

Closed-Form Stress Intensity Factor Solutions for Spot Welds in Various Types of Specimens

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ABSTRACT

Closed-form stress intensity factor solutions at the critical locations of spot welds in four types of commonly used specimens are obtained based on elasticity theories and fracture mechanics. The loading conditions for spot welds in the central parts of four types of specimens are first examined. The resultant loads on the weld nugget and the self-balanced resultant loads on the lateral surface of the central parts of the specimens are then decomposed into various types of symmetric and anti-symmetric parts. Closed-form structural stress and stress intensity factor solutions for spot welds under various types of loading conditions are then adopted from a recent work of Lin and Pan to derive new closed-form stress intensity factor solutions at the critical locations of spot welds in the four types of specimens.

INTRODUCTION

Resistance spot welding is widely used to join sheet metals in the automotive industry. These spot welds are subjected to complex multiaxial loads under service or crash conditions. The fatigue lives of spot welds in various types of specimens have been investigated by many researchers, for example, see Zhang [1]. Since a spot weld provides a natural crack or notch along the nugget circumference, fracture mechanics has been adopted to investigate the stress intensity factors at the critical locations of spot welds in order to investigate the fatigue lives of spot welds in various types of specimens [1-15].

Note that the stress intensity factors usually vary point by point along the circumference of spot welds in various types of specimens. In the literature, different formulae for stress intensity factor solutions were proposed, for example, by Pook [2], Radaj [4], Swellam et al. [10] and Zhang [11,12] for lap-shear specimens, by Pook [3] and Zhang [11,12] for coach-peel specimens and by Pook [3] for circular plates with connection and some bending dominant plate and beam configurations. As mentioned

in Zhang [11], computational results were available from Smith and Cooper [16] and Radaj [5] for lap-shear specimens and Wang and Ewing [17] for coach-peel specimens. However, these computational results are for their specimens of specific geometries and these results cannot be used for general applications of different specimen designs. Formulae such as those provided by Pook [2], Yuuki et al. [18], Swellam et al. [10] and Zhang [11] are needed for different designs of specimens. However, no comprehensive study has been conducted to validate these proposed formulae.

Wang et al. [13] conducted a three-dimensional finite element analysis of the mode I stress intensity factor solutions for spot welds in circular plates with connection and square-cup specimens based on axisymmetric and three-dimensional finite element models. The computational results are benchmarked to the known solution of Pook [3]. Later, Wang et al. [14] launched a parametric study of the stress intensity factor solutions at the critical locations of spot welds in lap-shear specimens based on the well benchmarked full three-dimensional finite element model of Wang et al. [13]. The computational results indicated that only the solution of Zhang [11] can accurately account for the dependence of the mode II stress intensity factors on the spot weld diameter and sheet thickness for the given range of the thickness and spot weld diameter. However, the computational results show a strong dependence of the mode I stress intensity factors on the specimen geometric parameters, especially the specimen width. Wang et al. [14] found that a generic analytical solution for spot welds under counter bending conditions is needed to model the width dependence of the mode I stress intensity factor in lap-shear specimens. Lin et al. [19] later developed a new analytical solution for a finite square plate under counter bending conditions. The new analytical solution can be used successfully to model the mode I stress intensity factor solutions from the computational results of Wang et al. [14]. With this new analytical solution for a rigid inclusion in a finite square plate under counter bending conditions, new analytical solutions with size

dependence for the four types of specimens can therefore developed in this paper for engineering applications of different specimen designs.

In this paper, closed-form stress intensity factor solutions at the critical locations of spot welds in four types of commonly used specimens are obtained based on elasticity theories and fracture mechanics. The loading conditions for spot welds in the central parts of four types of specimens are first examined. The resultant loads on the weld nugget and the self-balanced resultant loads on the lateral surface of the central parts of the specimens are then decomposed into various types of symmetric and anti-symmetric parts. Closed-form structural stress and stress intensity factor solutions for spot welds under various types of loading conditions are then adopted from Lin and Pan [20] to derive new closed-form stress intensity factor solutions at the critical locations of spot welds in the four types of specimens. The closed-form solutions for the four types of specimens are selectively validated by the three-dimensional finite element computational results that are reported in Lin and Pan [21].

CLOSED-FORM SOLUTIONS FOR FOUR TYPES OF SPOT-WELD SPECIMENS

Figure 1 shows halves of a lap-shear specimen, a square-cup specimen, a U-shape specimen and a coach-peel specimen. In this figure, the applied forces are marked by the bold arrows. These specimens are commonly used to obtain the mechanical properties of spot welds. For clear demonstration of spot weld locations, only halves of specimens are shown. The spot welds are idealized as circular cylinders and shown as half cylinders in the figure. The sheet metals used to make these specimens have the thickness t . As shown in the figures, these specimens have the specimen width $2b$ and the nugget diameter $2a$. The bold arrows represent the resultant forces F or $F/2$ applied to the specimens. In Figure 1(a), L represents the length of the upper or lower sheets of lap-shear specimens and V represents the overlap length. In Figure 1(d), the eccentricity distance between the applied force and the center of the spot weld in coach-peel specimens is also denoted as L . In this paper, new analytical stress intensity factor solutions at the critical locations of spot welds in four types of specimens are developed based on the central square parts of the specimens with a rigid inclusion under various types of loading conditions. We will concentrate on the upper halves of the four types of specimens to derive the closed-form stress intensity factor solutions. In the following derivations of the closed-form analytical solutions, the spot welds in the specimens are treated as rigid inclusions.

LAP-SHEAR SPECIMEN

According to the works of Radaj [4] and Radaj and Zhang [6-8], the loads of various types of specimens containing spot welds can be decomposed into several

symmetric and anti-symmetric parts based on the superposition principle of the linear elasticity theory. Figure 2 schematically shows the decomposition of the shear load of the central part of a lap-shear specimen. Based on the schematic of the lap-shear specimen shown in Figure 1(a), schematics of the central square parts of the specimens under various types of loads are shown as model A to F in Figure 2. As shown in the figure, we select the overlap length V equal to the plate width $2b$. In these schematics, two square plates with the width $2b$ and the thickness t represent the upper or lower parts of the overlap sheets and the rigid circular cylinder with the diameter $2a$ represents the spot weld. As shown in Figure 2, model A represents a spot weld under lap-shear loading conditions. For model A, F represents the total shear force of the resultant force uniformly applied to the edges of the square plates along the interfacial surface of the rigid circular cylinder. For model B, the shear force F applied along the interfacial plane is now changed to the membrane force F and the bending moment $Ft/2$ applied to the mid-planes of the upper and lower plates. The shear force F and the bending moment $Ft/2$ of a lap-shear specimen is then decomposed into four types of symmetric and anti-symmetric loads: counter bending (model C), central bending (model D), in-plane shear (model E), and tension (model F). The bending moments in models C and D have a magnitude of $Ft/4$, and the forces in models E and F have a magnitude of $F/2$. The decomposition of the shear load of lap-shear specimens was presented in Lin et al. [19].

After the decomposition of the shear load of lap-shear specimens, the structural stresses of models C, D, E and F can be derived based on the closed-form moment, force and stress solutions of these rigid inclusion models listed in Lin and Pan [20]. Substituting the structural stress solutions into the expressions of the stress intensity factor solutions listed in Lin and Pan [20] gives the stress intensity factor solutions for lap-shear specimens. The K_I and K_{II} solutions at the critical locations (points A and B as shown in Figure 1(a)) are given as

$$K_I = \frac{-\sqrt{3}F}{8b\sqrt{tXY}} [2b^2X + 4Y(a^4b^4 + b^8)] \quad (1)$$

$$K_{II} = \frac{F}{\pi a\sqrt{t}} \quad (2)$$

where X and Y are defined as

$$X = (-1 + \nu)(a^4 + b^4)^2 - 4a^2b^6(1 + \nu)$$

$$Y = a^2(-1 + \nu) - b^2(1 + \nu)$$

The K_{III} solution at the critical location (point B as shown in Figure 1(a)) is given as

$$K_{III} = \frac{-F}{\sqrt{2\pi a}\sqrt{t}} \quad (3)$$

SQUARE-CUP SPECIMEN

Figure 3 schematically shows the decomposition of the load of the central square part of a square-cup specimen. Based on the schematic of the square-cup specimen shown in Figure 1(b), schematics of the bottom and top square sheets of the upper and lower cups of the specimen under various types of loads are shown as model A to D in Figure 3. The top and bottom square sheets have the width $2b$. As shown in Figure 3, model A represents a spot weld in the central top and bottom square sheets of the specimen under opening loading conditions. $\tilde{F}$ represents the uniformly distributed transverse shear force applied to the four edges of the square plates. $\tilde{M}_c$ represents the uniformly distributed bending moment applied to the four edges of the square plates. Note that under opening loading conditions, the square-cup flange walls constrain the deformation of the square plates. Based on the observations of the experimental results and computational results, the flange provides a nearly clamped boundary condition to the four edges of the top and bottom square plates. Therefore, we apply the uniformly distributed bending moments $\tilde{M}_c$ to the four edges of the top and bottom square plates. Note that $\tilde{M}_c$ is caused by the constraint of the flanges of the square-cup specimen. As shown in the figure, the geometry and loading conditions of model A are symmetric with respect to the interfacial surface of the circular cylinder. Therefore, only the upper half of model A is considered. Model B represents the upper half of model A. In model B, F ($=8\tilde{F}b$) represents the resultant opening force applied to the center of the circular interfacial surface of the spot weld. Finally, the forces and moments of model B are decomposed into two types of symmetric loads: opening (model C) and equal bi-axial counter bending (model D).

The detailed derivations of the structural stresses and stress intensity factor solution for square-cup specimens are similar to those of lap-shear specimens and are not reported here. The K_I solution for square-cup specimens is given as

$$K_I = \frac{\sqrt{t}}{\sqrt{3}} \left\{ \frac{-3F[(a^2 - b'^2)(-1 + \nu) + 2b'^2(1 + \nu)\ln(b'/a)]}{2\pi^2[a^2(-1 + \nu) - b'^2(1 + \nu)]} - \frac{3f_c F[a^2 - b'^2 + 2a^2 \ln(b'/a)]}{2\pi^2(1 + \nu)(a^2 - b'^2)} \right\} \quad (4)$$

where b' is denoted as the equivalent radius for the equivalent circular plate model and is approximately equal to $2b/\sqrt{\pi}$ using the simple area equivalence rule.

f_c is a geometric factor which is a function of the geometric parameters of the square-cup specimen such as the detailed geometry of the flange, the specimen width and the plate thickness. Here, f_c is defined as 0 for the simply supported edge and 1 for the clamped edge. For the specimens of Lin et al. [15], f_c is determined as 0.837.

U-SHAPE SPECIMEN

Figure 4 schematically shows the decomposition of the load of the central square part of a U-shape specimen. Based on the schematic of the U-shape specimen shown in Figure 1(c), schematics of the bottom and top sheets of the upper and lower U parts of the specimen under various types of loads are shown as model A to F in Figure 4. As shown in Figure 4, model A represents a spot weld in the central square part of the specimen under opening loading conditions. Here, $\tilde{F}$ represents the uniformly distributed transverse shear force applied to the right and left edges of the square plates. $\tilde{M}_b$ represents the counter bending moment applied to the right and left edges of the square plates. For a U-shape specimen with a longer bottom sheet of the upper U part or a longer top sheet of the lower U part, the contribution from the additional moment $\tilde{M}_b$ ($=\tilde{F} \times (L - b)$) due to the opening force needs to be considered. As shown in the figure, the geometry and loading conditions of model A are symmetric with respect to the interfacial surface of the spot weld. Therefore, only the upper half of model A is considered. Model B represents the upper half of model A. In model B, F ($=4\tilde{F}b$) represents the resultant opening force applied to the center of the circular interfacial surface of the spot weld. The forces and moments of model B can be decomposed into two types of symmetric loads: counter bending (model C) and counter opening (model D). However, a closed-form solution for a square plate with a rigid inclusion under counter opening conditions is not available. Therefore, we take a statically equivalent approximate loading condition for the counter opening condition. The approximate counter opening condition can be decomposed into two types of symmetric loads: opening (model E) and counter bending (model F). In model F, the counter bending moment $\tilde{M}$ is required to eliminate the effects of the extra rotation caused by $\tilde{F}/2$ along the front and back edges of model E. The three-dimensional finite element computational results of Lin and Pan [21] indicate that the analytical solutions based on the statically equivalent approximation appear to agree well with the computational results.

The detailed derivations of the structural stresses and stress intensity factor solution for U-shape specimens are similar to those of lap-shear specimens and are not reported here. The K_I solution at the critical locations of U-shape specimens (points A and B as shown in Figure 1(c)) is given as

