

Application of Adaptive Kalman Filter for Estimation of Power Train Variables

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ABSTRACT

The paper presents the estimator design procedures for automotive power train systems based on the adaptive Kalman filter. The Kalman filter adaptation is based on a simple and robust algorithm that detects sudden changes of power train variables. The adaptive Kalman filter has been used to estimate the SI engine load torque and air mass flow, and also the tire traction force and road condition. The presented experimental results indicate that proposed estimators are characterized by favorable response speeds and good noise suppression abilities.

INTRODUCTION

The estimation of power train variables is important for power train/vehicle monitoring and diagnostics, and various control applications. For example, the load torque estimate can be effectively used for load torque compensation, and tire force/road condition estimation can improve traction control and vehicle dynamics control performance. The power train/vehicle estimators are based on appropriate power train/vehicle models and readily available measurements. In order to facilitate accurate and robust power train estimation, advanced estimators structures should be considered.

The application of adaptive Kalman filter methodology, proposed in [1], has been recently investigated by the authors for the purpose of load torque estimation [2, 3]. The basis of the adaptive Kalman filter is the first-order integrator-type model (i.e. engine inertia model) with load torque (input variable) treated as a stochastic disturbance variable at the process input, which is described by a second-order stochastic disturbance model. The proposed adaptive load torque estimator comprises a stationary (non-adaptive) Kalman filter tuned to obtain favorable noise rejection in the estimated load torque, and an adaptation mechanism to increase the Kalman filter feedback gains over a relatively short period of time if a sudden change of load torque has been detected. The adaptive Kalman filter tuning is based on a straightforward multi-step tuning procedure. The experimental results in [3] have indicated that the

utilization of the proposed adaptive Kalman filter approach yields a fast response of estimated load torque (i.e. good tracking ability) while maintaining favorable noise levels. The proposed adaptive Kalman filter estimator has also been effectively applied as a key part of an adaptive load torque compensator within idle speed control system (ISC system) [3].

It can be anticipated that, if some other power train variable can be described as an input disturbance variable, then the adaptive Kalman filter approach from [1-3] can be applied to those power train systems as well. References [4] and [5] have suggested that the first-order input disturbance model could be utilized as a basis for the design of input disturbance observers of exhaust gas recirculation (EGR) flow, and tire traction force, respectively.

As shown in [6], the adaptive Kalman filter methodology can also be attractive for the road condition estimation based on the estimation of tire static curve gradient for normal (low-slip) driving, which is directly related to the friction potential (i.e. road condition) [7]. The adaptive Kalman filter design in [6] is based on the linear regression model of tire static curve in low-slip region. However, the proposed road condition estimator requires the measurement (estimation) of applied wheel torque and wheel slip, which may not be available in all driving conditions. Another road condition estimation concept presented in [8] is based on the wheel speed resonant mode at approximately 40 Hz, which is caused by tire sidewall compliance. Since the damping ratio of the 40 Hz tire mode is inversely proportional to the low-slip tire static curve gradient, the information about the tire static curve gradient can be obtained from the properties of wheel speed signal only [8]. In order to estimate the 40 Hz mode damping ratio, a second-order autoregressive model (AR model) of tire vibrations is used as a basis for the estimator design [8]. The adaptive Kalman filter may be worth investigating as an alternative approach to the instrumental variable estimation method presented in [8].

This paper presents an overview of the theory underlying the adaptive Kalman filter approach, and design procedures of adaptive Kalman filters utilized for the

estimation of SI engine load torque and air mass flow, and tire traction force and road condition. For the purpose of adaptive Kalman filter design, appropriate stochastic process models are established. The proposed adaptive estimators are experimentally verified by utilizing an SI engine setup, a test vehicle equipped with different automotive sensors, and an experimental electrical vehicle equipped with an in-wheel high-bandwidth servomotor and high-precision wheel speed sensors.

PROCESS MODELS

This section presents process models of different automotive power train subsystems which are used as a basis for the estimator design.

MODEL OF SI ENGINE ROTATIONAL DYNAMICS

The design of adaptive Kalman-filter estimator of SI engine load torque in [2, 3] was based on the simple first-order inertia-type model of rotational dynamics $J\dot{\omega} = M - M_b$ (where ω is the engine speed, J is the engine inertia, while M and M_b are the engine torque and load torque respectively). The load torque M_b changes are not known in advance, so it is treated as a disturbance variable (modeled by a second order linear stochastic model). Since the manifold air mass flow (EGR flow) and tire friction may also be treated as disturbance variables, this model structure may be used as a basis for the design of respective Kalman filter-based estimators.

SI ENGINE INTAKE MANIFOLD

The estimation of EGR flow is usually based on the isothermal mean-value manifold model [4]:

$$\frac{dp}{dt} = \frac{RT}{V}(W_i + W_{EGR} - W_o), \quad (1)$$

where p is the manifold air pressure, T is the air temperature, V is the manifold volume, R is the gas constant and W_i , W_{EGR} , and W_o are intake air mass flow, port flow, and EGR flow respectively.

EGR flow can be estimated from the measurement of manifold air pressure p , intake air mass flow W_i , and reconstruction of port flow W_o (Eq. (1)). Since the particular engine setup does not include EGR ($W_{EGR} = 0$), the effectiveness of the air mass flow estimator is illustrated for the analogous problem of estimation of intake air mass flow W_i based on port flow reconstruction. Note also that the intake air mass flow estimation can be beneficial in practical applications (for diesel engines) in order to avoid the influence of air mass flow sensor drift, and low sensor accuracy at low flow rates [3].

The port air mass flow W_o is reconstructed by utilizing the so-called speed density equation (see [9] and references therein):

$$\hat{W}_o = \frac{V_d}{120RT}(s_1 p + s_o)\omega, \quad (2)$$

where V_d is the engine displacement volume, ω is the engine speed, and s_1 and s_o are the coefficients of the speed density equation for the particular engine.

DRIVEN WHEEL MODEL

The estimation of tire traction force can be based on a simple dynamic model of the driven wheel. It is assumed that the wheel driving torque M_m is known directly in the case of utilization of an in-wheel motor, or through the power train torque estimate in traditional engine-based approaches. The tire force can be reconstructed according to the following expression [5]:

$$F = \frac{M_m - J\dot{\omega}_w}{r_w} = F_{app} - \frac{J\dot{\omega}_w}{r_w}, \quad (3)$$

where $\dot{\omega}_w$ is the driven wheel angular acceleration, F and $F_{app} = M_m/r_w$ are the tire friction force and applied tire force, respectively, r_w is the driven wheel radius, and J is the equivalent wheel inertia.

LOW-SLIP TIRE-ROAD FRICTION EFFECTS

Low-slip tire static characteristics

The tire friction force is usually described by the tire static characteristic (tire static curve), i.e. the friction coefficient $\mu = F/F_z$ (F_z – wheel normal force) versus wheel slip s dependence. The longitudinal wheel slip s for the case of front wheel drive and no braking is defined as:

$$s = 1 - \frac{v_F}{r_F \omega_F} = 1 - \frac{r_R \omega_R}{r_F \omega_F}, \quad (4)$$

where v_F is the front wheel center speed (vehicle speed), ω_F and ω_R are front wheel and rear wheel speed, respectively, and r_R and r_F are the effective rear and front wheel radii.

If the rear wheel and front wheel radii are equal ($r_R = r_F$), the wheel slip equation (4) is reduced to the equation for practical slip s_m (measured slip):

$$s_m = 1 - \frac{\omega_R}{\omega_F}. \quad (5)$$

However, the wheel radii r_R and r_F may vary due to the difference in front and rear wheel pressure, normal load, or tire wear [6, 7]. Hence, the wheel slip calculated by Eq. (5) may be characterized by an error, which affects the accuracy of tire static curve reconstruction. This is especially important for the low-slip region (i.e. normal driving), where the wheel radii error is manifested in the form of slip bias [6, 7]:

$$\delta = s_m|_{\mu=0} \neq 0. \quad (6)$$

For the purpose of characterization of low-slip tire-road friction, the tire static curve can be approximated by a linear μ -slip relation [6, 7]:

$$\mu = k_g(s_m - \delta),$$

where k_g is tire static curve gradient at low wheel slips.

The experimental analysis of low-slip friction effects in [6, 7] has shown that the gradient of tire static curve can be ten times lower for icy road than for dry or wet asphalt. Fig. 1 shows the results of experimental estimation of low-slip tire static curves $\mu(s_m)$ from [10]. The tire force and wheel slip data are reconstructed based on Eqs. (3) and (5) respectively. The experimental static curve data for wet snow and wet ice are approximated in the least-squares sense by second-order polynomials (p_0 , p_1 and p_2 are the interpolation coefficients):

$$\mu(s_m) = p_2 s_m^2 + p_1 s_m + p_0, \quad (8)$$

while in the case of dry concrete data, a simpler straight-line fit ($p_2 = 0$) is found to be more appropriate. The interpolating curves are also shown in Fig. 1. The dissipation of tire static curve data points $\mu(s_m)$ is not pronounced, which points out to good accuracy of tire static curve reconstruction in the critical low-slip region.

Based on Eq. (8) the low-slip tire static curve gradients $k_g(s_m)$ are readily calculated as:

$$k_g(s_m) = \frac{d\mu(s_m)}{ds_m} = 2p_2 s_m + p_1. \quad (9)$$

Note that the zero-slip gradient $k_g(0) = d\mu(0)/ds_m$ is equal to the interpolation coefficient p_1 .

The static curve gradients for dry concrete, wet snow and wet ice are shown in Fig. 2. The results in Fig. 2 indicate that the zero-slip tire force gradient for dry concrete is 5-8 times larger for dry concrete compared to wet ice, and 3-5 times larger compared to wet or dry snow [10]. Hence, the information on low-slip tire static curve gradient can indeed be used for road condition estimation [6,7]. Note

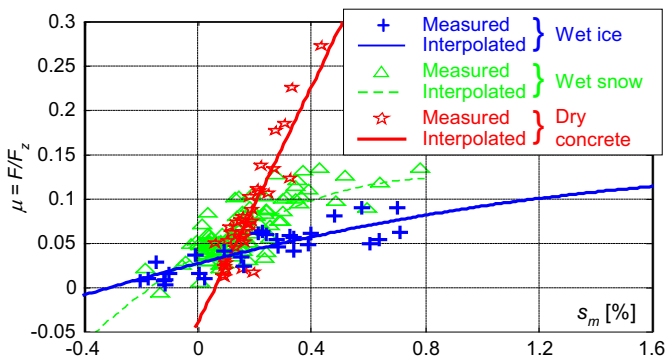


Fig. 1. Comparative experimental tire static curves for low wheel slips and straight-line driving on wet ice, wet snow and dry concrete.

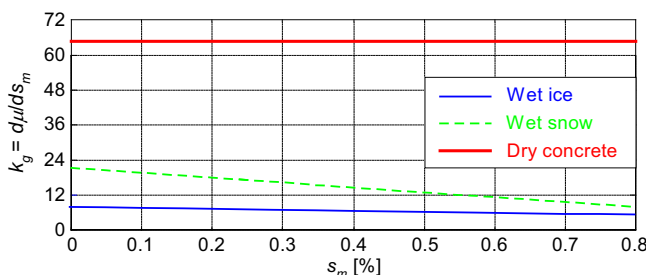


Fig. 2. Tire static curve gradient vs. wheel slip plots for dry concrete, wet snow and wet ice.

also that the gradient values do not overlap in a relatively wide range of low wheel slips s_m .

Tire sidewall torsional vibrations

Tire torsional vibrations are caused by the tire sidewall compliance effects, and can be excited by uneven road surface (so-called road noise). Fig. 3 shows the mass-spring model of torsional vibrations from [8]. It assumes that the wheel inertia can be separated into wheel rim inertia J_1 and tire belt inertia J_2 , linked by the tire sidewall compliance (characterized by its stiffness coefficient c). The road noise effect can be modeled as stochastic, white noise-like torque component ΔM_d [8]. Typical resonance frequency of tire torsional vibrations is around 40 Hz (the tire stiffness coefficient c varies with tire pressure [8]).

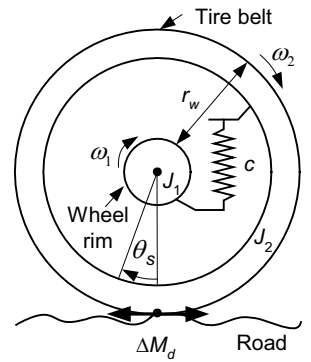


Fig. 3. Model of tire torsional vibrations

The tire torsional vibrations $\Delta\omega_1$ referred to the wheel rim side can be approximately described by the second-order oscillator model [8]:

$$G(s) = \frac{\Delta\omega_1}{\Delta M_d} \approx K_o \frac{2\zeta\Omega_n}{s^2 + 2\zeta\Omega_n s + \Omega_n^2}. \quad (10)$$

where the natural frequency of sidewall torsional vibrations $f_n = \Omega_n/2\pi$, the damping ratio ζ , and gain K_o are given in as follows (r_w - wheel radius):

$$f_n = \frac{1}{2\pi} \sqrt{\frac{c}{J_1}}, \quad \zeta = \frac{1}{2} \frac{J_1 + J_2}{\alpha r_w^2} \sqrt{\frac{c}{J_1}}, \quad K_o = \frac{1}{J_1 + J_2}.$$

The parameter α is the so-called longitudinal stiffness coefficient related to the tire static curve gradient $k_g = \Delta\mu/\Delta s$ as follows [8] (v - wheel longitudinal speed):

$$\alpha = k_g / v. \quad (11)$$

Namely, the damping ratio ζ of the tire torsional vibrations is inversely proportional to the tire static curve gradient. As a consequence, the wheel speed amplitude spectra around 40 Hz should exhibit different shapes of resonance peaks depending on the road condition. This has been confirmed in [8, 10] through estimation of wheel speed amplitude spectra and tire vibration damping ratio ζ .

The estimation of tire vibration damping ratio ζ is based in [8] on the second-order auto-regressive model (AR model) of band-pass filtered wheel speed signal ω_{bp} , which is given in the following form (k - sampling step):

$$(1 + a_1 q^{-1} + a_2 q^{-2}) \omega_{bp}(k) = \eta_c(k), \quad (12)$$

where q^{-1} is the discrete-time unit-delay operator and η_c represents colored road noise.

The wheel speed signal needs to be band-pass filtered to extract the interesting 40 Hz tire mode. Since the band-pass filtering of measured wheel speed signal ω_m effectively changes the properties of the stochastic perturbations at the process input, the colored road noise source η_c needs to be assumed instead of white road noise in Eq. (10) [8]. The parameters of AR model in Eq. (12) should be estimated by means of the instrumental variable identification method (IV method) [11].

Based on the analysis in [10], the following analytical relationships between the parameters of tire torsional vibration model f_n and ζ , and AR model parameters a_1 and a_2 are valid:

$$\zeta = \frac{1}{\sqrt{1 + 4 \left(\frac{\arctan \sqrt{4a_2/a_1^2 - 1}}{|\ln(a_2)|} \right)^2}}, \quad (13)$$

$$f_n = \frac{1}{4\pi T_s} |\ln(a_2)| \sqrt{1 + 4 \left(\frac{\arctan \sqrt{4a_2/a_1^2 - 1}}{|\ln(a_2)|} \right)^2}.$$

The results of estimation of the wheel speed amplitude spectra and 40 Hz mode damping ratios from [10] are shown in Fig. 4. The amplitude spectra in Fig. 4a clearly indicate the tire vibration mode resonant peaks located at approximately 40 Hz. The narrowest resonant peak (i.e. bandwidth) of the 40 Hz tire vibration mode is associated with the dry concrete surface. However, note that the wheel speed spectra comprise other resonant modes as well. These modes are also excited by road noise, and may notably interfere with the 40 Hz tire vibration mode.

The corresponding results of 40 Hz mode damping ratio estimation are shown in Fig. 4b. These results indicate that larger values of damping ratio ζ are obtained for road conditions with smaller values of low-slip tire static curve gradients (cf. Fig. 2). However, the estimated damping ratio ζ is not ideally inversely proportional to the low-slip tire static curve gradient k_g . Namely, the estimated damping ratio ζ for dry concrete is only about 50% smaller compared to wet ice, while the low-slip static curve gradient k_g for dry concrete is up to 8 times larger compared to wet ice (cf. Fig. 2). This could be caused by the relatively low road noise excitation, and the overlap distortion of 40 Hz mode due to its overlap with other tire modes ([10, 14], see also Fig. 4a).

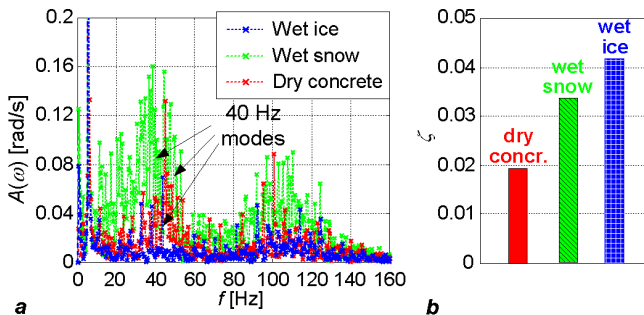


Fig. 4. Estimated wheel speed amplitude spectra for driving at 40 km/h on wet ice, wet snow and dry concrete (a), and corresponding estimated values of 40 Hz mode damping ratio (b).

ADAPTIVE KALMAN FILTER

Design and tuning procedures of different automotive power train estimators based on the adaptive Kalman filter are presented in this section.

BASIC KALMAN FILTER

It is assumed that the process is described by the linear, discrete-time SISO model, given in the following state-space form (k – sampling step) [12]:

$$\begin{aligned} \mathbf{x}(k) &= \mathbf{F}(k-1)\mathbf{x}(k-1) + \mathbf{G}(k-1)u(k-1) + \mathbf{\Omega}(k-1)\mathbf{v}(k-1), \\ y(k) &= \mathbf{H}(k)\mathbf{x}(k) + e(k), \end{aligned} \quad (14)$$

where $\mathbf{F}$, $\mathbf{G}$, $\mathbf{H}$, $\mathbf{\Omega}$ are the system matrix, input matrix, output matrix and state perturbation matrix, respectively, $\mathbf{x}$ is the process state vector, u and y are system scalar input and output signals respectively, $\mathbf{v}$ is the state perturbation vector, and e is the measurement noise (scalar-valued).

The state perturbation vector $\mathbf{v}$ and the measurement noise e are assumed to be mutually independent zero-mean Gaussian noise components, characterized by the state covariance matrix $\mathbf{Q}(k-1)$, and measurement noise variance $r(k)$, respectively.

The Kalman filter (estimator) for the process described by Eq. (14) is given by the following set of equations [12]:

$$\begin{aligned} \hat{\mathbf{x}}(k|k-1) &= \mathbf{F}(k-1)\hat{\mathbf{x}}(k-1|k-1) + \mathbf{G}(k-1)u(k-1), \\ \hat{e}(k|k-1) &= y(k) - \mathbf{H}(k-1)\hat{\mathbf{x}}(k|k-1), \\ \mathbf{P}(k|k-1) &= \mathbf{F}(k-1)\mathbf{P}(k-1|k-1)\mathbf{F}(k-1)^T \\ &\quad + \mathbf{\Omega}(k-1)\mathbf{Q}(k-1)\mathbf{\Omega}(k-1)^T, \\ \mathbf{K}(k) &= \frac{\mathbf{P}(k|k-1)\mathbf{H}(k)^T}{\mathbf{H}(k)\mathbf{P}(k|k-1)\mathbf{H}(k)^T + r(k)}, \\ \hat{\mathbf{x}}(k|k) &= \hat{\mathbf{x}}(k|k-1) + \mathbf{K}(k)\hat{e}(k|k-1), \\ \mathbf{P}(k|k) &= \mathbf{P}(k|k-1) - \mathbf{K}(k)\mathbf{H}(k)\mathbf{P}(k|k-1), \end{aligned} \quad (15)$$

where:

- $\hat{\mathbf{x}}(k|k-1)$, $\hat{\mathbf{x}}(k|k)$ – a-priori and a-posteriori state vector updates,
- $\mathbf{P}(k|k-1)$, $\mathbf{P}(k|k)$ – a-priori and a-posteriori error covariance matrix updates,
- $\mathbf{K}(k)$ – Kalman gain matrix,
- $\hat{e}(k|k-1)$ – a-priori prediction error.

By assuming time-invariant properties of the stochastic perturbations $\mathbf{v}$ and measurement noise e , the state covariance matrix $\mathbf{Q}$ and noise variance are time-invariant. Due to the fact that the individual components of $\mathbf{v}$ vector are mutually independent, the $\mathbf{Q}$ matrix also has a diagonal form.

The state covariance matrix $\mathbf{Q}$ and the measurement noise variance r are the design parameters of the Kalman