

Powertrain Torsional Model Development for On-Board Indicated Torque Estimation

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ABSTRACT

Effective and precise torque estimation is a great opportunity to improve actual torque-based engine management strategies. Modern ECU often already implement algorithms to estimate on-board the torque that is being produced by the engine, even if very often these estimation algorithms are based on look-up tables and maps and cannot be employed for example for diagnostic purposes.

The indicated torque estimation procedure presented in this paper is based on the measurement of the engine speed fluctuations, and is mainly based on two separated steps.

As a first step a torsional behavior model of the powertrain configuration is developed. The engine-driveline torsional model enables to estimate the indicated torque frequency component amplitude from the corresponding component of the instantaneous engine speed fluctuation. This estimation can be performed cycle by cycle and cylinder by cylinder.

As a second step the analysis of the relationship between the indicated torque mean value over an engine cycle and the amplitude of its frequency components, allowed defining the final estimation algorithm that reconstructs the indicated torque mean value starting from the instantaneous engine speed fluctuation analysis.

The developed approach has been applied with success to a diesel engine mounted on-board a vehicle. The obtained precision is compatible with on-board application like for example torque-based engine management strategies.

DEFINITIONS, ACRONYMS, ABBREVIATIONS

ORD Engine order

Z	Number of cylinders
Ω	Engine speed
Ω_{ORD}	Amplitude of the engine speed oscillation at engine order <i>ORD</i>
ϕ_{ORD}	Phase of the engine speed oscillation at engine order <i>ORD</i>
θ	Crank angle
θ_{TDCm}	m-th cylinder TDC crank angle position
θ_m	m-th cylinder crank angle position
r	Crank-slider mechanism radius
l	Connecting rod length
A_p	Piston area
$\lambda = r/l$	Crank-slider radius/connecting rod length ratio
$p_m(\theta_m)$	Relative pressure inside m-th cylinder
$f(\theta_m) = \sin(\theta - \theta_{TDCm}) + \frac{\lambda \sin(2(\theta - \theta_{TDCm}))}{2\sqrt{1 - \lambda^2 \sin^2(\theta - \theta_{TDCm})}}$	Crank slider mechanism function for the m-th cylinder
T_{indm}	Indicated torque for m-th cylinder
M_{eq}	Equivalent mass of the reciprocating masses
T_{rm}	Reciprocating torque for m-th cylinder
ω	Frequency
θ_{eng}	Engine angular position
θ_{load}	Load angular position
I_{eng}	Engine inertia
I_{load}	Load inertia
k_s	Engine-load connection stiffness
c_s	Engine-load connection damping
T_{load}	Load torque
T_{eng}	Torque applied to the engine
$F(j\omega)$	Engine-driveline system transfer function
$\hat{T}_{rm}$	Normalized reciprocating torque for m-th cylinder

TDC Top Dead Center
FRF Frequency Response Function
CAD Computer Aided Design
ECU Electronic Control Unit
RMSE Root Mean Square Error

INTRODUCTION

Modern torque based engine control systems usually require algorithms to evaluate on-board the torque that is being produced by the engine. In the next future if the application of in-cylinder pressure sensors will be demonstrated to be feasible and affordable for the entire vehicle life at reasonable costs, their use will constitute an important step forward from the indicated torque evaluation point of view. Up to that moment precise indicated torque evaluation has to be obtained using other information available on-board.

The approach developed in this paper makes use of the engine speed fluctuations measurement, performed using the same 60-2 teeth wheel, and the magnetic pick-up already present for control purposes. This makes the approach developed very affordable, since no additional sensors are required. Many studies, some of them from the same authors of this work, demonstrated the feasibility of estimating mean indicated torque starting from engine speed fluctuation measurement [1-18]. In particular, fluctuation measurement is needed at an order characteristic of the engine (mainly related to its number of cylinders and its firing order); usually the order at which the speed waveform over an engine cycle shows fluctuation of the highest amplitude is selected, even if, theoretically, the procedure could be applied to each excited component.

The approach is mainly based on two separated steps:

- the first one requires the development of a torsional behavior model of the system under study; the model enables the evaluation of the indicated torque fluctuations amplitude at the selected order, from the measurement of the corresponding engine speed fluctuations.
- as a second step, indicated torque mean value estimation is performed from its frequency components amplitude at the order of interest.

Next section will present the development of the torsional behavior model of the engine-vehicle system and the identification of the relationship between the indicated torque frequency components and its mean value. Particular care has been devoted to the evaluation of the engine orders over which the engine speed fluctuations are expected to be mainly developed and to the definition of the frequency range over which the model can be applied.

The following sections will then identify the existing relationship between indicated torque frequency components and its mean value and the results obtained applying the whole estimation procedure. The engine used throughout this paper is an L4 diesel engine mounted on-board a mid-size vehicle. The approach described and applied throughout the paper can be used also for other engine configurations (i.e. also to gasoline engines, to engines with a different architecture, to engines coupled with an eddy-current brake in a test cell), even if specific adaptations of the main path have to be adopted case by case to improve the estimation accuracy, as it will be discussed in more detail in the next sections.

TORSIONAL MODEL

The torsional model to be developed is required to estimate the indicated torque frequency component at a selected order, from the corresponding engine speed component. The model can be therefore usefully defined in the frequency domain, since both input and output belong to that domain. The frequency range over which the model has to work can be determined once the interesting orders have been selected, and knowing the engine operating speed range. The selection of the engine orders suitable for the torsional model development is made observing the engine speed waveform within an engine cycle, and in particular focusing the attention to the frequency components at which the engine speed fluctuations show the higher amplitude.

The orders to be taken into account depend mainly on the number of cylinders and on the firing order: for a Z cylinders engine with evenly-spaced combustions, for example, the order that will show the highest engine speed fluctuation amplitude is the $Z/2$. Other excited orders will be all the multiples of the main one (i.e. Z, $3Z/2$, $2Z$, ...), even though usually their amplitude is quite low, and represent a small fraction of the main one. If the combustions are not evenly spaced the choice of the orders to be used for the development of the indicated torque estimation procedure could be more difficult, but still it is possible to determine the ones at which the response in terms of engine speed fluctuations is expected to be higher. As already mentioned in the introduction, the approach could be theoretically applied to each excited frequency component, even if, in order to obtain a good signal to noise ratio, it is preferable to apply it to those with the highest amplitude (usually the lowest in frequency).

The roughness of diesel engine combustion enables the use of higher order frequency components to make the whole procedure more robust. If more than one engine order is significantly excited (like for example for diesel engines), the estimation procedure can be computed once for each order, obtaining in this way redundant results to be used to improve procedure robustness. If only one order presents significant amplitude,

usually the case for gasoline engines) cross checking the obtained results is not possible. Availability of significant excitations on high order components usually apply to diesel engines due to the roughness of diesel combustion, to the higher compression ratio, and to other factors that make the in-cylinder pressure (and consequently the indicated torque and its effects on the crankshaft – i.e. the engine speed fluctuations) of this kind of engines richer of high frequency content.

The frequency components selected in this work are those associated to the 2nd and the 4th order, since the engine considered is an L4 diesel engine with evenly-spaced combustions. The 4th order component proved to be significantly excited, being the ratio between the amplitudes of the indicated torque frequency components of the 4th order over those of the 2nd order at minimum equal to 0.45; the ratio has been evaluated over a series of steady-state tests covering the entire engine operating range.

Once the engine orders of interest have been selected it is possible to determine the frequency range in which the torsional model has to correctly describe the dynamic behavior of the system. For the engine taken into account (orders 2 and 4 are selected), running from 800 to 4500 rpm, the frequency range to be investigated goes from 26.7 to 300 Hz (26.7 to 150 Hz for order 2 and 53.3 to 300 Hz for order 4). The evaluation of the interesting frequency range is important for the correct definition of the dynamic model, since all the natural frequencies of the system out of this range can be neglected. For the engine taken into account the lowest natural frequency on the crankshaft is higher than 300 Hz, and therefore the crankshaft itself can be considered as a rigid body.

Either if the engine is mounted on-board a vehicle or it is mounted on a test bench the lowest resonance frequencies associated to the driveline are usually within the frequency range of interest. These conditions (crankshaft eigen-frequencies out of the frequency range of interest, and one or more driveline natural frequencies within this range) have been verified for many powertrain configurations investigated and is the main hypothesis of this study. If this condition is not verified restrictions to the application of the proposed approach have to be considered.

Based on the previous considerations only the natural frequencies on the driveline will be taken into account. The system torsional behavior can be described by means of an equivalent model with 2 or more inertias, one of them representing the engine, the number of inertias to be employed depending on the number of natural frequencies of the driveline, up to the frequency of interest.

Considering for the sake of simplicity the model with two inertias, dynamic equations of the system can be obtained as:

$$\begin{cases} I_{eng} \ddot{\theta}_{eng} = T_{eng} - k_s (\theta_{eng} - \theta_{load}) - c_s (\dot{\theta}_{eng} - \dot{\theta}_{load}) \\ I_{load} \ddot{\theta}_{load} = T_{load} - k_s (\theta_{load} - \theta_{eng}) - c_s (\dot{\theta}_{load} - \dot{\theta}_{eng}) \end{cases} \quad (1)$$

As already mentioned in the introduction, the input to the torsional model is expressed in the frequency domain, and therefore also the model can be usefully defined in that domain. Fourier expansion has been then applied to the signals in Equation (1) in order to obtain a relationship between the corresponding terms in the Fourier expansion series. The frequency associated to the i-th term of the Fourier expansion series is equal to $\omega = i\Omega/2 = ORD \cdot \Omega$ (i is the number of the harmonic over the engine cycle), for signals that can be considered periodic with period equal to an engine cycle (as it is for the torques and the angular positions of the inertias of the system of Figure 2). For the i-th terms of the torques and angular positions (T_{engi} , T_{loadi} , θ_{engi} and θ_{loadi}) it is therefore possible to write:

$$\begin{bmatrix} I_{eng} \omega^2 - j\omega c_s - k_s & j\omega c_s + k_s \\ j\omega c_s + k_s & I_{load} \omega^2 - j\omega c_s - k_s \end{bmatrix} \begin{bmatrix} \theta_{engi} \\ \theta_{loadi} \end{bmatrix} = \begin{bmatrix} -T_{engi} \\ -T_{loadi} \end{bmatrix} \quad (2)$$

Equation (2) could be seen as the relationship between the corresponding oscillations of torques and angular positions at frequencies $\omega = i\Omega/2$ for $i > 0$ (the engine order associated to the oscillation is $ORD = i/2$). Since the harmonic content of the load torque T_{load} is usually negligible (its dynamic is very slow), the terms T_{loadi} have been set equal to 0.

$$\begin{bmatrix} I_{eng} \omega^2 - j\omega c_s - k_s & j\omega c_s + k_s \\ j\omega c_s + k_s & I_{load} \omega^2 - j\omega c_s - k_s \end{bmatrix} \begin{bmatrix} \theta_{engi} \\ \theta_{loadi} \end{bmatrix} = \begin{bmatrix} -T_{engi} \\ 0 \end{bmatrix} \quad (3)$$

Focusing the attention on the relationship between the corresponding Fourier expansion coefficients (i.e. between the corresponding oscillation) of the crank angle position and the torque applied to the engine, it is possible to obtain:

$$\theta_{engi} = -\frac{1}{\omega^2} \cdot \frac{I_{load} \omega^2 - j\omega c_s - k_s}{I_{eng} I_{load} \omega^2 - (I_{eng} + I_{load})(j\omega c_s + k_s)} \cdot T_{engi} \quad (4)$$

Rewriting Equation (4) in terms of torque and speed it yields:

$$\dot{\theta}_{engi} = \frac{1}{\omega} \frac{I_{load} \omega^2 - j\omega c_s - k_s}{j(I_{eng} I_{load} \omega^2 - (I_{eng} + I_{load})(j\omega c_s + k_s))} T_{engi} \quad (5)$$

Equation (5) has been obtained for the simplest 2 inertias model case. Similar equations can be obtained if more complex torsional models have to be designed to correctly describe the system. The obtained equation will present in the case of a more complex system a higher number of parameters (inertias, stiffness and dampings). In any case it will be possible to describe the relationship

between the corresponding Fourier expansion coefficients of torque and speed using a Frequency Response Function (FRF), reported in the following as $F(j\omega)$.

$$\dot{\theta}_{engi} = \frac{1}{F(j\omega)} \mathbf{T}_{engi} \quad (6)$$

The Frequency Response Function $F(j\omega)$ represents the dynamic behavior of the system, and depends on the inertia-stiffness configuration of the system itself. Inertias and stiffness values can be determined starting from a CAD representation of the system, even if this approach is quite time consuming and requires full access to component CAD representation. In addition once FRF has been obtained usually it has to be verified and validated through specific experimental tests.

FRF identification is made in this work directly on an experimental basis, using engine speed and torque measurements during specifically designed tests. Rearranging Equation (6) it is in fact possible to see that FRF can be obtained as:

$$F(j\omega) = \frac{\mathbf{T}_{engi}}{\dot{\theta}_{engi}} \quad (7)$$

Amplitude and phase of the engine speed oscillation can be directly measured and, for the engine order of interest, it results:

$$\dot{\theta}_{engi} = \Omega_{ORD} e^{j\phi_{ORD}} \quad (8)$$

Engine torque T_{eng} is the result of all the torques applied to the cylinders, i.e. the difference between indicated and reciprocating torques. The same will apply also to the i -th harmonic component of the indicated and reciprocating torques that can be seen as the sum of the i -th harmonic components for all the cylinders. Considering indicated and reciprocating torque waveforms similar for all the cylinders, the Fourier representation of the torque harmonic components will differ only for the phase, whose value can be determined knowing the number of cylinders and the firing order (Equation (9) reports as an example what can be obtained for a Z cylinders engine with evenly spaced combustions).

$$\begin{aligned} \mathbf{T}_{indmi} &= \mathbf{T}_{ind1i} \cdot e^{jim2\pi/Z} \\ \mathbf{T}_{rmi} &= \mathbf{T}_{r1i} \cdot e^{jim2\pi/Z} \end{aligned} \quad (9)$$

$\mathbf{T}_{ind1i}$ is the i -th indicated torque harmonic component for cylinder 1; Indicated torque can be evaluated from the indicated pressure measurement for the same cylinder, through Equation (10) [19]:

$$T_{ind1}(\theta_1) = A_p r p_1(\theta_1) f(\theta_1) \quad (10)$$

Reciprocating torque can be expressed using the well known geometric expression [20]:

$$\begin{aligned} T_{rm}(\theta_m) &= M_{eq} r^2 f(\theta_m) \left[f(\theta_m) \ddot{\theta}_m + \frac{df(\theta_m)}{d\theta_m} \dot{\theta}_m^2 \right] = \\ &= M_{eq} \hat{T}_{rm}(\theta_m) \end{aligned} \quad (11)$$

where M_{eq} is the value of the reciprocating masses, and $\hat{T}_{rm}(\theta_m)$ represents the normalized reciprocating torque with respect to the reciprocating masses.

Amplitudes and phases of the frequency components of engine speed and engine torque have been evaluated as an example for a dynamic test performed in first gear whose speed and load variations are reported in Figures 1 and 2 respectively. As it is possible to observe from Figure 1, during the test a fast acceleration and deceleration has been performed, covering the whole engine speed operating range.

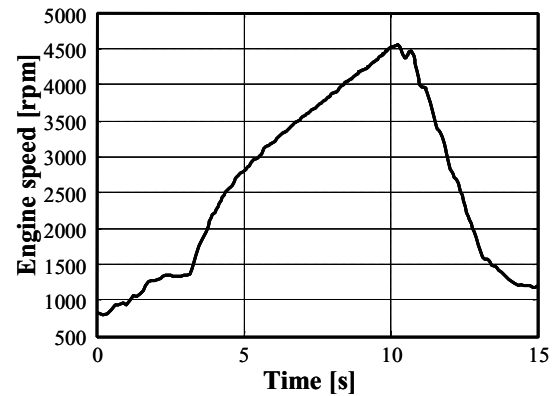


Figure 1: Engine speed variation during the test performed

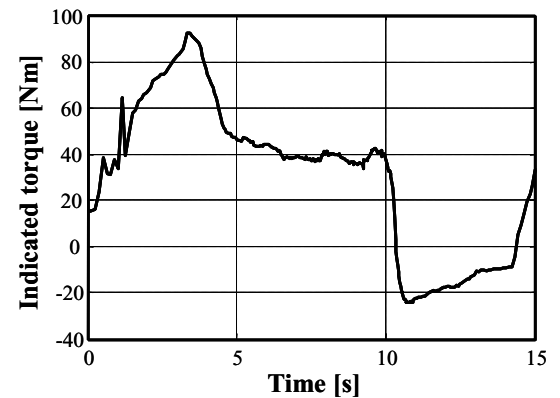


Figure 2: Indicated torque mean value variation during the test performed

Amplitudes and phases of the frequency components of engine torque and engine speed have been reported in