

Ride Comfort Evaluation through Analysis of Roll and Lateral Vehicle Behaviors Due to Road Input

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ABSTRACT

With the increasing demand for vehicle ride comfort, analysis of roll or lateral vehicle behavior in addition to the bounce or pitching behavior can provide further understanding for ride comfort evaluation. In this paper, vehicle behaviors over rough roads are analyzed by dynamics motion equations with additional specific road input. After consideration of this analytical model, it is confirmed that suspension and tire characteristics should be divided into slip angle change due to lateral and vertical force or stroke. Also, the effects of each characteristic to vehicle behavior are clarified at each vehicle velocity range by both calculations and experiments.

1. INTRODUCTION

Conventional analytical studies on vehicle ride comfort in the low-frequency range are focused on the bounce and pitch behaviors of vehicles. With the recent increasing demand for enhanced ride comfort, it is beneficial to analyze the roll and lateral behaviors of vehicles as well. Regarding vehicle dynamic responses to steering input, many recent research reports take account of the coupled vehicle motions in roll, lateral and yaw directions. On the other hand, virtually no research has been reported regarding vehicle dynamic responses to specific road input. By expanding the analytical model developed for vehicle dynamic responses to steering input, we theoretically analyzed vehicle dynamic responses to road input. The present paper reports the results of our analysis.

2. EQUATIONS OF MOTION

2.1 Analytical assumptions

- 1) Vehicle's unsprung mass is small enough for the vehicle to bounce following the road surface contour.
- 2) The vehicle is bi-laterally symmetrical in shape, and has linear tire and suspension characteristics.
- 3) Road input to the rear wheels is the same in magnitude as that to the front wheels, but is time-delayed due to wheelbase / vehicle velocity.

- 4) vehicle side-slip angle and roll angle is small enough to approximate $\sin x = x$, $\cos x = 1$.

2.2 Equations of motion

The following motion equations were created by replacing Segel motion equations ⁽¹⁾ with barycentric coordinate system for a vehicle ⁽²⁾, and adding the paragraph of vertical displacement inputs (opposite-phase components for right and left wheels) from the road surface.:

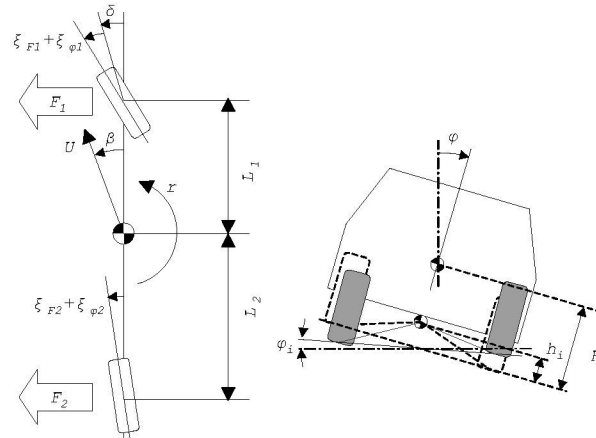


Fig.1 Analysis model

$$mU(\beta s + r) = F_1 + F_2 \quad (1)$$

$$I_z r s = F_1 L_1 - F_2 L_2 \quad (2)$$

$$I_x \phi s^2 = F_1 (H - h_1) + F_2 (H - h_2) + T \quad (3)$$

$$F_1 = K_1 \left(\delta - \beta - \frac{r L_1}{U} - \frac{\phi H s}{U} + \xi_{F1} + \xi_{\phi 1} \right) - mg \frac{L_2}{L} \phi_1 \quad (4)$$

$$F_2 = K_2 \left(-\beta + \frac{r L_2}{U} - \frac{\phi H s}{U} + \xi_{F2} + \xi_{\phi 2} \right) - mg \frac{L_1}{L} \phi_2 \quad (5)$$

$$T = -(K_{\phi 1} + C_{\phi 1} s)(\phi - \phi_1) - (K_{\phi 2} + C_{\phi 2} s)(\phi - \phi_2) + mg(H - h)\phi \quad (6)$$

$$L = L_1 + L_2, \quad h = \frac{h_1 L_2 + h_2 L_1}{L}$$

β : vehicle side-slip angle at center of gravity
 r : vehicle yaw angular velocity
 φ : vehicle roll angle
 U : vehicle longitudinal velocity
 m : vehicle mass
 I_z : yaw moment of inertia
 I_x : roll moment of inertia
 L_i : distance between vehicle center of gravity at each axle
 H : height of vehicle center of gravity
 h_i : height of roll center of each axle
 K_i : wheel cornering power of each axle (hereinafter CP)
 $K_{\varphi i}$: roll stiffness of each axle
 $C_{\varphi i}$: roll damping coefficient of each axle
 δ : steering angle
 φ_i : vertical displacement of each axle on the road surface
 $(\varphi_2 = \varphi_1 e^{-L/U s})$
 (Opposite-phase components for right and left wheels, in terms of roll angle)
 g : gravitational acceleration
 s : Laplace operator

(Suffix 1: front wheel; Suffix 2: rear wheel)

ξ_{Fi} represents change in slip angle due to lateral force F_i , and $\xi_{\varphi i}$ represents change in slip angle due to the relative roll angle between vehicle body and road surface $(\varphi - \varphi_i)$. ξ_{Fi} and $\xi_{\varphi i}$ can be described by the following expressions. Change in camber to road surface is expressed in terms of change in slip angle.

$$\xi_{Fi} = \left(p_{Fi} + \frac{q_{Fi}}{U} s \right) F_i \quad (7)$$

$$\xi_{\varphi i} = \left(p_{\varphi i} + \frac{q_{\varphi i}}{U} s \right) (\varphi - \varphi_i) \quad (8)$$

p_{Fi} : change in steering angle per unit lateral force
 q_{Fi} : lateral shift of wheel-ground contact point per unit lateral force
 $p_{\varphi i}$: change in steering angle per unit relative roll angle
 $q_{\varphi i}$: lateral shift of wheel-ground contact point per unit relative roll angle

In the case of vehicle dynamic response to steering input, $\varphi_i = 0$, and ξ_{Fi} and $\xi_{\varphi i}$ are expressed as functions of lateral force F_i . By substituting equations (7) and (8) in equations (4) and (5), it is possible to express the steering response using complex-equivalent cornering power (CP)⁽³⁾. In the case of vehicle dynamic response to a road input, ξ_{Fi} is a function of F_i and can be included in the complex-equivalent CP, but $\xi_{\varphi i}$ must be independently handled.

By transforming equations (4) and (5) based on the above condition and on the assumption that $\delta = 0$, F_i can be expressed as follows:

$$F_1 \cong K_1^* \left(-\beta - \frac{rL_1}{U} - \frac{\varphi H s}{U} + \alpha_1^* (\varphi - \varphi_1) \right) - mg \frac{L - L_1}{L} \varphi_1 \quad (9)$$

$$F_2 \cong K_2^* \left(-\beta + \frac{rL_2}{U} - \frac{\varphi H s}{U} + \alpha_2^* (\varphi - \varphi_2) \right) - mg \frac{L - L_2}{L} \varphi_2 \quad (10)$$

$$K_i^* = \frac{K_{ei}}{1 + \tau_i s}, \quad \alpha_i^* = \left(p_{\varphi i} + \frac{q_{\varphi i}}{U} s \right)$$

$$K_{ei} = \frac{K_i}{1 - K_i p_{Fi}}, \quad \tau_i = -\frac{K_i}{1 - K_i p_{Fi}} \frac{q_{Fi}}{U}$$

In the above expressions, K_i^* represents the complex-equivalent CP that takes account of lateral-force compliance only, and can be expressed with equivalent CP K_{ei} and time constant of lateral force τ_i (delay time of lateral force to tire slip angle); α_i^* represents the steering characteristic resulting from change in vertical stroke of the suspension. $p_{\varphi i}$ corresponds to suspension geometry characteristics of roll steer coefficient etc.; $q_{\varphi i}$ corresponds to roll-center height etc.

2.3 Examination of the equation of motion

When equations (6), (9) and (10) are substituted in equation (3) and put in order, the following equation of roll motion can be obtained:

$$a_{\varphi} \varphi + a_{\beta} \beta + a_r r = b \quad (11)$$

$$a_{\varphi} = I_x s^2 + \left\{ (C_{\varphi 1} + C_{\varphi 2}) s + K_{\varphi 1} + K_{\varphi 2} - mg(H - h) \right\} + \frac{\{ K_1^* (H - h_1) + K_2^* (H - h_2) \} H}{U} s - \{ \alpha_1^* K_1^* (H - h_1) + \alpha_2^* K_2^* (H - h_2) \}$$

$$a_{\beta} = K_1^* (H - h_1) + K_2^* (H - h_2)$$

$$a_r = \frac{K_1^* L_1 (H - h_1) - K_2^* L_2 (H - h_2)}{U}$$

$$b = \frac{(C_{\varphi 1} s + K_{\varphi 1}) \varphi_1 + (C_{\varphi 2} s + K_{\varphi 2}) \varphi_2}{U} - \{ \alpha_1^* K_1^* (H - h_1) \varphi_1 + \alpha_2^* K_2^* (H - h_2) \varphi_2 \} - Mg \frac{L_2 (H - h_1) \varphi_1 + L_1 (H - h_2) \varphi_2}{L}$$

The actual meaning of each term of the above expressions is as follows:

- 1) 2nd term of expression for a_{φ} and 1st term of expression for b : roll moments due to suspension roll stiffness and roll damping, respectively
- 2) 3rd term of expression for a_{φ} : roll damping due to lateral shift of wheel to ground contact point resulting from roll motion
- 3) 4th term of expression for a_{φ} and 2nd term of expression for b : roll moment due to suspension geometry
- 4) 3rd term of expression for b : roll moment due to lateral force caused by road surface inclination
- 5) a_{β} and a_r : roll moments due to coupling with the planar motion of vehicle

For typical vehicle characteristics, the effect of terms 3) and 4) on roll behavior is not significant and can be

ignored. Numerical calculation was conducted to study the effect of terms 2) and 5). Fig. 2 shows the result of the calculation.

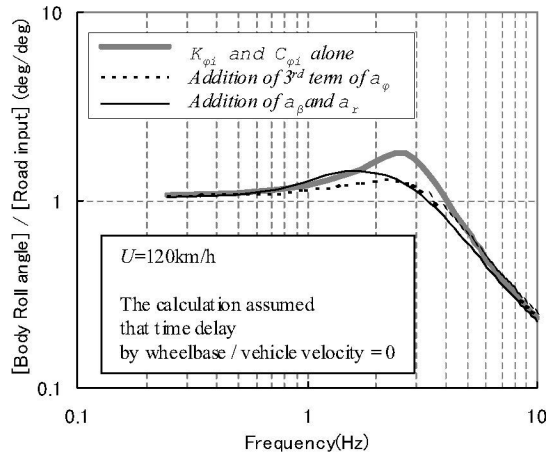


Fig.2 Effect of lateral forces for roll movement

Compared with the case where suspension roll stiffness and damping alone are considered, roll behavior, calculated with roll damping due to lateral shift of the wheel to ground contact point also taken into account, decreases at around the roll resonance frequency of 2.5 Hz. In the case where the roll moments due to coupling with the planar motion of the vehicle are additionally taken into account, the roll behavior increases at around the yaw or lateral resonance frequency of 1.5 Hz.

To facilitate examination of the lateral and yaw motions of vehicle, the term for roll angle ϕ has been placed on the input side or right side of the equation to obtain the following equations of motion:

$$\begin{bmatrix} mUs + K_1^* + K_2^* & mU + \frac{K_1^* L_1 - K_2^* L_2}{U} \\ K_1^* L_1 - K_2^* L_2 & I_{ss} + \frac{K_1^* L_1^2 + K_2^* L_2^2}{U} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} = \begin{bmatrix} F_{10} + F_{20} \\ F_{10} L_1 - F_{20} L_2 \end{bmatrix} \quad (12)$$

$$F_{10} = -K_1^* \frac{H}{U} \phi_s + K_1^* \alpha_i (\phi - \phi_i) - Mg \frac{(L - L_i) \phi_i}{L} \quad (13)$$

The 1st term of the equation (13) describes the lateral force due to the lateral shift of the wheel to ground contact point associated with roll motion, and indicates that larger lateral force is generated at a lower vehicle velocity. The 2nd term describes the lateral force due to change in steering angle associated with suspension geometry. The 3rd term describes the lateral force due to road surface inclination angle. The expression on the left side of the equation (12) is exactly the same as that of the equation of motion for steering response, except that suspension geometry is not taken into account in complex-equivalent CP K_i^* . The lateral and yaw responses can be expressed by the product of the excitation force calculated by the equation (13) and the same transfer characteristic as for the steering response. Calculation results of lateral and yaw response to roll angle ϕ are shown in Figs. 3 and 4. It is confirmed that the influence of rear wheel equivalent CP is large as well

as steering response (Fig. 3). Also lateral and yaw behaviors increase by delay of rear wheel lateral force at around resonance frequency of 1.5 Hz (Fig. 4).⁽²⁾

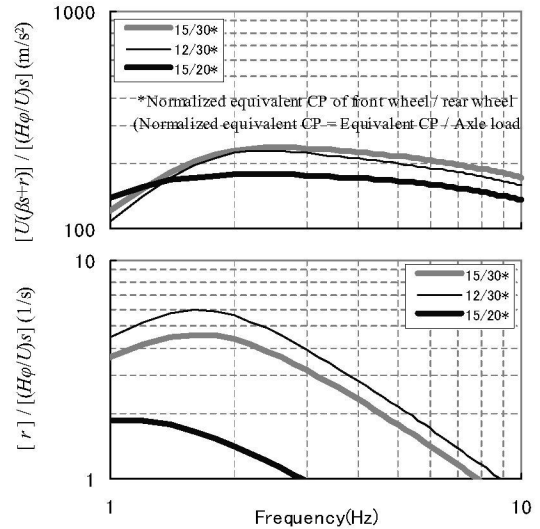


Fig. 3 Effect of Equivalent CP (Calculation)

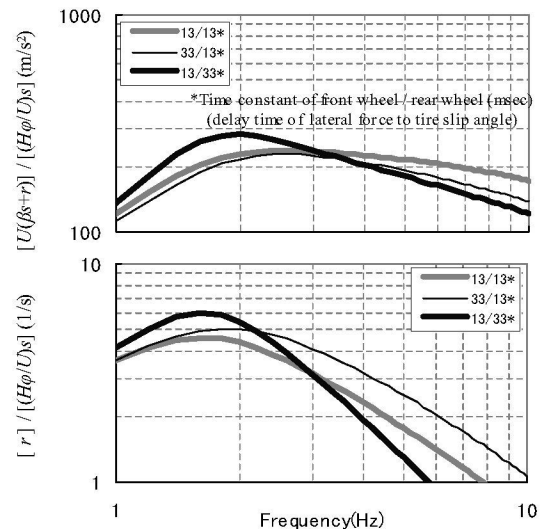


Fig. 4 Effect of Time constant (Calculation)

3. EXPERIMENTAL VERIFICATION

To verify the analytical model, an on-road experiment was conducted under the condition specified in Table 1. Figs. 5 and 6 compare the experimental results and calculation results [the results of solving equations (11) and (12) for ϕ , β and r] of the behavior of sprung body at vehicle velocities of 25 and 120 km/h, respectively. At both vehicle velocities, the calculation results coincide well with the experimental results, confirming the effectiveness of our analytical model.