

Stability and Control Considerations of Vehicle-Trailer Combination

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ABSTRACT

In this paper, dynamics and stability of an articulated vehicle in the yaw plane are examined through analysis, simulations, and vehicle testing. Control of a vehicle-trailer combination using active braking of the towing vehicle is discussed. A linear analytical model describing lateral and yaw motions of a vehicle-trailer combination is used to study the effects of parameter variations of the trailer on the dynamic stability of the system and limitations of different control strategies.

The results predicted by the analytical model are confirmed by testing using a vehicle with a trailer in several configurations. Design of the trailer makes it possible to vary several critical parameters of the trailer. The test data for vehicle with trailer in different configurations is used to validate the detailed non-linear simulation model of the vehicle-trailer combination. Analysis and the simulation model are used to examine the advantages and limitations of two active brake control methods: the uniform braking and the direct yaw moment (DYM) control of the towing vehicle.

INTRODUCTION

Handling behavior of articulated vehicles is more complex and less predictable than that of non-articulated vehicles. This poses challenges to non-expert drivers of light vehicles, who occasionally tow recreational trailers and may not have good understanding of dynamic complexities of a vehicle-trailer combination. The task of controlling the vehicle becomes the most difficult when the system becomes unstable. Apart from rollover, which is not discussed in this paper, the articulated vehicle may experience two types of instability in the yaw plane. The first one is a divergent instability such as jackknifing, in which the hitch angle increases without experiencing oscillations. This occurs when, at a particular operating condition, the understeer gradient of a vehicle-trailer combination becomes negative and the speed is above critical velocity. The second type is dynamic in nature and may lead to oscillatory response with increasing amplitude known as snaking or sway.

The influences of physical parameters of the vehicle and trailer on both types of instability have been investigated in detail using both validated analytical models and vehicle testing [1-6]. It has been shown [2, 5] that the divergent type of instability depends only on the parameters of the towing vehicle and on the vertical hitch load. The latter is affected by the trailer mass and the location of the trailer center of gravity in the longitudinal direction. Reducing the hitch load, or equivalently moving the trailer center of gravity rearward, improves the stability margin with respect to divergent motion. In contrast, stability with respect to the snaking motion depends on the parameters of both the towing vehicle and the trailer. More specifically, the system becomes unstable beyond a certain speed. This speed decreases, thus rendering the system less stable, as: 1) the mass of the trailer (relative to the vehicle's mass) increases, 2) the center of gravity of the trailer moves rearward, 3) the moment of inertia of the trailer increases, 4) cornering stiffness of trailer tires decreases, 5) cornering stiffness of the vehicle's rear tires decreases 6) the distance from the vehicle rear axle to the hitch point increases, 7) vehicle wheelbase decreases. Even for a specific vehicle-trailer combination, most of these parameters can be greatly altered by changing parameters that are in control of the user, such as weight distribution or tire type and tire pressure. Hence, stability of the vehicle-trailer combination cannot be guaranteed by selecting the passive design parameters, thus necessitating an active control approach.

Several active control methods have been proposed to improve handling and stability of a vehicle-trailer combination. For example, active rear steering of the towing vehicle was considered by Kageyama and Nagai [7] and active front and rear steering by Deng et al. [8]. Active control of trailer braking was investigated by Lugner et al. [9] and by Fernandez and Sharp [10], while Kimbrough et al. [11] considered active control of both vehicle and trailer braking. Aside from active vehicle brake control, these types of systems are seldom available in production vehicles or trailers. Furthermore, the proposed control strategies typically required a prior knowledge of trailer parameters and the variables describing trailer states (e.g. hitch angle, rate, trailer yaw rate or lateral acceleration of trailer center of gravity) which necessitated

additional sensors. More recently, cost effective solutions to improve stability and handling of vehicle-trailer combinations have been proposed and implemented by utilizing only brake actuators and sensors from the towing vehicle's electronic stability control (ESC) system. In essence, two control methods have been considered: one is a symmetric braking of the towing vehicle with the objective of bringing the vehicle speed below the critical speed (e.g. [12]); the second is control of the yaw moment of the towing vehicle via asymmetric braking [13]. Since no direct feed-back information about the trailer motion or even knowledge of trailer parameters is available, maintaining robustness of the control method with respect to variation in trailer parameters is one of the most important issues for systems intended for production.

In this paper, dynamics, stability, and stabilizing control of the articulated vehicle is investigated using first an analytical model and then a combination of testing and full-vehicle simulations.

ANALYTICAL MODEL OF VEHICLE-TRAILER COMBINATION

For the purpose of studying the yaw plane dynamics of the vehicle-trailer combination, a single track 4-th order model illustrated in Figure 1 is selected. It is the simplest model, which describes both types of instabilities and permits studying of the effects of system parameters and different control strategies on the dynamic behavior of the system. Nearly identical models have been considered in the past [1, 3, 5, 6] and have been shown to represent the handling dynamics very well and to achieve excellent match between the values predicted by the model and the test results.

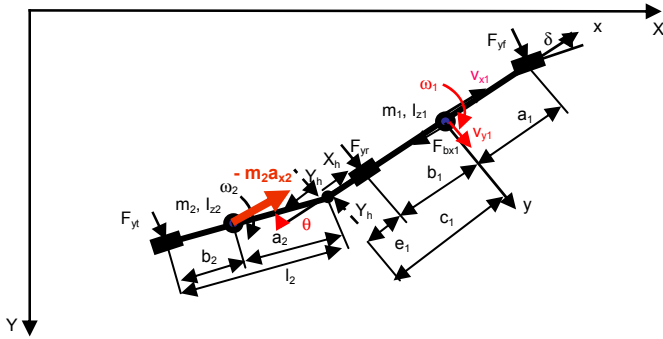


Figure 1: Simplified Model of Vehicle with Trailer in the Yaw Plane

In the model, the tires of each axle are represented as a single tire. The front wheel steering angle, δ , the hitch angle, θ , and the tire slip angles are assumed to be small. In addition, the effects of aerodynamic drag forces and deceleration on lateral dynamics are ignored. In the figure, m_1 and m_2 are the masses of the vehicle and the trailer, respectively, I_{z1} and I_{z2} are the yaw moments of inertia of the vehicle and the trailer about the respective yaw axes passing through the centers of mass. The symbols a_i and b_i are the distances of the vehicle front and rear axle to the

center of mass, e_i is the distance of the rear axle of the vehicle to the hitch point and $c_i = b_i + e_i$ is the distance of the hitch point to the vehicle center of mass. Symbols a_2 and b_2 denote the distances of the trailer center of mass to the hitch point and trailer axle, respectively, and $l_2 = a_2 + b_2$ is the distance of the trailer axle to the hitch point. The yaw rates of the vehicle and trailer are denoted by ω_1 and ω_2 , respectively, and lateral velocities at their centers of mass are v_{y1} and v_{y2} , respectively. Under the small angle assumption, the longitudinal velocities of vehicle and trailer are the same $v_{x1} = v_{x2} = v_x$ and are denoted by v_x . F_{yf} , F_{yr} and F_{yt} are the lateral tire forces per axle acting at front and rear axles of the vehicle and the trailer axle, respectively. Y_h represents the hitch force in the lateral direction. Additionally, the vehicle may be subjected to the yaw moment ΔM_{z1} resulting from active asymmetric braking of the towing vehicle, which is considered the control input. Under the above conditions and assumptions, the equations of lateral and yaw motions of the vehicle and trailer are:

$$m_1 a_{y1} = F_{yf} + F_{yr} - Y_h \quad (1a)$$

$$I_{z1} \dot{\omega}_1 = F_{yf} a_1 - F_{yr} b_1 + Y_h c_1 + \Delta M_{z1} \quad (1b)$$

$$m_2 a_{y2} = Y_h + F_{yt} \quad (1c)$$

$$I_{z2} \dot{\omega}_2 = Y_h a_2 - F_{yt} b_2 \quad (1d)$$

Here, a_{y1} and a_{y2} are the lateral accelerations at the vehicle and trailer centers of mass. The following kinematic relationships hold:

$$\omega_2 = \omega_1 + \dot{\theta} \quad (2a)$$

$$a_{y1} = \dot{v}_{y1} + v_x \omega_1 \quad (2b)$$

$$a_{y2} = \dot{v}_{y1} + v_x \omega_1 - c_1 \dot{\omega}_1 - a_2 (\dot{\omega}_1 + \ddot{\theta}) \quad (2c)$$

The tire forces per axle are assumed to be proportional to the tire slip angles:

$$F_{yf} = -C_{yf} \alpha_f = -C_{yf} \left(\frac{v_{y1} + a_1 \omega_1}{v_x} - \delta \right) \quad (3a)$$

$$F_{yr} = -C_{yr} \alpha_r = -C_{yr} \frac{v_{y1} - b_1 \omega_1}{v_x} \quad (3b)$$

$$F_{yt} = -C_{yt} \alpha_t = -C_{yt} \left(\frac{v_{y1} - (c_1 + l_2) \omega_1 - l_2 \theta}{v_x} - \theta \right) \quad (3c)$$

C_{yf} , C_{yr} , and C_{yt} denote the cornering stiffness values for the tires of the vehicle's front axle, rear axle and the trailer axle, respectively. Combining equations (1) through (3) yields a system of linear equations, which is written in a matrix form:

$$\mathbf{M} \dot{\mathbf{x}} = \mathbf{D}(v_x) \mathbf{x} + \mathbf{E} u + \mathbf{F} \delta \quad (4)$$

$\mathbf{M}$, $\mathbf{D}$, $\mathbf{E}$, and $\mathbf{F}$ are the system matrices which are given in the Appendix. While matrices $\mathbf{M}$, $\mathbf{E}$, and $\mathbf{F}$ are

constant, matrix **D** is a function of vehicle speed, v_x . The state vector, $\mathbf{x}$, and the control input, u , are given by

$$\mathbf{x} = \begin{bmatrix} v_{y1} & \omega_1 & \dot{\theta} & \theta \end{bmatrix}^T, \quad u = \Delta M_{z1} \quad (5)$$

By pre-multiplying equation (4) on both sides by $\mathbf{M}^{-1}$, the state equation is obtained

$$\dot{\mathbf{x}} = \mathbf{A}(v_x)\mathbf{x} + \mathbf{B}u + \mathbf{G}\delta \quad (6)$$

where

$$\mathbf{A}(v_x) = \mathbf{M}^{-1}\mathbf{D}(v_x), \quad \mathbf{B} = \mathbf{M}^{-1}\mathbf{E}, \quad \mathbf{G} = \mathbf{M}^{-1}\mathbf{F} \quad (7)$$

Matrix **A** is a function of vehicle speed, v_x , while matrices **B** and **G** are constant.

It is known that the linear bicycle model of a vehicle without a trailer is described by the state equation of the same form as equation (6), specifically

$$\dot{\mathbf{x}}_v = \mathbf{A}_v(v_x)\mathbf{x}_v + \mathbf{B}_v u_v + \mathbf{G}_v \delta \quad (8)$$

The matrices in equation (8) are given in the Appendix; the control input is the same as before and the state vector is

$$\mathbf{x}_v = \begin{bmatrix} v_{y1} & \omega_1 \end{bmatrix}^T \quad (9)$$

The analytical model of a vehicle-trailer combination described here is used primarily to study the effect of vehicle and trailer parameters on the stability of the system at the onset of potential instability, when amplitudes of motion are small and the linear assumptions are approximately satisfied. While extensions of the model are possible by employing nonlinear tire models and avoiding the small angle assumption, in this paper a combination of vehicle testing and simulations based on a nonlinear full vehicle with trailer model are used to validate the analytical results.

STABILITY ANALYSIS

In this section, stability of the vehicle-trailer combination is investigated using the model described in the previous section. Although the concept of stability appears intuitively obvious, it is necessary to make two important points. First, any system of at least the 2nd order may experience two types of instability: a static or divergent type of instability, in which the variables describing the system increase exponentially in magnitude without oscillations and a “dynamic” instability in which the variables experience oscillations with increasing amplitude. Changes in some parameters which increase static stability may reduce the dynamic stability and vice versa. Second, from an engineering point of view, it is not only important to maintain system stability, but a certain degree of stability (stability margin) is necessary. For example, a system that is stable but highly oscillatory may be difficult to control.

A good example of these points is the handling of a vehicle without a trailer. In a steady-state turn, the yaw rate of the vehicle, ω_1 , is

$$\omega_1 = \frac{v_x \delta}{l_1 + K_u v_x^2} \quad (10)$$

Here, l_1 is the vehicle wheel-base. The understeer gradient, K_u , is given by [14]

$$K_u = \frac{m_1(C_{yf}b_1 - C_{yr}a_1)}{C_{yf}C_{yr}l_1} = \frac{1}{g} \left(\frac{W_f}{C_{yf}} - \frac{W_r}{C_{yr}} \right) \quad (11a,b)$$

In the above, W_{fr} denote the front and rear axle static loads (due to gravity forces) and g is the gravity acceleration. If the understeer gradient is negative, vehicle yaw response diverges when the vehicle speed is above the critical speed $\sqrt{-l_1 / K_u}$. Thus, increasing the understeer gradient improves static (steady-state) stability of the vehicle.

At the same time, since the vehicle model is a 2nd order system, which is typically under-damped throughout most of the speed range, it is characterized by the natural frequency and the damping ratio. The latter, which is a good indicator of dynamic stability, is given by the following function of speed [14]

$$\zeta(v_x) = \frac{\zeta_0}{\sqrt{1 + v_x^2(K_u / l_1)}} \quad (12)$$

Here, ζ_0 represents the damping ratio at zero speed. Hence, the larger the understeer gradient is, the faster the rate at which the damping coefficient decreases with speed. At sufficiently high speed, the yaw response of the vehicle can become highly oscillatory when K_u is large. While increasing the understeer gradient improves static stability, it can reduce dynamic stability at high speeds.

STATIC STABILITY – In order to study the static stability of a vehicle-trailer combination, the system described by equation (4) without the control and steering inputs is considered in steady-state condition ($d\mathbf{x}/dt = 0$). That is

$$\mathbf{D}(v_x)\mathbf{x} = 0 \quad (13)$$

Solving the above equation yields the steady-state values of the state variables. In particular, vehicle yaw rate, ω_1 , is given by

$$\omega_1 = \frac{v_x \delta}{l_1 + (K_u - \Delta K_u)v_x^2} \quad (14)$$

K_u is the understeer gradient of the vehicle alone and ΔK_u is the change in the understeer gradient due to the presence of trailer. It is given by

$$\Delta K_u = \frac{m_2 b_2 [C_{yf}(a_1 + c_1) + C_{yr} e_1]}{C_{yf} C_{yr} l_1 l_2} \quad (15)$$

By observing that the changes in the static vertical loads of the vehicle front and rear axles due to the vertical hitch load are

$$\Delta W_f = -\frac{m_2 b_2 e_1 g}{l_1 l_2}; \quad \Delta W_r = \frac{m_2 b_2 (e_1 + l_1) g}{l_1 l_2} \quad (16)$$

it can be shown that the understeer gradient of the vehicle trailer combination is given by

$$K_u - \Delta K_u = \frac{1}{g} \left(\frac{W_{ft}}{C_{yf}} - \frac{W_{rt}}{C_{yr}} \right) \quad (17)$$

$W_{ft} = W_f + \Delta W_f$ and $W_{rt} = W_r + \Delta W_r$ are the static loads of the front and rear vehicle axle, respectively, when a trailer is present. Thus, equations (14) and (17) are identical to the corresponding equations for the vehicle without a trailer. It follows from equations (15) or (17) that the understeer gradient is reduced, as compared to that of the vehicle alone, when $b_2 > 0$ (i.e. when center of mass of the trailer is ahead of the trailer axle), it increases when $b_2 < 0$ and remains the same when $b_2 = 0$. Thus in a typical case of $b_2 > 0$, the presence of a trailer reduces the static stability of the articulated vehicle and increases steady state yaw response. It should be noted, however, that contrary to what is often assumed, the cornering stiffness values do not remain constant, but change with the changes in axle normal load. If the change in cornering stiffness were proportional to the normal load, there would be no change in the understeer gradient regardless of the dimension b_2 . In reality, the cornering stiffness values change less than proportionally with normal load [14], resulting in changes in understeer gradient, which are directionally the same, but significantly smaller than those predicted under the assumption of constant cornering stiffness values.

DYNAMIC STABILITY – In order to study the inherent dynamic stability of a vehicle-trailer combination, consider a state equation of the free system (i.e. equation (6) with the control and steering inputs both equal to 0) under the assumption of constant speed:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} \quad (18)$$

The system eigenvalues (poles) are the eigenvalues of matrix $\mathbf{A}$, that is, the roots of the characteristic equation

$$\det(s\mathbf{I} - \mathbf{A}) = 0 \quad (19)$$

Here, $\mathbf{I}$ denotes an identity matrix and s is the Laplace operand which is the unknown variable in equation (19). Since $\mathbf{A}$ is a matrix of dimension 4x4 with all real coefficients, equation (19) is a 4-th order equation with real coefficients. The roots are therefore either real or complex occurring in conjugate pairs. For the system to

be asymptotically stable, all the roots must have negative real parts. If any of the roots has a positive real part, the system becomes unstable. Since matrix $\mathbf{A}$ is speed dependent, the positions of the eigenvalues also change with speed. A convenient way of visualizing the changes in system eigenvalues and the degree of stability with speed is to plot the damping coefficients and the damped natural frequencies as functions of vehicle speed. If the system has a complex pair of poles at

$$s_{1,2} = -d \pm j\omega_d \quad (20)$$

where j is the imaginary unit, d is the damping coefficient, and ω_d is the damped natural frequency, then the damping ratio, ζ , and the un-damped natural frequency, ω_n , are

$$\zeta = \frac{-d}{\sqrt{d^2 + \omega_d^2}}, \quad \omega_n = \sqrt{d^2 + \omega_d^2} \quad (21)$$

A negative damping ratio ζ indicates an unstable system. As a 4-th order system, the vehicle with trailer model has 2 pairs of complex conjugate poles with one damping ratio for each pair for the total of two. In contrast, the vehicle without trailer possesses only one pair of complex conjugate poles and one damping ratio. The damping ratios for the vehicle alone and the vehicle with trailer are plotted as functions of speed in Figure 2. The numerical values of parameters correspond to a large pick-up truck with a 2000 kg (~ 4400 lbs) trailer.

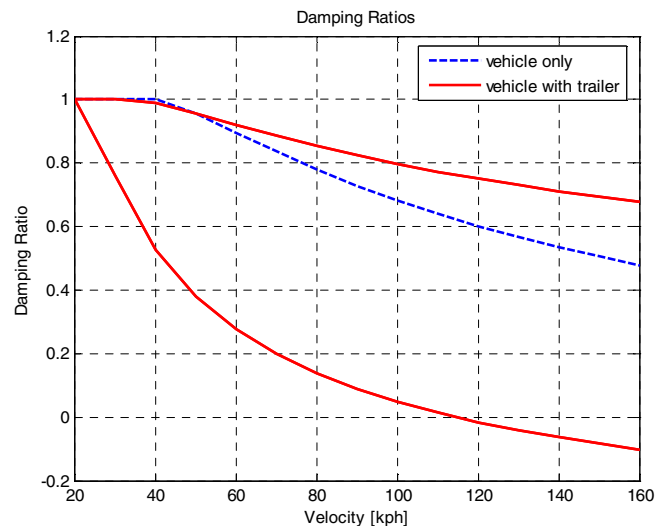


Figure 2: Damping Ratios of Vehicle Alone and Vehicle with Trailer as Functions of Speed

The vehicle without a trailer has the damping ratio which decreases with speed from 1 to about 0.5 at 160 kph (~ 100 mph), implying well damped response in the entire range of speeds. For the vehicle with trailer, one pair of eigenvalues has a slightly higher damping ratio than for the vehicle alone. The other pair of eigenvalues, in contrast, has a low damping ratio (near 0) at a speed above 60 kph (~ 37 mph) and becomes negative at about