

Simulation of a Valve Train Using Non-Smooth Mechanics

Robert Huber and Heinz Ulbrich

Institute of Applied Mechanics, Technical University Munich

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ABSTRACT

Nowadays, multi-body systems theory including bilateral and unilateral constraints is comparatively well established by means of set-valued force laws. Although methods of non-smooth mechanics enable a highly efficient modeling, they are not conventionally used in industrial practice. Therefore in the present paper a valve train including hydraulic elements like a hydraulic lash adjuster is modeled using above mentioned methods.

A main focus is laid upon the treatment of contact problems and two different models are investigated. Contacts in multi-body dynamics are classically described using spring and damper elements minimizing penetration. This approach results in stiff differential equations with unintentional high eigenfrequencies and long computing times as well as uncertainties in the parameters for contact stiffness and damping. Whereas in rigid contact models the contact is supposed to be completely stiff leading to non-smooth systems. The contact forces are subject to set-valued force laws describing the physical properties of the contact, namely the condition of non-penetration. Thus, stiff differential equations can be avoided and efficient models of these systems are obtained. For the rigid model discussed herein, only one interpretable physical parameter describing the dissipation of energy has to be adjusted.

The dynamics of the system are described in terms of a measure differential equation augmented by projection functions representing set-valued force laws. Integration is done by a half-explicit time-stepping scheme. The set-valued laws are solved by methods of convex analysis.

The concepts of non-smooth mechanics are adopted to hydraulic components. Using these methods a hydraulic

lash adjuster is modeled and the simulation is compared to experimental results. Finally a non-smooth model of a valve train is investigated briefly and the different contact models are compared.

INTRODUCTION

In combustion engines valve trains are used to control the combustion process by activating inlet and exhaust valves as well as fuel injectors. For this purpose the camshaft rotary motion is converted into a reciprocating motion at the valves. A timing belt or a timing chain links the camshaft to the crankshaft so that the valves are in sync with the pistons. Steadily increasing requirements concerning efficiency and environmental interests need a more and more sophisticated control of the combustion process like active controllable valve trains.

The valve train typically consists of the camshaft, valves, valve springs, rocker arms and hydraulic systems like for example hydraulic lash adjusters (HLA) or cam phasing systems. Hence, to describe the dynamical behaviour of modern valve trains in an accurate way, the interaction between the different physical domains – mechanics, hydraulics and control electronics – have to be integrated in one consistent mathematical model. In order to account for all important dynamical effects, a detailed contact model is necessary. Contacts in multi-body dynamics are classically described using non-linear spring and damper elements minimizing penetration. This approach results in stiff differential equations with unintentional high eigenfrequencies. Whereas in rigid contact models the contact forces are subject to set-valued force laws and stiff differential equations can be avoided.

Due to the multitude of different contacts within a valve train - namely coil collisions, closing valves, contact be-

tween cam and roller, hydraulic models etc. - it is obvious that valve trains are suitable systems for applying non-smooth mechanics [1].

Due to the low compressibility of oil and the very small volume of the high pressure chamber hydraulic lash adjusters lead to stiff differential equations within the valve train equations. Since the compressibility of the high pressure chamber is very small, it can be modeled as an incompressible hydraulic volume. This assumption leads to set valued laws similar to the set valued force laws in non-smooth mechanics.

NON-SMOOTH MULTI-BODY DYNAMICS

A brief overview is given for the equations describing the dynamics of systems with bi- and unilateral constraints e.g. contacts, friction and frictional impacts. A main focus is laid upon the treatment of contact problems. Two different ways describing constrained systems are presented: smooth and non-smooth mechanics.

Classically, contacts in multi-body dynamics are described using spring and damper elements minimizing penetration of the bodies coming into contact. Leading to continuous time dependent state variables these systems are called smooth. For rigid contact models the stiffness in the contact zone is assumed to be infinite. Therefore impacts due to closing contacts lead to jumps in the velocities of the system. The related systems are called non-smooth [2].

EQUATIONS OF MOTION First the attention is focused on multibody systems without any constraints. The basic equations of motion write

$$M(q) \dot{u} = h(u, q, t) \quad (1)$$

The mass matrix M depends on the vector of generalized positions q . h denotes the vector containing all smooth external and gyroscopical forces depending on q , generalized velocities $u = T \dot{q}$ and time t . For constraint systems equation (1) must be extended with constraining forces using the Lagrange multiplier method :

$$M \dot{u} = h + W \lambda \quad (2)$$

The constraint matrix $W = [W_N, W_T]$ is a transformation from the space of constraints to the configuration space and can be interpreted as the normal and tangential directions of the contact forces in the configuration space. The vector $\lambda = [\lambda_N, \lambda_T]^T$ combines the magnitudes of the normal and tangential contact forces.

Equation (2) expresses the dynamics of a constrained multibody system during an impact free phase at the acceleration level. Equations of motion relating accelerations to forces are not suitable to describe motions with impacts. A more appropriate formulation can be achieved by replacing (2) by a finite dimensional measure differen-

tial equation [3, 4]:

$$M du = h dt + W d\Lambda \quad (3)$$

The measure for the velocities $du = \dot{u} dt + (u^+ - u^-) d\eta$ is split in a continuous part $\dot{u} dt$ of smooth dynamics and a discrete part $(u^+ - u^-) d\eta$ at the discontinuity points e.g. impacts. u^+ and u^- denotes the velocities before and after the discontinuities, which are weighted by the sum of the DIRAC delta functions $d\delta_i$ at the discontinuities t_i :

$$d\eta = \sum_i d\delta_i \quad d\delta_i = d\delta(t - t_i)$$

$$\int_T f(t) d\delta_i = \begin{cases} f(t_i) & \text{if } t_i \in T, \\ 0 & \text{else.} \end{cases} \quad (4)$$

The reaction measure $W d\Lambda$ of contacts is decomposed by the generalized force directions $W = W(q)$ and the magnitudes $d\Lambda$. In analogy to the velocity measure, the reaction measure $d\Lambda$ contains smooth forces λ due to persisting contacts as well as discrete impulses Λ due to collisions of bodies at the impact times t_i :

$$d\Lambda = \lambda dt + \Lambda d\eta.$$

Integrating equation (3) under consideration of the DIRAC delta function (4) yields the classical equations of motion

$$M \dot{u} = h + W \lambda \quad (5)$$

for a constrained system of smooth dynamics as well as the impact equations

$$M_i (u_i^+ - u_i^-) = W_i \Lambda_i \quad (6)$$

being valid at times t_i of impact.

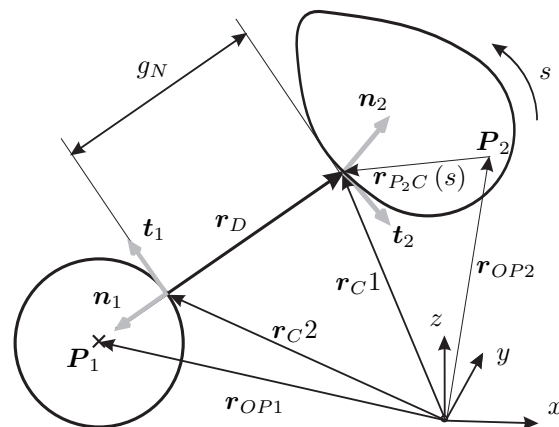


Figure 1: Kinematics of the contact between two bodies e.g. cam and roller.

CONTACT KINEMATICS For the description of the contact between two bodies, the contact geometry needs to be analyzed, no matter which contact model is used to describe the corresponding contact. The potential contact points r_C on the contours need to be determined as well as the gap distance g_N and the relative velocities $\dot{g}_N$ and

$\dot{g}_T$. These information is used for the evaluation of the force laws as well as for the calculation of the generalized force directions $\mathbf{W}$.

Contours are described by a position vector $\mathbf{r} = \mathbf{r}(\mathbf{q}, s)$, the contour normal $\mathbf{n} = \mathbf{n}(\mathbf{q}, s)$ and the associated tangent $\mathbf{t}_1 = \mathbf{t}_1(\mathbf{q}, s)$, all depending on the generalized position of the associated body and on the contour parameters s . Potential contact points can be identified by the necessary contact condition, that the difference vector $\mathbf{r}_D = \mathbf{r}_{C1} - \mathbf{r}_{C2}$ between potential contact points on both bodies is perpendicular to both tangents. As an example the evaluation of the contact kinematics between a circle and a contour e.g. the contact between cam and roller or rocker and hydraulic lash adjuster is pointed out briefly (see Figure 1). A detailed description can be found in [5]. The contours are interpolated with cubic splines. A single point on the contour can be referenced by the scalar contour parameter s . The normal and tangential vectors $\mathbf{n}$ and $\mathbf{t}$ are obtained using standard geometry formulations [6]. The contour parameter for potential contacts can be calculated by solving

$$\mathbf{f}(\mathbf{q}, s) = \begin{pmatrix} \mathbf{r}_D^T \mathbf{t}_1 \\ \mathbf{r}_D^T \mathbf{t}_2 \end{pmatrix} = 0 \quad (7)$$

The distance between the two bodies results from $g_N = \mathbf{r}_D^T \mathbf{n}_2$. Based on the contact points the JACOBIANS $\mathbf{J} = (\partial \mathbf{r}_{Ci} / \partial \mathbf{q})^T = (\partial \dot{\mathbf{r}}_{Ci} / \partial \mathbf{u})^T$ between the system-space and the generalized coordinates of the two bodies are developed. The generalized force directions write :

$$\begin{aligned} \mathbf{W}_N^T &= \mathbf{n}_1^T \mathbf{J}_{C1} + \mathbf{n}_2^T \mathbf{J}_{C2} \\ \mathbf{W}_T^T &= \mathbf{t}_1^T \mathbf{J}_{C1} + \mathbf{t}_2^T \mathbf{J}_{C2} \end{aligned} \quad (8)$$

FORCE LAWS Solving the equations of motion requires the knowledge of the contact reactions λ respectively for non-smooth system Λ at the discontinuities. To establish different force laws a decomposition of the contact force $\mathbf{F}_C = \mathbf{n} \lambda_N + \mathbf{t} \lambda_T$ in normal and tangential direction is used. The scalar λ_N is the absolute value of the contact force in normal direction and λ_T is the scalar friction force in tangential direction.

Single valued force laws: Spring-Damper-Elements

A commonly used approach to model a unilateral contact considering elasticity is to use spring and damper elements to minimize penetration. Therefore λ_N is split into a force λ_{Nc} resulting from the spring and the damping influence λ_{Nd} . Adding a spring between the bodies with a very high stiffness pushes them apart as soon as the normal distance g_N becomes negative. The force is zero when the two bodies are separated. Using linear elements the force law writes

$$\lambda_{Nc} = \begin{cases} -c g_N & \text{if } g_N < 0 \\ 0 & \text{else} \end{cases} \quad (9)$$

More sophisticated force laws, for example to apply contact stiffness according to HERTZ [7], are obtained by us-

ing an exponential force law for $g_N < 0$

$$\lambda_{Nc} = -k g_N^e \quad (10)$$

with stiffness parameter k and exponent e . For the damping value λ_{Nd} a linear correlation for $g_N < 0$ is applied:

$$\lambda_{Nd} = d \dot{g}_N \quad (11)$$

For numerical reasons to avoid discontinuities in λ_{Nd} at the beginning of penetration, d is set to zero for $g_N = 0$ and is increased with increasing penetration up to a maximum value.

For bilateral contacts equivalent force laws could be stated leading to common spring-damper elements.

For considering friction in closed contacts COULOMB's law is used. With the normal force λ_N and the relative tangential velocity $\dot{g}_T$ the scalar friction force λ_T is given by

$$\dot{g}_T = 0 \Rightarrow |\lambda_T| \leq \mu |\lambda_N| \quad (12a)$$

$$\dot{g}_T \neq 0 \Rightarrow \lambda_T = -\frac{\dot{g}_T}{|\dot{g}_T|} \mu |\lambda_N| \quad (12b)$$

To use this discontinuous model of friction forces in smooth systems integrable by standard ODE solvers, a regularized form of the COULOMB's law is needed. One possibility is to smooth the set-valued force law of COULOMB as follows :

$$|\dot{g}_T| < 0 \Rightarrow |\lambda_T| = -\frac{\dot{g}_T}{|\dot{g}_T|} \mu (1 - e^{-\frac{|\dot{g}_T|}{v_{ref}}}) \lambda_N \quad (13a)$$

$$|\dot{g}_T| > 0 \Rightarrow \lambda_T = -\frac{\dot{g}_T}{|\dot{g}_T|} \mu |\lambda_N| \quad (13b)$$

In comparison to (12), equation (13) does not permit real sticking. A tangential friction force requires a tangential velocity at the contact point. In Figure 4 both friction forces are depicted.

Set valued force laws: Rigid Contact

In the previous section contacts were modeled using spring and damper elements. Due to the high stiffness of the contacts, this approach results in stiff differential equations with unintentional high eigenfrequencies and long computing times as well as uncertainties in the parameters for contact stiffness and damping. Therefore another possibility to model contacts, namely the rigid approach, is discussed. For the rigid approach the contact is described by a discrete point model where the stiffness in the contact zone is assumed to be infinite resulting in a constraint. Mathematically this means that an explicit functional relation with respect to the state variables no longer exist. The contact reactions are described by set valued laws. For unilateral contacts the assumption of no penetration can be described by the SIGNORINI-FICHERA-condition

$$g_N \geq 0, \quad \lambda_N = 0, \quad g_N \lambda_N = 0 \quad (14)$$