

Theoretical Framework for Modeling Spot Welds under Various Types of Loading Conditions

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ABSTRACT

The theoretical framework and closed-form stress intensity factor solutions in terms of the structural stresses for spot welds under various types of loading conditions are presented based on elasticity theories and fracture mechanics. A mechanics description of loading conditions for a finite plate with a rigid inclusion is first presented. The loading conditions of interest are the resultant loads on the inclusion in a plate and the surface tractions on the lateral surface of a plate. The surface tractions on the lateral surface of the plate can be decomposed into a load-balanced part and a self-balanced part. The resultant loads on the inclusion and the self-balanced resultant loads on the lateral surface are then decomposed into various types of symmetric and anti-symmetric parts. Based on the elasticity theories, closed-form moment, force and stress solutions are derived for a plate with a rigid inclusion subjected to various types of loading conditions. Based on the J integral for a strip model, closed-form analytical stress intensity factor solutions for spot welds joining two sheets of equal thickness are derived in terms of the structural stresses around a rigid inclusion in a plate under various types of loading conditions. The closed-form solutions presented in this paper are used as the basis to develop new analytical stress intensity factor solutions for spot welds in various types of specimens presented in a subsequent paper.

INTRODUCTION

Resistance spot welding is widely used to join sheet metals in the automotive industry. These spot welds are subjected to complex multiaxial loads under service or crash conditions. The fatigue lives of spot welds in various types of specimens have been investigated by many researchers, for example, see Zhang [1]. Since a spot weld provides a natural crack or notch along the nugget circumference, fracture mechanics has been adopted to investigate the stress intensity factors at the

critical locations of spot welds in order to investigate the fatigue lives of spot welds in various type of specimens [1-14].

Pook [2] indicated that for a class of transversely loaded configurations consisting of two thin plates or beams joined over part of their common plane under symmetric loading conditions, the energy release rate or the stress intensity factor at a crack tip depends on the bending moment acting to the beam or plate in the vicinity of the crack tip. Wang et al. [12] conducted a three-dimensional finite element analysis of circular plates with connection under opening loading conditions. The computational results indicate that the stress intensity factor along the crack front can be correlated very well with the analytical solutions based on the bending moments or the corresponding structural stresses for thin plates with connection. Wang et al. [13] conducted a three-dimensional finite element analysis of a nearly square large lap-shear specimen. The computational results indicated that the stress intensity factor at the critical locations agrees very well with the analytical solution based on the bending moments and the membrane forces or the corresponding structural stresses. The results of Wang et al. [12,13] suggest that using the bending moments and the membrane forces or the corresponding structural stresses to obtain the stress intensity factors for spot welds assumed as rigid inclusions in thin plates can be quite accurate.

The closed-form solutions for thin plates with rigid inclusions under shear, central bending, counter bending, and opening loading conditions were obtained by Muskhelishvili [15], Reißner [16], Goland [17], Timoshenko and Woinowsky-Krieger [18], and Lin et al. [14], respectively. These solutions except those of Goland [17] and Lin et al. [14] were used by Zhang [1,10,11] to obtain the structural stresses at several critical locations of spot welds in various types of specimens and automotive structures, where the spot welds were treated as rigid in the analytical or numerical solution procedures. Rupe et al. [19,20] used the

element model whereas Salvini et al. [21,22] and Vivio et al. [23] used a spot weld assembly finite element model to represent a spot weld to obtain the resultant forces and moments through the spot weld for fatigue life estimations. Their analytical solutions are obtained from those for a rigid inclusion in a circular plate with clamped boundary conditions along the edge of the plate. In this paper, all analytical solutions needed are derived again here using the stress function approach and the Kirchhoff plate theory in order to develop new analytical solutions for spot welds in various types of commonly used specimens in a subsequent paper [24]. From this viewpoint, this paper can be considered as a comprehensive review paper where most of relevant closed-form analytical solutions for a plate with a rigid inclusion under various types of loading conditions are derived and presented in a consistent manner. These analytical solutions are useful for engineers to estimate the fatigue lives of spot welds under various types of loading conditions.

In this paper, the theoretical framework and closed-form stress intensity factor solutions in terms of the structural stresses for spot welds under various types of loading conditions are presented based on elasticity theories and fracture mechanics. A mechanics description of loading conditions for a finite plate with a rigid inclusion is first presented. The loading conditions of interest are the resultant loads on the inclusion in a plate and the surface tractions on the lateral surface of a plate. The surface tractions on the lateral surface of the plate can be decomposed into a load-balanced part and a self-balanced part. The resultant loads on the inclusion and the self-balanced resultant loads on the lateral surface are then decomposed into various types of symmetric and anti-symmetric parts. Based on the elasticity theories, closed-form moment, force and stress solutions are derived for a plate with a rigid inclusion subjected to various types of loading conditions. Based on the J integral for a strip model, closed-form analytical stress intensity factor solutions for spot welds joining two sheets of equal thickness are derived in terms of the structural stresses around a rigid inclusion in a plate under various types of loading conditions. The closed-form solutions presented in this paper are used as the basis to develop new analytical stress intensity factor solutions for spot welds in various types of specimens presented in a subsequent paper [24].

A SPOT WELD IN A FINITE PLATE UNDER GENERAL LOADING CONDITIONS

According to Zhang [10,11], Wang et al. [13] and Lin et al. [14], the closed-form stress solutions for a rigid inclusion under the self-balanced surface tractions of the plate are also important to obtain the analytic solutions of the stress intensity factors for spot welds. In order to consider both the resultant loads and self-balanced surface tractions, Figure 1 shows the free body diagram

of an upper half weld nugget in an upper sheet. The upper sheet has the thickness t and the nugget has the diameter $2a$. The Cartesian coordinate system is centered at the center of the upper half nugget located on the middle plane of the sheet. The resultant loads on the lower surface of the upper half nugget are denoted as $\mathbf{F}$ and $\mathbf{M}$. The surface traction on the plate lateral surface is denoted as $\mathbf{T}_U$ which can be decomposed into the load-balanced part $\mathbf{T}_U^{Load}$ and the self-balanced part $\mathbf{T}_U^{Self}$. The load-balanced traction $\mathbf{T}_U^{Load}$ shown in Figure 1 is in equilibrium with $\mathbf{F}$ and $\mathbf{M}$. The resultant force $\mathbf{F}$ and the resultant moment $\mathbf{M}$ acting at the origin of the Cartesian coordinate system are now decomposed into three resultant forces F_x , F_y and F_z and three resultant moments M_x , M_y and M_z , respectively. The self-balanced resultant force $\mathbf{F}^s$ and the self-balanced resultant moment $\mathbf{M}^s$ due to the self-balance traction $\mathbf{T}_U^{Self}$ on the plate lateral surface can now be decomposed into two uniform tensile forces $\tilde{F}_x^s$ and $\tilde{F}_y^s$, two uniform shear forces $\tilde{F}_{xy}^s$ and $\tilde{F}_{yx}^s$, two uniform bending moments $\tilde{M}_x^s$ and $\tilde{M}_y^s$, and two uniform twisting moments $\tilde{M}_{xy}^s$ and $\tilde{M}_{yx}^s$.

The units of the forces, $\tilde{F}_x^s$, $\tilde{F}_y^s$, $\tilde{F}_{xy}^s$, and $\tilde{F}_{yx}^s$, and moments, $\tilde{M}_x^s$, $\tilde{M}_y^s$, $\tilde{M}_{xy}^s$, and $\tilde{M}_{yx}^s$, with tilde signs are force per unit length and moment per unit length, respectively. In the following, we investigate the stresses, moments and transverse shear forces near a spot weld in a sheet under various types of loading conditions based on elasticity theories. Note that, the spot weld is assumed as a rigid inclusion in the following.

CLOSED-FORM SOLUTIONS FOR RESULTANT FORCES AND MOMENTS ON INCLUSIONS

Figure 2 shows plates with rigid inclusions subjected to various types of resultant loads to the inclusion and subjected to different types of boundary conditions: (a) a shear force F_x , (b) an opening force F_z and a clamped edge, (c) an opening force F_z and a simply supported edge, (d) a central bending moment M_x and a clamped edge, (e) a central bending moment M_x and a simply supported edge, and (f) a twisting moment M_z . Closed-form solutions for plates with inclusions under different resultant forces and moments are summarized in the following. The solutions are needed to derive the new closed-form stress intensity factor solutions for spot welds in various types of specimens as detailed in a subsequent paper [24].

SHEAR FORCES F_x AND F_y

$$\begin{aligned}\sigma_{rr} &= F_x \left[-\frac{(3+\nu)}{4\pi r t} + \frac{(1+\nu)a^2}{4\pi^3 t} \right] \cos \theta \\ \sigma_{r\theta} &= F_x \left[\frac{(1-\nu)}{4\pi r t} + \frac{(1+\nu)a^2}{4\pi^3 t} \right] \sin \theta \\ \sigma_{\theta\theta} &= F_x \left[\frac{(1-\nu)}{4\pi r t} - \frac{(1+\nu)a^2}{4\pi^3 t} \right] \cos \theta\end{aligned}\quad (1)$$

Note that the stresses for the spot weld under the resultant shear force F_y can be obtained by substituting $\theta + \pi/2$ for θ in Equation (1).

OPENING FORCE F_z

For clamped edge:

$$\begin{aligned}\tilde{M}_r &= \frac{-F_z}{4\pi^2(a^2-b^2)} \left\{ b^2 \left[-a^2(-1+\nu) + r^2(1+\nu) \right] \ln(b/a) \right. \\ &\quad \left. + (a^2-b^2)r^2 \left[1 + (1+\nu) \ln(r/a) \right] \right\} \\ \tilde{M}_\theta &= \frac{-F_z}{4\pi^2(a^2-b^2)} \left\{ b^2 \left[a^2(-1+\nu) + r^2(1+\nu) \right] \ln(b/a) \right. \\ &\quad \left. + (a^2-b^2)r^2 \left[\nu + (1+\nu) \ln(r/a) \right] \right\} \\ \tilde{M}_{r\theta} &= 0, \quad \tilde{Q}_r = \frac{-F_z}{2\pi a}, \quad \tilde{Q}_\theta = 0\end{aligned}\quad (2)$$

Based on the classical Kirchhoff plate theory, the corresponding stresses are given as

$$\begin{aligned}\sigma_{rr} &= \frac{M_r z}{I}, \quad \sigma_{\theta\theta} = \frac{M_\theta z}{I}, \quad \tau_{r\theta} = \frac{M_{r\theta} z}{I} \\ \tau_{rz} &= \frac{Q_r}{2I} \left(\frac{t^2}{4} - z^2 \right), \quad \tau_{z\theta} = \frac{Q_\theta}{2I} \left(\frac{t^2}{4} - z^2 \right)\end{aligned}\quad (3)$$

For simply supported edge:

$$\begin{aligned}\tilde{M}_r &= \frac{F_z}{4\pi^2 \left[a^2(-1+\nu) - b^2(1+\nu) \right]} \left\{ a^2(b^2-r^2)(-1+\nu) \right. \\ &\quad \left. + b^2(1+\nu) \left[a^2(-1+\nu) - r^2(1+\nu) \right] \ln(b/a) \right. \\ &\quad \left. + r^2 \left[b^2(1+\nu)^2 - a^2(-1+\nu^2) \right] \ln(r/a) \right\} \\ \tilde{M}_\theta &= \frac{-F_z \left\{ (-1+\nu) \left[-b^2 r^2(1+\nu) + a^2(b^2+r^2\nu) \right] \right.}{4\pi^2 \left[a^2(-1+\nu) - b^2(1+\nu) \right]} \\ &\quad \left. + b^2(1+\nu) \left[a^2(-1+\nu) + r^2(1+\nu) \right] \ln(b/a) \right. \\ &\quad \left. + r^2(1+\nu) \left[a^2(-1+\nu) - b^2(1+\nu) \right] \ln(r/a) \right\}\end{aligned}$$

$$\tilde{M}_{r\theta} = 0, \quad \tilde{Q}_r = \frac{-F_z}{2\pi a}, \quad \tilde{Q}_\theta = 0 \quad (4)$$

The corresponding stresses can be derived by substituting Equation (4) into Equation (3).

CENTRAL BENDING MOMENTS M_x AND M_y

For clamped edge:

$$\begin{aligned}\tilde{M}_r &= \frac{M_x \left\{ a^2 \left[b^2(-1+\nu) - r^2(1+\nu) \right] + r^2 \left[-b^2(1+\nu) + r^2(3+\nu) \right] \right\} \sin \theta}{4\pi^3(a^2+b^2)} \\ \tilde{M}_{r\theta} &= \frac{M_x(a^2-r^2)(b^2-r^2)(1-\nu) \cos \theta}{4\pi^3(a^2+b^2)} \\ \tilde{M}_\theta &= \frac{M_x \left\{ a^2 \left[b^2(1-\nu) - r^2(1+\nu) \right] + r^2 \left[-b^2(1+\nu) + r^2(1+3\nu) \right] \right\} \sin \theta}{4\pi^3(a^2+b^2)} \\ \tilde{Q}_r &= \frac{M_x(a^2+b^2+2r^2) \sin \theta}{2\pi^2(a^2+b^2)} \\ \tilde{Q}_\theta &= \frac{-M_x(a^2+b^2-2r^2) \cos \theta}{2\pi^2(a^2+b^2)}\end{aligned}\quad (5)$$

For simply supported edge:

$$\begin{aligned}\tilde{M}_r &= \frac{M_x(b^2-r^2)}{4\pi^3 \left[a^4(-1+\nu) - b^4(3+\nu) \right]} \left[a^4(-1+\nu^2) \right. \\ &\quad \left. - a^2(b^2+r^2)(-3+2\nu+\nu^2) + b^2 r^2(3+4\nu+\nu^2) \right] \sin \theta \\ \tilde{M}_{r\theta} &= \frac{M_x(a^2-r^2)(1-\nu)}{4\pi^3 \left[a^4(-1+\nu) - b^4(3+\nu) \right]} \left\{ a^2 \left[r^2(1-\nu) + b^2(1+\nu) \right] \right. \\ &\quad \left. + b^2 \left[r^2(1+\nu) - b^2(3+\nu) \right] \right\} \cos \theta \\ \tilde{M}_\theta &= \frac{M_x}{4\pi^3 \left[a^4(-1+\nu) - b^4(3+\nu) \right]} \left\{ a^4(b^2+r^2)(1-\nu^2) \right. \\ &\quad \left. + b^2 r^2(1+\nu) \left[b^2(3+\nu) - r^2(1+3\nu) \right] \right. \\ &\quad \left. + a^2 \left[b^4(-3+2\nu+\nu^2) + r^4(-1-2\nu+3\nu^2) \right] \right\} \sin \theta \\ \tilde{Q}_r &= \frac{M_x}{2\pi^2 \left[a^4(-1+\nu) - b^4(3+\nu) \right]} \left\{ a^4(-1+\nu) + 2a^2 r^2(-1+\nu) \right. \\ &\quad \left. - b^2 \left[2r^2(1+\nu) + b^2(3+\nu) \right] \right\} \sin \theta \\ \tilde{Q}_\theta &= \frac{M_x \left\{ -a^4(-1+\nu) + 2a^2 r^2(-1+\nu) \right.}{2\pi^2 \left[a^4(-1+\nu) - b^4(3+\nu) \right]} \\ &\quad \left. + b^2 \left[-2r^2(1+\nu) + b^2(3+\nu) \right] \right\} \cos \theta\end{aligned}\quad (6)$$

Note that the moments and shear forces for the resultant moment M_y can be obtained by substituting $\theta + \pi/2$ for θ in Equations (5) and (6). The corresponding stresses