrices are defined in terms of modal co-ordinates (corresponding to the superelement modes) and physical co-ordinates (corresponding to the grid points on the superelement boundaries). CMS is advantageous because there are fewer modal co-ordinates than physical co-ordinates – perhaps only one percent as many. In addition, CMS can utilize modal test data for sub-systems, thereby increasing the accuracy of the overall analysis. In the superelement creation, a care must be taken to select the superelement boundary grids, which depend on the boundary(constrain) connection points between the structures, the input loading points, and the output response points. When superelements are used in a static solution, a static reduction process is performed to replace the physical superelement model with reduced matrices that have the same properties. In dynamic analysis a reduction must also be performed for all superelements. In static analysis no approximation is introduced by using superelements. That is, the full static response of the superelement can be represented by the reduced matrices. In statics, the fixed-boundary solution added on to the boundary solution provides the exact solution to the problem. Unfortunately, the reduction methods in dynamics are not exact, as it is in the static solutions. In dynamic reduction the equivalent (exact interior motion) requires calculating all of the modes of each superelement, which is normally prohibitive. Therefore, approximate reduction techniques are used.

In dynamic reduction, there is the assumption that the solution for each superelement can be represented by using superposition of a series of *shape* functions. If this approach is compared to the approach used in static analysis, the static part of the transformation is identical; however, instead of the fixed-boundary solution, there is a dynamic transformation matrix that must be multiplied by the motion of a set of generalized coordinates to obtain the solution for the interior DOFs of the superelement. There are two classes of reduction used for superelements in dynamics: static reduction and dynamic reduction. Dynamic reduction is the more accurate approach for most problems, but dynamic reduction also requires more computational effort and some user judgment.

CONDENSATION

The standard eigenproblem can be written as follows:

$$Ku = \lambda Mu \quad (1)$$

Where K and M are the stiffness and mass matrices, respectively, of a finite element model of a structure.

Both matrices are assumed to be large and sparse, and are symmetric and positive definite.

DYNAMIC CONDENSATION

Dynamic reduction of superelements is similar to the transformation of a structure into modal coordinates. However, the transformation involves the use of both static and dynamic shapes. There are two types of dynamic reduction available when using superelements: Generalized Dynamic Reduction and Component Modal Synthesis. The first step is to generate the static transformation matrix, regardless of the reduction approach used. In dynamics, a dynamic transformation can be performed in addition to the static transformation. A series of dynamic shape-functions is found and used to dynamically transform the superelement properties to the exterior DOFs. When either dynamic reduction approach is used, the program calculates dynamic shapes based on the boundary conditions that have been selected. By partitioning the degrees of freedom into variables $\mathbf{u}_e$ (i.e. exterior or also called interface DOF) to be kept and variables $\mathbf{u}_i$ (i.e., interior or also called local DOF) to be dropped, equation (1) can be written, after reordering the terms, as follows:

$$\begin{bmatrix} K_{ii} & K_{ie} \\ K_{ei} & K_{ee} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_i \\ \mathbf{u}_e \end{Bmatrix} = \lambda \begin{bmatrix} M_{ii} & M_{ie} \\ M_{ei} & M_{ee} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_i \\ \mathbf{u}_e \end{Bmatrix} \quad (2)$$

Solving first of the two equations in (2) yields

$$\mathbf{u}_i = -(K_{ii} - \lambda M_{ii})^{-1} (K_{ie} - \lambda M_{ie}) \mathbf{u}_e \quad (2a)$$

and substituting in the second equation in (2), the exactly condensed eigenproblem can be written as,

$$(K_{ei} - \lambda M_{ei})(K_{ii} - \lambda M_{ii})^{-1} (K_{ie} - \lambda M_{ie}) \mathbf{u}_e - K_{ee} \mathbf{u}_e + \lambda M_{ee} \mathbf{u}_e = 0 \quad (2b)$$

STATIC CONDENSATION

The static condensation is an application of Gauss elimination. The basic assumption of static condensation in the calculation of frequencies and mode shapes is that the mass of the structure can be lumped at only some specific degrees of freedom without much effect on the accuracy of the frequencies and mode shapes of interest. Neglecting the dynamic part of the first equation in (2) and solving for $\mathbf{u}_e$ yields

$$\mathbf{u}_i = -K_{ii}^{-1} K_{ie} \mathbf{u}_e \quad (3a)$$

and substituting into the second equation in (2) yields the statically condensed eigenproblem, introduced independently by Guyan [28] and Irons [29],

$$\tilde{K}_{ee} \mathbf{u}_e = \lambda \tilde{M}_{ee} \mathbf{u}_e \quad (3b)$$

where $\tilde{K}_{ee}$ is the Schur complement of K_{ii} .

$$\tilde{K}_{ee} = K_{ee} - K_{ei} K_{ii}^{-1} K_{ie} \quad (3c)$$

and

$$\tilde{M}_{ee} = M_{ee} - K_{ei} K_{ii}^{-1} M_{ie} - M_{ie} K_{ii}^{-1} K_{ie} + K_{ei} K_{ii}^{-1} M_{ii} K_{ii}^{-1} K_{ie} \quad (3d)$$

The slave degrees of freedom are assumed to depend quasistatically on the master degrees of freedom.

SUBSTRUCTURING OF EIGENPROBLEMS

Consider one level versions of substructuring methods. Assume that the structure is partitioned into r substructures connecting to each other through the master variables on the interfaces only. The rows and columns of stiffness K and mass M matrices can be reordered as follows:

$$K = \begin{bmatrix} K_{ii_1} & O & . & . & . & O & K_{ie_1} \\ O & K_{ii_2} & . & . & . & O & K_{ie_2} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ O & O & . & . & . & K_{ii_r} & K_{ie_r} \\ K_{ei_1} & K_{ei_2} & . & . & . & K_{ei_r} & K_{ee} \end{bmatrix} \quad (4a)$$

and

$$M = \begin{bmatrix} M_{ii_1} & O & . & . & . & O & M_{ie_1} \\ O & M_{ii_2} & . & . & . & O & M_{ie_2} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ O & O & . & . & . & M_{ii_r} & M_{ie_r} \\ M_{ei_1} & M_{ei_2} & . & . & . & M_{ei_r} & M_{ee} \end{bmatrix} \quad (4b)$$

where $j=1,2,\dots,r$. K_{ii_j} is the interior degrees of freedom stiffness matrix corresponding to the j th substructure, and K_{ie_j} is the interaction of the exterior degrees of freedom and the j th substructure.

For the static condensation problem,

$$\tilde{K}_{ee} = K_{ee} - \sum_{j=1}^r K_{ei_j} K_{ii_j}^{-1} K_{ie_j} \quad (5a)$$

$$\tilde{M}_{ee} = M_{ee} - \sum_{j=1}^r M_{ee_j} \quad (5b)$$

where,



$$M_{ee_j} = K_{ei_j} K_{ii_j}^{-1} M_{ie_j} + M_{ei_j} K_{ii_j}^{-1} K_{ie_j} - K_{ei_j} K_{ii_j}^{-1} M_{ii_j} K_{ii_j}^{-1} K_{ie_j} \quad (5c)$$

The submatrices corresponding to the individual substructures can be determined independently from smaller subproblems and in parallel.

A TRANSFORMATION OR PROJECTION APPROACH

The transformation methods comprise a group of eigensystem solution procedures that employ the basic properties of the eigenvectors in the matrix Φ ,

$$\Phi^T K \Phi = \Lambda \quad (6a) \quad \text{and} \quad \Phi^T M \Phi = I \quad (6b)$$

where $\Phi = [\varphi_1, \dots, \varphi_n]$, $\Lambda = \text{diag}(\lambda_1)$, and I is the identity(or unit) matrix with only zero elements except for its diagonal entries, which are unity. Since the matrix Φ of order $n \times n$, which diagonalizes K and M in the way given in (6a) and (6b) is unique. The basic scheme is to reduce K and M into diagonal form using successive pre- and post- multiplication by matrices P^T and P , respectively. By transforming the matrix K to block diagonal form using block Gaussian elimination, i.e., applying the congruent transformation with

$$P = \begin{bmatrix} I & -K_{ii}^{-1} K_{ie} \\ 0 & I \end{bmatrix} \quad (7)$$

obtain the equivalent form as follows:

$$[P^T K P, P^T M P] = \begin{bmatrix} K_{ii} & 0 \\ 0 & \tilde{K}_{ee} \end{bmatrix}, \begin{bmatrix} M_{ii} & \tilde{M}_{ie} \\ \tilde{M}_{ei} & \tilde{M}_{ee} \end{bmatrix} \quad (8)$$

Note that K_{ii} and M_{ii} stay unchanged, $\tilde{K}_{ee}$ and $\tilde{M}_{ee}$ are defined in equations (3c) and (3d), respectively,

and $\tilde{M}_{ie} = M_{ie} - M_{ii} K_{ii}^{-1} K_{ie} = \tilde{M}_{ei}^T$. Neglecting all rows and columns in equation (8) corresponding to slave degrees of freedom, i.e., projecting equation (1) to the

subspace spanned by columns of $\begin{bmatrix} -K_{ii}^{-1} K_{ie} \\ I \end{bmatrix}$, one

obtains the following with the method of static condensation introduced by Guyan[28] and Irons[29]as,

$$\tilde{K}_{ee} v = \lambda \tilde{M}_{ee} v \quad (9)$$

In order to model the deviation from quasistatic behavior thereby improving the approximation properties of static condensation, consider the eigenvalue problem given below:

$$K_{ii} \Phi = M_{ee} \Phi \Omega \quad \text{and} \quad \Phi^T M_{ii} \Phi = I \quad (10)$$

Where Ω is a diagonal matrix containing the eigenvalues.

CRAIG-BAMPTON FORM

For the component mode synthesis, the most commonly used approach is the Craig-Bampton [22] method, which is also known as fixed boundary dynamic reduction. After applying any multipoint constraints and all physical constraints to the superelement, the component matrices have been reduced to the set of superelement matrices and are partitioned into two sets of degrees-of-freedom (DOFs). The first set represents the boundary (exterior) points and the second set is the remaining interior DOFs. At this point a set of constraint modes is generated. Each constrained mode represents the motion of the superelement resulting from moving one boundary DOF 1.0 unit while holding the other boundary DOFs fixed. Therefore, there is one constrained mode for each boundary DOF. By changing the basis for the interior DOFs to a modal one, i.e., applying the further congruent transformation $\text{diag}\{\Phi, I\}$ to equation (8), one gets

$$\begin{bmatrix} \Omega & 0 \\ 0 & \tilde{K}_{ee} \end{bmatrix} \begin{bmatrix} I & \Phi^T \tilde{M}_{ie} \\ \tilde{M}_{ei} \Phi & \tilde{M}_{ee} \end{bmatrix} \quad (11)$$

In structural dynamics, equation (6) is called Craig-Bampton form of the eigenvalue problem of equation (1) corresponding to the partitioning in equation (2). In terms of linear algebra, it results from block Gaussian elimination to reduce K to block diagonal form, and diagonalization of the block K_{ii} using a spectral basis.

COMPONENT MODE SYNTHESIS

The component mode synthesis (CMS) has been developed to a large extent as a natural consequence of the analysis procedure followed in practice when large and complex structures are analyzed. The general practical procedure is that different groups perform the analyses of different components of the structure under consideration. The advantage of considering separately the components of the structure is primarily one of time scheduling; i.e., the separate groups can work on the analysis and designs of the components at the same time. It is primarily for this reason that the component mode synthesis is very appealing in the analysis and design of large structural systems.

The term component modes is used to signify a class of *Ritz vectors*, or *assumed modes*, that are used a *basis vectors* in describing the displacement of points within a substructure or component. By selecting some eigenmodes according to eigenvalues which do not exceed a cut-off threshold (however, in a recent paper Bai and Lia [30] suggested a different choice based on a moment-matching analyses) and dropping the rows and columns in equation (11) corresponding to the other modes, one arrives at the CMS method of Hurty[21] and Craig and Bampton [22]. Hence, if the