

# Analysis on Yield Tightening of Threaded Fasteners

Sayed A. Nassar, Xianjie Yang and Saravanan Ganeshmuthy  
Fastening and Joining Research Institute, Oakland University

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## ABSTRACT

In this paper, an emphasis is put on describing the elastic and plastic deformation behavior of the bolted joint. The bolt material is assumed to be plastic hardening. A nonlinear combined stress model is established for a typical bolted joint for the purpose of studying its behavior under a yield tightening. The combined effect of axial and torsional stresses in the tightened threaded fastener is considered. A new approach for yield tightening under ideal plastic bolt was proposed, and the effects of the thread and bearing frictional coefficients on the clamp load prediction are evaluated. The prediction precision of deformation behavior of the bolted joint under yield tightening for the strain hardening bolt material are studied experimentally.

## 1. INTRODUCTION

Torque control and torque-angle control approaches are widely used in bolted joint assemblies [1-6]. The real coefficients of friction for the bolted joints are unknown. When the torque-only control method is used for tightening a fastener, there is a significant scatter in the torque-tension correlation due to normal variation in the frictional coefficients. The friction coefficients have a great effect on the torque-tension relationship [7-8]. It is to be pointed out that the torque-to-yield control method is mostly used in critical applications [4-6]. An alternative tightening approach is modified torque-angle controlled tightening over the yield point of a bolt. It generates higher levels of preload than elastic tightening, and the preload scatter is greatly reduced. The effect of the friction coefficients is also reduced. However, the analysis on the precision of the preload prediction for the torque to yield control approach is expected further so that it can guidance the usage of the approach in bolted joint assembly.

The traditional yield control approach predicts the clamp load by using the yield stress and the cross area, but not considering the effect of the shear stress caused by the twist torque. In fact, when the thread friction coefficient is relatively large, significant effect of the twist torque on the clamp load should be considered.

In the paper, an elastic/plastic hardening model for the bolt material is assumed. An elasto-plastic analysis on the threaded fastener joint assembly is conducted under tightening, in which the clamp force and tightening torque are subjected to the bolts. Based on the elasto-plastic analysis of the bolt, a new methodology for the determination of the tightening clamping force under different tightening with different thread friction coefficient was proposed, in which the effect of some variables on the amount of clamp load is investigated. The angle of turn is determined based on the elastic/plastic analysis regarding the effect of the twist torque on the bolt. The theoretical clamp load versus the angle of turn and the twist torque versus the angle of turn are compared with the corresponding experimental results. The experimental verification is carried out.

## 2. RELATIONSHIP BETWEEN CLAMPING FORCE AND TORQUE

Under tightening torque, the static torque equilibrium for the thread bolt is given as following.

$$T = T_b + T_p + T_t = T_b + T_{tp} \quad (1)$$

where  $T$ ,  $T_b$ ,  $T_p$  and  $T_t$  are the tightening torque, the bearing friction torque, the pitch torque and thread friction torque, respectively. The torque-tension relationship is given by [8] as follows

$$T = F \left\{ \mu_b r_b + r_t \frac{tg\beta + \mu_t \cos \beta \sqrt{1 + tg^2 \alpha + tg^2 \beta}}{1 - \mu_t \sin \beta \sqrt{1 + tg^2 \alpha + tg^2 \beta}} \right\} \quad (2)$$

where  $F$ ,  $\mu_b$ ,  $r_b$ ,  $r_t$ ,  $\mu_t$ ,  $\alpha$  and  $\beta$  are the clamping force, the bearing friction coefficient, the effective bearing radius, the effective thread radius, half of the thread profile angle, and the lead helix angle, respectively.

In the paper, the bearing and thread contact pressures are assumed to be uniform. Accordingly, for a standard hexagonal head M12  $\times$  1.75 bolt ( $\gamma_b = 1.6$ ,  $\gamma = 1.22$ ), Table 1 gives the values  $r_t = 5.895\text{mm}$ ,  $r_e = 9.44\text{mm}$ ,  $r_{\min} = 4.925\text{mm}$ ,  $r_{\max} = 6\text{mm}$ , the effective bearing radius  $r_b = 7.75\text{mm}$ , and the thread radius is  $r_t = 5.5\text{mm}$ .

## 3. A Novel Torque-Angle Approach To Control Fastener Elastic/Plastic Tightening

### 3.1 Elastic deformation behavior of bolted joint assembly

In fastener engineering, one of the control methods of bolt tightening uses the Torque-Angle approach [2-3], in which the automated assembly tool is programmed to turn certain number of degrees past a threshold (snug) torque value. This approach achieves a more reliable clamping force, especially when compared with the Torque-only control method, which highly sensitive to the variations in the frictional coefficients for different fasteners with different surface treatment and heat-treatment [2-3]. Therefore, an analysis of the Torque-Angle approach for elastic/plastic tightening is provided so that the reliability of the approach is evaluated.

Experimental data on the torques, angle of turn, and clamp load versus time are collected during tightening steps. Until a threshold (snug) torque level is reached past the initial free spinning, the experimental  $T-\theta$  and  $F-\theta$  curves are nonlinear. For a linear elastic bolted joint system, the  $T-\theta$  and  $F-\theta$  curves are linear past the threshold torque value. Under the elastic deformation assumption, the bolt compliance  $C_B$ , joint compliance  $C_J$  can be assumed to be constant during the elastic tightening process in which the friction coefficients remain constant. On the other hand, the plastic deformation behavior of the bolt needs to be taken into

account if the plastic tightening occurs. If the total elongation of bolt is  $\Delta l_b$ , the relative angle  $\theta_{TN}$  of turn between the but and bolt threads is given by

$$\theta_{TN} = 360(C_J F + \Delta l_b) / p \quad (3)$$

where  $p$  is the thread pitch.

If the tightening is elastic, we have

$$\Delta l_b = C_b F \quad (4)$$

Rotational clearances in the socket and/or tool drive will lead to an additional rotational angle. The total clearance  $\theta_{cl}$  contributes to the initial nonlinear signature of the joint represented by the  $T - \theta$ ,  $T_{ip} - \theta$  and  $F - \theta$  curves. Additionally, if a linear elastic deformation is assumed, the tightening machine's torsional compliance  $C_{TM}$  and the bolt torsional compliance  $C_{TB}$  remain constant, the follow torque-angle relationship may be used

$$\theta = \theta_{cl} + \theta_M + \theta_B + \theta_{TN} = \theta_{cl} + C_{TM} T + \theta_B + \theta_{TN} \quad (5)$$

$$T_{ip} = \frac{r_i \frac{\mu_{ik} \cos \beta \sqrt{1 + t g^2 \alpha + t g^2 \beta} + t g \beta}{1 - \mu_{ik} \sin \beta \sqrt{1 + t g^2 \alpha + t g^2 \beta}}}{\mu_{bk} r_b + r_i \frac{\mu_{ik} \cos \beta \sqrt{1 + t g^2 \alpha + t g^2 \beta} + t g \beta}{1 - \mu_{ik} \sin \beta \sqrt{1 + t g^2 \alpha + t g^2 \beta}}} T = \psi T \quad (6)$$

where  $\theta_{cl}$  is the initial angle of turn caused by all clearances, which is not dependent on  $T$ ,  $\theta_M$  is the elastic angle of twist of the drive of the machine,  $\theta_B$  is the twist angle of the bolt.

$$\theta_B = C_{TB} T_{ip} \quad (7)$$

$$\theta_{TN} = \frac{360}{p} (C_B + C_J) F \quad (8)$$

On the other hand, if the tightening is plastic, the  $\theta_B$  and  $\theta_{TN}$  can be determined by using plastic deformation model of the bolted joint assembly.

### 3.2 Plastic deformation behavior of the bolted joints

The deformation assumption is shown in the following:

(1). The axial strain is uniform in the cross-section area, that is,

$$\varepsilon(r) = \varepsilon \quad (9)$$

(2). The shear strain is proportional to the radius, that is,

$$\gamma = \gamma(r) = \frac{r \theta_B}{L} \quad (10)$$

Under the plastic tightening, the yield criterion is

$$\sigma^2 + 3\tau^2 = f^2(p) \quad (11)$$

The incremental plastic flow rule is

$$d\varepsilon = \frac{d\sigma}{E} + dp \frac{\sigma}{\sigma_{eq}} \quad (12)$$

$$d\gamma = \frac{d\tau}{G} + dp \frac{3\tau}{\sigma_{eq}} \quad (13)$$

$$dp = \frac{\alpha d\sigma + 3\alpha d\tau}{\sigma_{eq} f'(p)} \quad (14)$$

Considering Eq.(14), combined Eq.(12) with Eq.(13), we can get

$$\frac{d\varepsilon - \frac{d\sigma}{E}}{\frac{d\tau}{G} - \frac{3\tau}{\sigma_{eq}}} = \frac{\sigma}{3\tau} \quad (15)$$

To simplify the derivation, let  $E = 3G$ . From Eqs.(12-14), the incremental stress components under plastic loading are

$$d\tau = \frac{E [E\sigma^2 + f'(p)\sigma_{eq}^2] d\gamma - 3E\sigma d\varepsilon}{3 [E + f'(p)]\sigma_{eq}^2} \quad (16)$$

$$d\sigma = E d\varepsilon \left[ 1 - \frac{\sigma^2 E}{\sigma_{eq}^2 [f'(p) + E]} \right] - \frac{E^2 \sigma d\gamma}{\sigma_{eq}^2 [f'(p) + E]} \quad (17)$$

The stress status is different at different radius, the initial yielding occurs when yielding at the points at the external radius. After that, if the radius at the elastic/plastic boundary is  $r_c$ , and the initial yield stress is  $\sigma_s = f(0)$ , we have

$$\sigma^2(r_c) + 3\tau^2(r_c) = \sigma_s^2 \quad (18)$$

In the elastic region of the cross-section of bolt,  $0 \leq r \leq r_c$ , we have

$$d\sigma = E d\varepsilon \quad (19)$$

$$d\tau = G d\gamma = \frac{1}{3} E d\gamma \quad (20)$$

In the plastic region of the cross-section  $r_c \leq r \leq a$ , we have

$$d\tau(r_i) = \frac{E [E\sigma^2(r_i) + f'(p_i)\sigma_{eq}^2(r_i)] d\gamma - 3E\sigma(r_i)\tau(r_i) d\varepsilon}{3 [E + f'(p_i)]\sigma_{eq}^2(r_i)} \quad (21)$$

$$d\sigma(r_i) = E d\varepsilon \left[ 1 - \frac{\sigma^2(r_i) E}{\sigma_{eq}^2(r_i) [f'(p_i) + E]} \right] - \frac{E^2 \sigma(r_i)\tau(r_i) d\gamma}{\sigma_{eq}^2(r_i) [f'(p_i) + E]} \quad (22)$$

$$d\varepsilon^p(r_i) = d\varepsilon - \frac{d\sigma(r_i)}{E} \quad (23)$$

$$d\gamma^p(r_i) = d\gamma - \frac{d\tau(r_i)}{G} \quad (24)$$

$$dp(r_i) = \sqrt{[d\varepsilon^p(r_i)]^2 + \frac{[d\gamma^p(r_i)]^2}{3}} \quad (25)$$

Based on the numerical trapezoid integration, the axial bolt force  $F$  and twist torque  $T_{ip}$  are given by

$$F = \int_0^a 2\pi r \sigma(r) dr = E\pi r_c^2 + \pi \left[ r_c E\varepsilon + a\sigma(a) + 2 \sum_{i=2}^{2^{n-1}} r_i \sigma(r_i) \right] \quad (26)$$

$$T_{ip} = \int_0^a 2\pi r^2 \tau(r) dr = \frac{1}{2} G\theta_B \pi r_c^4 + \pi \left[ G\theta_B r_c^3 + a^2 \tau(a) + 2 \sum_{i=2}^{2^{n-1}} r_i^2 \tau(r_i) \right] \quad (27)$$

$$t = (a - r_c) / 2^{n-1} \quad (28)$$

$$\Delta l_b = \varepsilon' L \quad (29)$$

$$\theta_B = \gamma_{r=a} L / a \quad (30)$$

where  $t$  is the numerical integration step length, and  $n$  is the step number.

If  $r_c = 0$ , the whole section is in the plastic deformation, Eqs.(21)-(25) are also effective for the stress determination of cross section only if  $r_c = 0$ . The strain hardening material of the bolt will lead to

$$F = \int_0^a 2\pi r \sigma(r) dr = \pi \left[ a\sigma(a) + 2 \sum_{i=2}^{2^{n-1}} r_i \sigma(r_i) \right] \quad (31)$$

$$T_{ip} = \pi \left[ a^2 \tau(a) + 2 \sum_{i=2}^{2^{n-1}} r_i^2 \tau(r_i) \right] \quad (32)$$

$$t = a / 2^{n-1} \quad (33)$$

The bolt elongation  $\Delta l_b$  and bolt twist angle  $\theta_B$  can be determined by Eqs.(29) and (30).