

Static and Dynamic Instabilities of Electrostatic Actuated MEMS Devices

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ABSTRACT

Fast and accurate characterization of stability regions and operational range with respect to pull-in voltage and displacement is critical in the design and development of MEMS resonators and switches. This paper presents a mathematical and computational procedure for modeling and analysis of static and dynamic instabilities of capacitive microdevices employing resonant microbeams. The mathematical model consists of a nonlinear microbeam under distributed electrostatic actuation and squeeze film damping. The coupled system is described by the nonlinear beam equation and a modified compressible Reynolds equation to account for the rarefied gas in the narrow gap between the microbeam and substrate. The Differential Quadrature Method (DQM) is used to discretize partial differential equations of motion and solve for static deflection, natural frequencies, static pull-in voltage, and quality factors for various encapsulation air pressures and applied DC voltages. A reduced order model is employed to study the dynamics of the microbeam under combined DC and AC voltages. It is shown that there is possible to trigger pull-in at a DC voltage much lower than static pull-in voltage. This combination can be used to minimize driving voltage of MEMS switches; however, it has to be avoided in the case of MEMS resonators to prevent the failure of the device.

INTRODUCTION

Electrostatic actuation and sensing principles are common in various MEMS devices such as resonators, filters, switches, tunable capacitors, comb drives, capacitive pressure sensors, and micro pumps [1-9]. Microbeams, microplates, and membranes are basic components used to achieve sensing or actuation encountered in most of the capacitive MEMS devices. Recent advancement in the wireless and satellite communications has led to an increased interest in the development of high-performance RF MEMS (radio frequency) devices that use as basic components MEMS resonators, filters, switches, and variable capacitors [9]. Potential applications in the automotive industry of this class of microsystems include

communication and navigation systems and radars. RF-MEMS switches have several advantages compared to semiconductor switches such as near-zero power consumption, low insertion loss, good isolation, high frequency range, and high Q-factor [10-12]. However, there are some disadvantages and a significant research effort has been directed to make RF MEMS switches competitive. One of the major concerns in the development of switches is reduction of actuation voltage. RF MEMS switches require relatively high actuation voltage (10-80 V) for reliable operation and have relatively low switching speed (high pull-in time). In addition, long-term reliability is yet under the acceptable limits and packaging cost is currently high because of stringent requirements of near-hermetic seals, inert atmosphere, and low humidity [10-12].

RF MEMS switches consist of a bridge (fixed-fixed) or a cantilever (fixed-free) microstructure (beam, plate or membrane) and use elastic deformation of the microstructure to achieve a short circuit. The force necessary to achieve elastic deformation is the electrostatic capacitor force resulting from a voltage difference between microstructure (electrode) and substrate. Similarly, RF MEMS mechanical resonators consist of a bridge microstructure and rely on the resonance of a structure which is deflected by a DC bias and driven by AC harmonic voltage. The DC component results in an electrostatic force which deflects the microstructure to a new equilibrium position while the AC component vibrates the microstructure around the equilibrium position. Coupling of two or more resonators can be used to obtain filters [13,14]. The applied DC voltage or the combined electrical load has an upper limit beyond which the electrostatic force exceeds the elastic restoring force leading to the collapse of the microstructure. This phenomenon is called pull-in instability and the associated voltage is pull-in voltage. The operational range of MEMS resonators is limited by pull-in instability that leads to the collapse of the microstructure and hence the failure of the device. In the case of MEMS switches, the pull-in effect is intentionally created and a low pull-in voltage is desired.

Many studies have addressed the pull-in phenomenon [15-21]. An accurate prediction of pull-in voltage is challenging because of nonlinearity of the electrostatic force, existence of fringing fields, squeeze film damping, non-ideal boundaries of anchors, and residual stress resulting from the manufacturing process. Most of the previous studies that addressed the pull-in phenomenon investigated the stability and the static deflection of the microstructure. Recently, dynamic pull-in phenomenon has received increasing attention [18,19].

Controlling the amount of damping is essential in achieving the desired performance of the microdevice [22]. During oscillations of the microstructure, the gas is pumped and sucked underneath the microstructure resulting in additional viscous forces called squeeze-film damping forces. In order to lower the energy loss due to gas-structure interaction, the silicon microstructures are encapsulated at very low pressure. For certain applications a quality factor (Q-factor), which characterizes the amount of damping in the system, of 30-50 can be acceptable [9]. For many applications, however, a quality factor of a few thousands is required. Very high quality factors in the range of 1.000-10.000 and as high as 100.000 have been obtained for vacuum encapsulated micromachined MEMS resonators [4,12]. The Reynolds equation is usually used to solve for the air pressure distribution. In its standard form, the Reynolds equation is derived based on the assumption that the fluid is a continuum media. However, the low pressure and small air gap make the assumption of continuum media no longer applicable. Consequently, the standard incompressible Reynolds equation is no longer valid [22, 23]. In microsystems, air rarefaction is included by considering the slip flow boundary conditions. A widely used method to incorporate the slip flow boundary conditions is by introducing the effective viscosity coefficient [23].

Despite significant progress in the manufacturing of microsystems, analytical and computational tools still lag the growth of the technology. Different approaches for modeling and simulations of multi-domain interaction in MEMS have been reported in the literature. The full-scale finite element method (FEM) has been used for electro-mechanical coupling or for air damping alone, and rarely for coupled simulation implying electro-mechanical-fluidic fields. Although FEM is adequate for static analyses, the simulation time is still a serious drawback of this method for dynamic analyses of coupled fields.

This paper proposes an efficient analytical and numerical procedure to investigate static and dynamic behavior of capacitive resonators and switches. The model consists of a nonlinear microbeam electrostatically actuated by a combination of a DC and AC voltages coupled with Reynolds equation. The model accounts for the electrostatic force of the capacitor, restoring force of the elastic microplate, residual stress, squeeze-film damping, and nonlinearities due to mid-

plane stretching and electrostatic force. First, the modeling procedure to determine static deflection, natural frequency, and pull-in voltage using the Differential Quadrature Method (DQM) is presented. Then, the same numerical procedure is used to discretize the coupled elastic-fluidic-electric domains and solve for natural frequency of the coupled system and quality factors for both low and high static deflections. Finally, a reduced order model is proposed to study dynamics of the microbeam under superharmonic and primary resonant AC excitation resulting in pull-in at DC loads much lower than the static pull-in voltage. Simulations are performed for clamped and cantilever microbeams. The results are compared to theoretical and experimental data available in the literature.

MODEL DESCRIPTION

Consider a microbeam (Figure 1) of length l , width b , and thickness h actuated by an electrical load $V_{DC} + V_{AC} \cos(\Omega t)$ where V_{DC} is the polarization voltage and V_{AC} and Ω are the amplitude and frequency of the AC voltage, respectively. The microbeam is separated from the silicon substrate by a small air gap. The voltage difference between the microbeam and substrate results in an electrostatic force that actuates the microbeam. Structural deformations result in a significant moving of air, which in turn generates a viscosity force (squeeze film damping force) against the microbeam. The equation of motion that governs the transverse displacement $w(x, t)$ of the microbeam is written as

$$EI \frac{\partial^4 \hat{w}}{\partial \hat{x}^4} + \hat{c} \frac{\partial \hat{w}}{\partial \hat{t}} + \rho b h \frac{\partial^2 \hat{w}}{\partial \hat{t}^2} - \left[\beta \frac{EA}{2l} \int_0^l \left(\frac{\partial \hat{w}}{\partial \hat{x}} \right)^2 d\hat{x} + \hat{N} \right] \frac{\partial^2 \hat{w}}{\partial \hat{x}^2} = -\hat{P}b + \frac{1}{2} \epsilon_0 \epsilon_r b \frac{(V_{DC} + V_{AC} \cos(\Omega t))^2}{(d - \hat{w})^2} \quad (1)$$

where $\hat{x}$ is the coordinate along the length of the microbeam, $\hat{t}$ is time, E is Young's modulus, $I = bh^3/12$ is the area moment of inertia, $\hat{c}$ is structural viscous damping, d is the capacitor initial gap width, ρ is the material density, $A = bh$ is the cross section area, $\hat{N}$ is the tensile ($\hat{N} > 0$) or compressive ($\hat{N} < 0$) applied axial force, $\hat{P}(\hat{x}, \hat{t})$ is the pressure per unit length due to air in the gap, ϵ_0 is the dielectric constant of the vacuum, ϵ_r is the relative dielectric constant of the gap medium to the air gap. Axial load, $\hat{N}$, is associated with the residual stress, $\sigma_{residual}$, due to manufacturing process through relation $\sigma_{residual} = \hat{N}/A$ where A is the cross section area of the microbeam. In MEMS residual stress is usually tensile but, due to temperature variations, it can also be compressive. Axial load, $\hat{N}$,

can be also associated to an applied load in order to control the natural frequency of the microbeam.

The first three terms in the equation (1) are the microbeam stiffness, structural viscous damping and inertia forces, respectively. The integral term represents the in-plane stretching of the microbeam because of non-moving supports in the case of clamped-clamped microbeam ($\beta=1$). This term disappears in the case of cantilever microbeam ($\beta=0$) since there is no in-plane stretching. The term $\hat{P}b$ is the squeeze film damping force. The last term in the equation (1) is the electrostatic force of the capacitor assuming a complete overlap of the microbeam and the substrate. Fringing fields and elasticity of supports (anchors) are neglected.

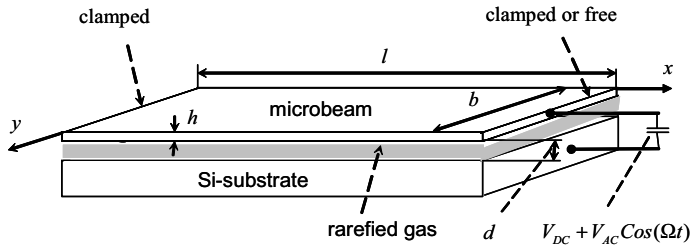


Figure 1. Schematics of an electrostatic actuated microbeam under low encapsulation air pressure.

For the clamped-clamped microbeam, the boundary conditions are

$$\hat{w}(0, \hat{t}) = \hat{w}(l, \hat{t}) = 0 \text{ and } \frac{\partial \hat{w}(0, \hat{t})}{\partial \hat{x}} = \frac{\partial \hat{w}(l, \hat{t})}{\partial \hat{x}} = 0. \quad (2)$$

For the clamped-free (cantilever) microbeam, the boundary conditions are

$$\hat{w}(0, \hat{t}) = 0, \quad \frac{\partial \hat{w}(0, \hat{t})}{\partial \hat{x}} = 0, \quad \frac{\partial^2 \hat{w}(l, \hat{t})}{\partial \hat{x}^2} = 0 \text{ and } EI \frac{\partial^3 \hat{w}(l, \hat{t})}{\partial \hat{x}^3} - \hat{N} \frac{\partial \hat{w}(l, \hat{t})}{\partial \hat{x}} = 0. \quad (3)$$

The gas pressure under the microbeam is described by the Reynolds equation for compressible and isothermal flow. Assuming small pressure variations across the beam width, the side leakage from the gap spacing is neglected and, therefore, the pressure gradient across the microbeam width is zero. With this assumptions, Reynolds equation is written as

$$\frac{\partial}{\partial \hat{x}} \left(H(\hat{x}, \hat{t})^3 \hat{P}(\hat{x}, \hat{t}) \frac{\partial \hat{P}(\hat{x}, \hat{t})}{\partial \hat{x}} \right) = 12\eta \left(H(\hat{x}, \hat{t}) \frac{\partial \hat{P}(\hat{x}, \hat{t})}{\partial \hat{t}} + \hat{P}(\hat{x}, \hat{t}) \frac{\partial H(\hat{x}, \hat{t})}{\partial \hat{t}} \right) \quad (4)$$

where $H(\hat{x}, \hat{t})$ is the varying distance between the beam and substrate, $\hat{P}(\hat{x}, \hat{t})$ is the gas pressure, and η is the viscosity of the fluid. Equation (4) is derived from the Navier-Stokes equation and continuity equation for an isothermal ideal gas imposing a number of assumptions [24]. The main assumption is that the gas is considered as continuum media. The validity of this assumption is given by the Knudsen number, K_n , defined as

$$K_n = \frac{\lambda}{H} \quad (5)$$

where $\lambda = P_0 \lambda_0 / P_a$ is the molecular mean free path of the gas, P_0 and λ_0 are the pressure and molecular mean free path at typical ambient conditions, respectively. The high values of Knudsen number imply a non-continuous gas domain.

In the case of low encapsulation pressure characteristics for the microsensors, the assumption of continuum media used in deriving Reynolds equation (4) no longer applies. Therefore, equation (4) is modified by including an effective viscosity coefficient η_{eff} to account for the rarefied gas. In this study, the effective viscosity of Veijola et al. [23] is used

$$\eta_{eff} = \frac{\eta}{1 + 9.63 K_n^{1.159}}. \quad (6)$$

By introducing the Knudsen number, the resulting equation accounts for the slip flow at the microbeam boundaries.

The microbeam transverse deflection, $\hat{w}(\hat{x}, \hat{t})$, is composed of a static component, $\hat{w}_s(\hat{x})$, due to DC voltage and a dynamic component, $\hat{u}(\hat{x}, \hat{t})$ due to AC voltage

$$\hat{w}(\hat{x}, \hat{t}) = \hat{w}_s(\hat{x}) + \hat{u}(\hat{x}, \hat{t}). \quad (7)$$

In order to linearize the Reynolds equation (4), the varying distance between the plate and substrate is written as a function of the initial gap, d , and the transverse oscillations of the microbeam, $\hat{w}$,

$$H(\hat{x}, \hat{t}) = d - \hat{w}_s(\hat{x}) - \hat{u}(\hat{x}, \hat{t}) \text{ and } \hat{P}(\hat{x}, \hat{t}) = P_a + \bar{P}(\hat{x}, \hat{t}) \quad (8)$$

where P_a is the encapsulation pressure.

It should be noted that linearization of pressure equation (4) is done about the static deflected position of the microbeam and not about the static undeflected position. Therefore, the model is valid for both small and large deformations of the microbeam which are caused by small and large applied DC voltages.

Substituting equation (8) into (4), the Reynolds equation including boundary slip conditions is obtained as follows

$$\frac{\partial}{\partial \hat{x}} \left[(d - \hat{w}_s(\hat{x}))^3 P_a \frac{\partial \bar{P}}{\partial \hat{x}} \right] = 12\eta_{eff} \left[(d - \hat{w}_s(\hat{x})) \frac{\partial \bar{P}}{\partial \hat{t}} - P_a \frac{\partial \hat{u}(\hat{x}, \hat{t})}{\partial \hat{t}} \right] \quad (9)$$

Air rarefaction resulting in a non-continuous fluid domain is taken into account by using the modified Reynolds equation (9) and effective viscous coefficient (6).

Assuming a zero flux at clamped edges and zero pressure at free edges, the pressure boundary conditions of the clamped-clamped microbeam are

$$\frac{\partial \hat{P}(0, \hat{t})}{\partial \hat{x}} = \frac{\partial \hat{P}(l, \hat{t})}{\partial \hat{x}} = 0. \quad (10)$$

Similarly, the pressure boundary conditions of the cantilever microbeam are

$$\frac{\partial \hat{P}(0, \hat{t})}{\partial \hat{x}} = 0 \text{ and } \hat{P}(l, \hat{t}) = 0. \quad (11)$$

Introducing the non-dimensional parameters

$$x = \frac{\hat{x}}{l}, \quad t = \hat{t} \sqrt{\frac{EI}{\rho b h l^4}}, \quad T = \sqrt{\frac{\rho b h l^4}{EI}}, \quad w = \frac{\hat{w}}{d},$$

$$u = \frac{\hat{u}}{d}, \quad P = \frac{\bar{P}}{P_a}, \quad c = \hat{c} \frac{l^2}{\sqrt{EI \rho b h}},$$

$$N = \frac{\hat{N}^2}{EI}, \quad P_{nom} = \frac{P_a b l^4}{EI d}, \quad \sigma = \frac{12\eta_{eff} l^2}{d^2 P_a T}, \quad \alpha_2 = 6 \left(\frac{d}{h} \right)^2,$$

$$\alpha_2 = \frac{6\varepsilon_r \varepsilon_0 l^4}{E h^3 d^3}, \quad \alpha_3 = \frac{\hat{N}^3}{EI}, \quad \Omega = \hat{\Omega} \sqrt{\frac{\rho b h l^4}{EI}} \quad (12)$$

equations (1) and (9) are written in non-dimensional form as follows

$$\frac{\partial^4 w}{\partial x^4} + c \frac{\partial^2 w}{\partial t^2} + \frac{\partial^2 w}{\partial t^2} - [\beta \alpha_1 \Gamma(w, w) + N] \frac{\partial^4 w}{\partial x^4}$$

$$= -P_{nom} P + \alpha_2 \frac{(V_{DC} + V_{AC} \cos(\Omega t))^2}{(1-w)^2} \quad (13)$$

$$\frac{\partial}{\partial x} \left[(1-w_s)^3 \frac{\partial P}{\partial x} \right] = \sigma \left[(1-w_s) \frac{\partial P}{\partial t} - \frac{\partial u}{\partial t} \right] \quad (14)$$

where

$$\Gamma(f_1(x, t), f_2(x, t)) = \int_0^1 \frac{\partial f_1}{\partial x} \frac{\partial f_2}{\partial x} dx. \quad (15)$$

The boundary conditions of the transverse displacement (2,3) and pressure (10,11), are written in non-dimensional form as follows

- clamped-clamped microbeam

$$w = 0, \quad \frac{\partial w}{\partial x} = 0, \quad \text{and} \quad \frac{\partial P}{\partial x} = 0 \text{ at } x = 0 \text{ and } x = l \quad (16)$$

- clamped-free microbeam

$$w = 0, \quad \frac{\partial w}{\partial x} = 0, \quad \frac{\partial P}{\partial x} = 0 \text{ at } x = 0$$

and

$$\frac{\partial^2 w}{\partial x^2} = 0, \quad \frac{\partial^3 w}{\partial x^3} - \alpha_3 \frac{\partial w}{\partial x} = 0, \quad P = 0 \text{ at } x = l \quad (17)$$

STATIC RESPONSE OF THE MICROBEAM TO A DC VOLTAGE

Static deflection of the microbeam under DC voltage is obtained by setting time dependent terms, time derivatives, AC forcing term, and squeeze film damping force term to zero into equation (13) and (7)

$$\frac{d^4 w_s}{dx^4} - [\beta \alpha_1 \Gamma(w_s, w_s) + N] \frac{d^4 w_s}{dx^4} = \alpha_2 \frac{V_{DC}^2}{(1-w_s)^2} \quad (18)$$

where $\Gamma(w_s, w_s) = \int_0^1 \frac{dw_s}{dx} \frac{dw_s}{dx} dx$. Boundary conditions

(16) and (17) corresponding to static deflection of the microbeam can be written as

- clamped-clamped microbeam

$$w_s = 0 \text{ and } \frac{dw_s}{dx} = 0 \text{ at } x = 0 \text{ and } x = l \quad (19)$$

- clamped-free microbeam

$$w_s = 0, \quad \frac{dw_s}{dx} = 0 \text{ at } x = 0 \text{ and}$$

$$\frac{d^2 w_s}{dx^2} = 0 \text{ and } \frac{d^3 w_s}{dx^3} - \alpha_3 \frac{dw_s}{dx} = 0 \text{ at } x = l. \quad (20)$$

Equation (18) and boundary conditions (19) or (20) are numerically solved using the Differential Quadrature Method (DQM). DQM is a numerical method for solving non-linear partial differential equations originally proposed by Bellman and Casti [25]. DQM was applied in solid mechanics and vibrations [26, 27], fluid dynamics [28, 29], and static and dynamic analysis of shaped microbeams [30-32]. DQM was employed for the first time in [33] to solve coupled structural, fluidic and

electrostatic domains. The basic idea of DQM is that the partial derivatives of a function with respect to a spatial coordinate at a given point are approximated as a weighted linear sum of the function values at all discrete points in the domain. In addition, the higher order derivatives are expressed as a recurrence expression of lower derivatives.

Consider the discretization of the one-dimensional domain using N_x grid points. The n th derivative of the transverse displacement, w , at point $x = x_i$ can be approximated as

$$\left. \frac{\partial^n w(x, t)}{\partial x^n} \right|_{x=x_i} = \sum_{k=1}^{N_x} A_{ik}^{(n)} w(x_k, t). \quad (21)$$

For the first order derivatives, the weighting coefficients are given by

$$A_{ij}^{(1)} = \frac{\prod_{v=1, v \neq i}^{N_x} (x_i - x_v)}{(x_i - x_k) \prod_{v=1, v \neq k}^{N_x} (x_k - x_v)}; \quad i, k = 1, 2, \dots, N_x; \quad i \neq k. \quad (22)$$

The higher order derivatives are given by the following recurrence equations

$$A_{ik}^{(r)} = r \left(A_{ii}^{(r-1)} A_{ik}^{(1)} - \frac{A_{ik}^{(r-1)}}{(x_i - x_k)} \right) \quad (23)$$

$$r = 2, 3, \dots, N_x - 1; \quad i, k = 1, 2, \dots, N_x; \quad i \neq k.$$

The diagonal terms of the weighting coefficients are given by

$$A_{ii}^{(r)} = - \sum_{v=1, v \neq i}^{N_x} A_{iv}^{(r)}, \quad i = 1, 2, \dots, N_x; \quad 1 \leq r \leq N_x - 1 \quad (24)$$

The accuracy of DQM strongly depends on the grid point distribution [34]. For the present analysis, the unequally spaced grid point distribution is used [27]

$$x_i = \frac{1}{2} \left(1 - \cos \frac{i-1}{N_x-1} \pi \right). \quad (25)$$

Using discretization rules (21-25) equation (18) is written as

$$\begin{aligned} & \sum_{j=1}^{N_x} (A_{ij}^{(4)} - N A_{ij}^{(2)}) w_j + \beta \alpha_1 \sum_{j=1}^{N_x} \sum_{k=1}^{N_x} \sum_{l=1}^{N_x} C_j A_{jk}^{(2)} A_{il}^{(2)} w_j w_k w_l \\ & = \alpha_2 \frac{V_{DC}^2}{(1 - w_i)^2}, \quad i = 1, 2, \dots, N_x. \end{aligned} \quad (26)$$

The coefficients C_j are derived by applying Newton-Cotes formula [27] to the integral term, Γ , using unequally spaced grid points (25)

$$C_j = \int_0^1 \prod_{\substack{v=1 \\ v \neq j}}^{N_x} \frac{\xi - \xi_v}{\xi_j - \xi_v} d\xi. \quad (27)$$

Discretized boundary conditions (19) and (20) are

- clamped-clamped microbeam

$$w_1 = \sum_{j=1}^{N_x} A_{1j}^{(1)} w_j = w_{N_x} = \sum_{j=1}^{N_x} A_{N_x j}^{(1)} w_j = 0 \quad (28)$$

- clamped-free microbeam

$$\begin{aligned} w_1 &= \sum_{j=1}^{N_x} A_{1j}^{(1)} w_j = 0 \quad \text{and} \\ \sum_{j=1}^{N_x} A_{N_x j}^{(2)} w_j &= \sum_{j=1}^{N_x} A_{N_x j}^{(3)} w_j - \alpha_3 \sum_{j=1}^{N_x} A_{N_x j}^{(3)} w_j = 0. \end{aligned} \quad (29)$$

The discretized equations of electrostatic actuated microbeam (26) and corresponding boundary conditions (28 and 29) are solved numerically for maximum deflection, natural frequency, and static pull-in voltage. The method is validated by comparison to theoretical and experimental results available in the literature.

First, the convergence of DQM is analyzed. Figure 2 shows the effect of mesh refinement on the maximum static deflection, $w_{Max} = w(x = l/2)$, at the middle point of the clamped microbeam on applied constant DC voltage. It is seen that the DQM provides a very good convergence even when using a very coarse mesh. Microbeam deflection is predicted with acceptable accuracy using only 9 grid points. The characteristics of the microbeam used in the simulation are taken from Choi and Lovell [35] $l = 400 \mu m$, $b = 45 \mu m$, $h = 2 \mu m$, $d = 1 \mu m$, and $N = 0$. There is a threshold of DC load beyond which the electrostatic force exceeds the restoring mechanical force leading to the collapse of the microbeam. The critical voltage associated with this threshold is called pull-in voltage and the structural instability associated with the structure collapse is called pull-in instability. The solution shown in Figure 2 features a stable (lower) branch which increases with an increase of applied electrostatic force (applied voltage) and an unstable (upper) branch which decreases such that the two branches collide with each other at the static pull-in instability. At pull-in, the rate change of the deflection of a point on the beam with respect to applied voltage (or electrostatic force), $\partial w_{Max} / \partial (V_{DC})$, approaches infinity.

Figure 3 shows the variation of the first natural frequency with applied DC load for the same microbeam as in Figure 2. The effect of electrostatic force shifts the