

Numerical Simulation of Vehicle Motion including Non-Holonomic Constraints by Extended Baumgarte's Method

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ABSTRACT

In this paper, we propose the non-holonomic constraint equations for the wheel roll motion when the wheel leans to the road, which are based on the assumption that the wheel rolls without a slip as if it is the circular cone with the apex which is the intersection of the normal directed downward from the wheel center to the road surface. The present assumption is applied to the numerical simulations of the bicycle motion and the rollover motion of the four-wheeled vehicle, and is proven to work sufficiently for both problems.

INTRODUCTION

The motion of a two-wheeled vehicle such as a bicycle or motorcycle is generally unstable unless the rider controls the vehicle. The rollover of a four-wheeled vehicle is a very dangerous motion. Therefore, a large number of researches have been devoted to the two-wheeled vehicle stability and the rollover motion of four-wheeled vehicles. However, there are fundamental difficulties on solving these problems from the viewpoint of multi-body dynamics. Recently, Lot and Massaro [1] developed a tire model, in which the contact between the 3-dimensional road surface and the tire carcass is taken into account. It simulates the motorcycle motion when it crosses road bumps or enters in the 3-dimensional curve. Nishimura et al. [2] simulated the response of the motorcycle to the impulsive steering torque. Chase and Wielenga [3] simulated the rollover motion of a four-wheeled vehicle using ADAMS.

In this paper, we propose the non-holonomic constraint equations for the wheel roll motion when the wheel leans to the road, which are based on the assumption that the

wheel rolls without a slip as if it is the circular cone with the apex which is the intersection of the normal directed downward from the wheel center to the road surface. In the next step, in order to validate the present assumption, these constraint equations are applied to the numerical simulations of the bicycle motion under self-steering condition, and then the rollover motion of the four-wheeled vehicle when it travels only by the two side wheels at high steer angles.

The present bicycle model consists of a frame, front fork connected to steering, front and rear wheel, so that the motion is governed by the nine Lagrange equations with two holonomic and four non-holonomic constraint equations. The differential-algebraic equations are solved by Baumgarte's constraint stabilization method [4] which we need to extend in order to deal with both holonomic and non-holonomic constraint equations simultaneously.

The present four-wheeled vehicle model with leaf spring suspensions consists of a car body, front axle, rear axle, four wheels, and four spring/damper units. The governing equations of the rollover consist of the eighteen Lagrange equations, eight holonomic constraint equations, and four non-holonomic constraint equations. The differential-algebraic equations of the rollover are solved by extended Baumgarte's constraint stabilization method.

NON-HOLONOMIC CONSTRAINT EQUATIONS FOR ROLL MOTION OF SINGLE WHEEL

In this section, we propose the non-holonomic constraint equations for the wheel roll when the wheel leans to the road. We consider that the wheel has a sharp rim and

rolls on the xy plane as shown in Fig.1. We denote the position of the wheel center by $C[x,y,z]$, the orientation by 3-1-2 Euler angles $[\phi, \theta, \psi]$, and the position of the contact point by $P[u,v,0]$. From Fig.2 (a) and (b), x , y and z are expressed as

$$x = u + r \sin \theta \sin \phi \quad (1)$$

$$y = v - r \sin \theta \cos \phi \quad (2)$$

$$z = r \cos \theta \quad (3)$$

where r denotes the wheel radius.

Differentiations of Eq.(1) and (2) give the following expressions.

$$\dot{x} = \dot{u} + r \dot{\phi} \sin \theta \cos \phi + r \dot{\theta} \cos \theta \sin \phi \quad (4)$$

$$\dot{y} = \dot{v} + r \dot{\phi} \sin \theta \sin \phi - r \dot{\theta} \cos \theta \cos \phi \quad (5)$$

The point Q in Fig.2 represents the intersection of the normal directed downward from the wheel center C to the road surface. Now, we assume that the wheel rolls instantaneously without a slip on a circle with the center Q. Δu , Δv , $\Delta \phi$, and $\Delta \psi$ denote the increments of u , v , ϕ , and ψ during Δt as depicted in Fig.2(c), respectively. Then the above assumption is expressed as follows.

$$-R \Delta \phi = r \Delta \psi \quad (6)$$

where R denotes the distance between P and Q.

On the other hand, Δu and Δv are calculated as

$$\Delta u = -R \{\sin(\phi + \Delta \phi) - \sin \phi\} \approx R \Delta \phi \cos \phi \quad (7)$$

$$\Delta v = R \{\cos(\phi + \Delta \phi) - \cos \phi\} \approx R \Delta \phi \sin \phi \quad (8)$$

By the usage of (6), expressions (7) and (8) are modified as

$$\Delta u = r \cos \phi \Delta \psi \quad (9)$$

$$\Delta v = r \sin \phi \Delta \psi \quad (10)$$

Dividing both sides of Eq.(9) and (10) with Δt , the derivatives $\dot{u}$ and $\dot{v}$ are derived as

$$\dot{u} = r \cos \phi \dot{\psi} \quad (11)$$

$$\dot{v} = r \sin \phi \dot{\psi} \quad (12)$$

Substituting Eq.(11) and (12) into Eq.(4) and (5), the two non-holonomic constraint equations are derived as

$$\dot{x} = r(\dot{\phi} \sin \theta \cos \phi + \dot{\theta} \cos \theta \sin \phi + \dot{\psi} \cos \phi) \quad (13)$$

$$\dot{y} = r(\dot{\phi} \sin \theta \sin \phi - \dot{\theta} \cos \theta \cos \phi + \dot{\psi} \sin \phi) \quad (14)$$

Equations (13) and (14) are the final constraint conditions for the wheel roll based on the present circular cone hypothesis.

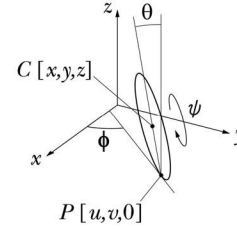


Fig.1 position and orientation of wheel

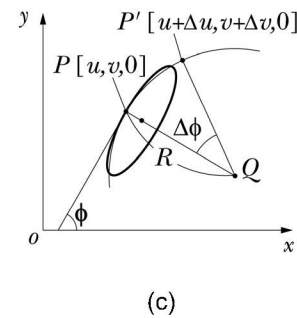
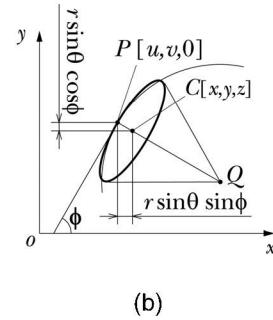
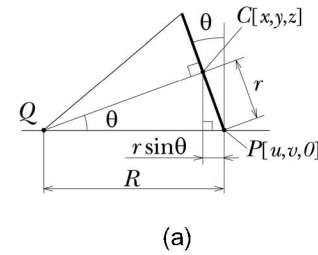


Fig.2 wheel motion according to artificial cone hypothesis: (a) artificial cone for wheel trajectory; (b) relationship between contact point P and wheel center C; (c) instantaneous trajectory of wheel

BICYCLE MODEL

In order to validate the present assumption about the contact between the wheel and road, we consider

numerical simulations of the bicycle motion under the self-steering condition.

The bicycle model consists of a frame, front fork connected to steering, front wheel, and rear wheel as shown in Fig.3. The front fork connects to the frame via a revolute joint. The front and rear wheel also connect to the front fork and frame via revolute joints, respectively. The rider is assumed to rest on the frame. The same assumption described in the previous section is applied extendedly to the contact between the two wheels and road. X , Y and Z denote the coordinates of the bicycle gravity center G , and the Euler angles ϕ_B , θ_B and ψ_B the orientation of the frame as depicted in Fig.4. Euler angles ϕ_f , θ_f and ψ_f represent the orientation of the front wheel. The rotation angle of the rear wheel is defined by ψ_r . The front and rear wheel contact with the road at P_f and P_r , respectively. The configuration of the bicycle is depicted in Fig.5.

Considering the constraint condition, the Lagrangian of this system is expressed as

$$L = \frac{1}{2} M (\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2) + \frac{1}{2} \omega_B^T (\mathbf{I}_B + \mathbf{I}') \omega_B + \frac{1}{2} \omega_f^T \mathbf{I}_w \omega_f + \frac{1}{2} \omega_r^T \mathbf{I}_w \omega_r - MgZ \quad (15)$$

where M denotes the mass of the bicycle including the rider, $\mathbf{I}_B$, $\mathbf{I}_w$ the inertia tensors of the frame and wheel, ω_B , ω_f , ω_r the angular velocity vectors of the frame, front wheel and rear wheel, respectively. The mass of the front fork is neglected for simplicity, and then $\mathbf{I}'$ represents the total inertia tensor of the frame, front wheel and rear wheel about the gravity center G .

Now, let the coordinates of the front and rear wheel center be $[x_f, y_f, z_f]$ and $[x_r, y_r, z_r]$. Then, according to the circular cone hypothesis, the following equations have to be satisfied;

$$\dot{x}_f - r(\dot{\phi}_f \sin \theta_f \cos \phi_f + \dot{\theta}_f \cos \theta_f \sin \phi_f + \dot{\psi}_f \cos \phi_f) = 0 \quad (16)$$

$$\dot{y}_f - r(\dot{\phi}_f \sin \theta_f \sin \phi_f - \dot{\theta}_f \cos \theta_f \cos \phi_f + \dot{\psi}_f \sin \phi_f) = 0 \quad (17)$$

$$\dot{x}_r - r(\dot{\phi}_r \sin \theta_r \cos \phi_r + \dot{\theta}_r \cos \theta_r \sin \phi_r + \dot{\psi}_r \cos \phi_r) = 0 \quad (18)$$

$$\dot{y}_r - r(\dot{\phi}_r \sin \theta_r \sin \phi_r - \dot{\theta}_r \cos \theta_r \cos \phi_r + \dot{\psi}_r \sin \phi_r) = 0 \quad (19)$$

On the other hand, according to the geometrical relationships between the bicycle gravity center and two wheel centers, $[x_f, y_f, z_f]$ and $[x_r, y_r, z_r]$ are expressed as

$$\begin{bmatrix} x_f & y_f & z_f \end{bmatrix}^T = \begin{bmatrix} X & Y & Z \end{bmatrix}^T + \mathbf{A} \begin{bmatrix} a & 0 & -c \end{bmatrix}^T \quad (20)$$

$$\begin{bmatrix} x_r & y_r & z_r \end{bmatrix}^T = \begin{bmatrix} X & Y & Z \end{bmatrix}^T + \mathbf{A} \begin{bmatrix} -b & 0 & -c \end{bmatrix}^T \quad (21)$$

where

$$\mathbf{A} = \begin{bmatrix} \begin{pmatrix} \cos \phi_B \cos \psi_B \\ -\sin \phi_B \sin \theta_B \sin \psi_B \\ \sin \phi_B \cos \psi_B \\ +\cos \phi_B \sin \theta_B \sin \psi_B \\ -\cos \theta_B \sin \psi_B \end{pmatrix} & \begin{pmatrix} -\sin \phi_B \cos \theta_B \\ \cos \phi_B \cos \theta_B \\ \sin \theta_B \end{pmatrix} & \begin{pmatrix} \cos \phi_B \sin \psi_B \\ +\sin \phi_B \sin \theta_B \cos \psi_B \\ \sin \phi_B \sin \psi_B \\ -\cos \phi_B \sin \theta_B \cos \psi_B \\ \cos \theta_B \cos \psi_B \end{pmatrix} \end{bmatrix} \quad (22)$$

Differentiating Eq.(20) and (21) with respect to time, Eq.(23) and (24) are obtained.

$$\begin{bmatrix} \dot{x}_f & \dot{y}_f & \dot{z}_f \end{bmatrix}^T = \begin{bmatrix} \dot{X} & \dot{Y} & \dot{Z} \end{bmatrix}^T + \dot{\mathbf{A}} \begin{bmatrix} a & 0 & -c \end{bmatrix}^T \quad (23)$$

$$\begin{bmatrix} \dot{x}_r & \dot{y}_r & \dot{z}_r \end{bmatrix}^T = \begin{bmatrix} \dot{X} & \dot{Y} & \dot{Z} \end{bmatrix}^T + \dot{\mathbf{A}} \begin{bmatrix} -b & 0 & -c \end{bmatrix}^T \quad (24)$$

Elimination of $\dot{x}_f$, $\dot{y}_f$, $\dot{x}_r$, $\dot{y}_r$ from Eq.(16) ~ (19) by the usage of Eq.(23) and (24) leads to the non-holonomic constraint equations

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} & b_{17} & b_{18} & b_{19} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} & b_{27} & b_{28} & b_{29} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} & b_{37} & b_{38} & b_{39} \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} & b_{46} & b_{47} & b_{48} & b_{49} \end{bmatrix} \dot{\mathbf{q}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (25)$$

where $\mathbf{q} = [X \ Y \ \phi_B \ \theta_B \ \psi_B \ \phi_f \ \theta_f \ \psi_f \ \psi_r]^T$ is the generalized coordinates vector, and the matrix elements b_{ij} are functions of $\mathbf{q}$.

As for the vertical coordinate, we have the following equation easily from the third component of Eq.(21).

$$Z = z_r - b \cos \theta_B \sin \psi_B + c \cos \theta_B \cos \psi_B \quad (26)$$

Substituting Eq.(26) into Eq.(15), Z is eliminated from the Lagrangian.

In addition to Eq.(25), we need the following two holonomic constraint equations to close the system:

$$G_1 \equiv r \cos \theta_f - r \cos \theta_B + (a+b) \cos \theta_B \sin \psi_B = 0 \quad (27)$$

$$G_2 \equiv \cos \theta_f \sin(\psi_B - \chi) \sin(\phi_f - \phi_B) + \cos \theta_f \sin \theta_B \cos(\psi_B - \chi) - \sin \theta_f \cos \theta_B \cos(\psi_B - \chi) = 0 \quad (28)$$

The equation (27) states that the vertical distance between P_f and G is equal to that between P_r and G . The equation (28) represents that the rotation axis of the front wheel must be perpendicular to the direction of the front fork.

The Lagrange equations of the bicycle motion under these constraints are expressed finally as

