

Directional Drilling Tool Simulation and System Design

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ABSTRACT

In order to develop from feasibility, and validate from first principles control strategies for down-hole directional drilling tools for the oil industry a non-linear, transient coupled system model has been developed. This paper reports on the basic physics and operation of the directional drilling tools and incorporates these principles into a simulation environment which includes sensor models (accelerometers and magnetometers) so that realistic azimuth and inclination measurements can be simulated in a fully coupled way with the dynamics of the tool. The drilling simulator environment is implemented in Matlab/Simulink® thus enabling control algorithms and sub-system models (bit mechanics etc) to be incorporated into the simulation environment as required. A case study of a vertical drilling algorithm is included in the paper to demonstrate the use of such a drilling simulator for development of control algorithms for trajectory control of directional drilling tools.

INTRODUCTION

The hydrocarbon industry today faces the prospect of finding ever dwindling reserves of oil for an ever expanding market whilst the remaining reserves are proving more difficult to exploit. A key factor in dealing with these problems has been the development of directional drilling tools in the last decade or so. Oil wells are conventionally drilled with thread assembled flexible steel sections (typically 30 ft in length) of hollow steel shaft rotating with a drill bit at the end, the assembled collection of sections typically being referred to as the drill string. The drill string is hollow not only because that is a more efficient means of designing what is essentially a torsional prop shaft, but also because drilling mud is pumped down the center of the drill string and re-circulated about the annular clearance between the drilled hole in the formation and the outside diameter of the drill string. The drilling mud serves a number of purposes, not least of which is removing the drilling spoil as it is created by the bit as it machines into the formation [1]. The mud also serves as the actuation and communication medium for directional drilling tools. A variation on this principle is that of a mud motor, where the torque rotating the steering mechanism is provided by a mud motor down hole as opposed to being

transmitted from the surface via the drill string. In this way the majority of the drill string is non-rotating, except that portion of it down hole from the mud motor. The mud motor essentially generates torque as a result of mud being pumped through it, where the mud motor itself can be viewed as a inverted Archimedes screw device.

The directional drilling tools developed in recent years enable the drill string to be deflected and hence the well bore propagation direction to be controlled. This enables the well bore to extend substantially laterally (sometimes thousands of feet) from the point of insertion of the tool on the surface. Hence, for example, well bore trajectories can be guided out under the sea bed whilst the drilling rig is on shore, and many other advantageous situations, there by greatly improving the economics of exploiting reservoirs that may have been deemed uneconomic only a few years ago. The dynamics of the machining and well propagation problem are highly complex and it is the case that it is a challenge to control directional drilling tools and measure many of the key variables and responses down hole. However, the need to better control directional drilling tools downhole given the expense incurred in maintaining and manning the drilling process and the time verses money aspect of the exercise implies there is a definite commercial advantage to be gained from fast and accurate drilling to achieve a required well trajectory [2].

Many of the system trajectory models used for directional drilling tools have tended to be rather complex and created with the aim of capturing in high fidelity the trajectory response of directional drilling tools. In order to achieve this, the dynamics and kinematics of the system have to be taken into account in some detail. The dynamics of the system stem from the structural, contact and machining interactions of the steering mechanism with the formation. This has lead to the widespread use of the finite element method to solve the drill string bending and mechanism contact problems in a coupled way [3,4,5,6] when modeling directional drilling tools. Another example of work modeling directional drilling tools from a physical standpoint is presented by Downton [7] where, starting from kinematic considerations, a delay differential equation and hence transfer function is derived describing the curvature response of a directional drilling tool in terms of the position of the stabilizer contact points with the formation. This

modeling approach incorporates a bit machining model and a beam bending finite element analysis to solve for well trajectory whilst obeying the kinematic constraints. The approach is quasi static in that the beam bending analysis is solved for as a series of static analyses at each time step, the bit reaction forces computed and from the bit model and displacement components the trajectory solved for accordingly.

A recent paper has presented another physically based generic directional drilling modeling approach where by the drill string assembly and directional drilling tool is broken into segments and each segment solved for in terms of Lubinski circular shaft buckling equations [8]. These governing equations are solved for segment by segment along the length of the modeled drill string leaving two unknowns, which are then solved iteratively. It is claimed that this approach is robust and numerically efficient in comparison to the more standard finite element approaches previously stated. There are also examples in the literature where empirical models have been developed to attempt to describe directional drilling tool behaviour. Typically a simple non-physically based model is used, sometimes only based on kinematics alone, where the parameters in the model are fitted to field data off line using probabilistic methods [9]. Probabilistic techniques have also been used to model high fidelity BHA models that include shock and vibration transient analyses [10]. Purely kinematics based models have also been developed for well plan design, using the assumption of circular arc deflections of the drill string and empirically derived design constants for specific tools [11].

The aim of this paper is to derive a generic system model in 2D and 3D that captures all the fundamental behaviour of directional drilling tools whilst being as simple and tractable as possible. The physics and formulation of the model is aimed at being scalable in complexity so that the same principles can be used to implement large and complex drilling simulations in bespoke software packages if need be, but also to be capable of implementing small systems models in standard PC software packages such as Excel, MathCad & Matlab/Simulink. The formulation given allows for the linearization of the system model and reduction to state space format and hence enables the use of standard modern time domain control techniques widely used and accepted in the systems and control research community [12,13].

This paper starts by stating the key assumptions made in deriving the well propagation trajectory model. These assumptions being necessary in order to reduce the complexity of the system model to an extent to where computationally expensive iterative and discretisation techniques are no longer necessary. These assumptions are then used to help state the basic methodology behind the derivation. Following this a general 2D derivation of a state space realisation for the well propagation model is given followed by an extension of these principles to give a general 3D state space realisation. For the 2D case, a tool specific derivation is

given using the generic 2D state space formulation approach derived earlier. Using the tool specific model key conclusions are drawn relating to well propagation stability, and the expected form of the transient response. Additionally, a general discussion is given on the usage of directional drilling tools in the light of the observed behaviour from the model. Finally, typical directional drilling tool system responses for a vertical drilling example are included to demonstrate the general operation of directional drilling tools and how the formulation detailed in this paper has been utilised to simulate this.

1. Derivation assumptions

1. Only the first four contact points (including the bit) between the formation and the BHA are significant in well bore trajectory propagation. The justification for this is linked to assumption 8 in that beyond four contact points back from the bit the lateral reaction forces and reacted moment on the drill string are totally reacted by the formation and not by the flexure of the drill string and steering mechanism.
2. The torsional dynamics of the entire drill string are secondary and can be ignored from a well trajectory propagation perspective. Hence the formulation given in this paper applies equally to the case where torque is transmitted from the surface down hole via the drill string or down hole using a mud motor.
3. The self weight of the tool and the drill string has been ignored and its effect on well bore trajectory assumed negligible.
4. The drill string and steering mechanism components behave elastically.
5. Inertial dynamics of the drill string, steering mechanism and system fluids (drilling mud) are ignored.
6. Only point contact between the drill string, steering mechanism and the formation is assumed.
7. The off collar axis angle the bit points in can be taken as small and effectively the ROP normal to the bit can be taken as being parallel to the collar axis. However, the off collar axis angle does effect the magnitude of the tangential reaction force on the bit, and hence the lateral machining rate of the bit.
8. The section of the drill string active in well propagation is treated as a simply supported beam subject to a uniform bending moment such that any lateral point loads on the beam are reacted by the formation rather than the drill string 'beam' itself. In this way only the moment reacted back from the steering mechanism deflects the drill string and dictates the well propagation direction.
9. The well propagation datum (inertial datum) is taken as the end of the drill string at the 1st stabiliser prior to the directional drilling mechanism itself.

10. The derivation applies equally to point or push the bit steering mechanism types. For a point the bit mechanism the direction the well bore propagates is governed by the weight on the bit and mechanism geometry. For a push the bit tool the direction the well bore propagates is governed by both the steering mechanism geometry and an actuation force between the tool and the formation. A specific example for a point the bit type mechanism is given in this paper.

2. Generic model derivation

Consider figure 1 where a basic 2D schematic of a BHA for a directional drilling tool is shown.

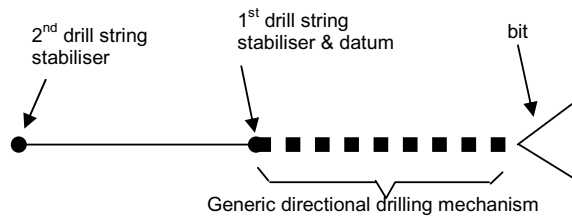


Figure 1: Generic directional drilling tool schematic

The approach is based on viewing the last section of the drill string between the 1st and 2nd stabilisers as being deflected solely by the moment applied by the directional drilling mechanism attached to the 1st stabiliser of the drill string. The reactions at the 1st and 2nd stabilisers are assumed to be reacted only by the formation. In this way the analysis of the deflection of the drill string can be greatly simplified and the general moment curvature relationship for a beam can be utilised to couple the kinematics and dynamics of the well bore propagation problem. This being the case the well known moment curvature relationship can relate the applied moment, M , on the drill string beam to its deflection in terms of path curvature, ρ (reciprocal of radius of curvature, $1/R$), and beam material, E (Young's Modulus), and cross sectional properties, I (second moment of area), hence:

$$\rho = \frac{1}{R} = \frac{\partial \alpha}{\partial x} = \frac{M}{EI} \quad 1$$

If the steering mechanism applied moment M reacted by the drill string can be computed in terms of the weight on the bit, F_w , and the steering mechanism actuator force F_a (zero in the case of a point the bit system, assumption 10) then from equation 1 the resulting path curvature, $\partial \alpha / \partial x$, can be evaluated. From a knowledge of the steering mechanism, if the normal reaction force on the bit can be evaluated, then from the bit properties the well propagation rate in the direction normal to the bit can be found, and from assumption 7 assumed to be the

same as the collar axis well propagation rate magnitude and direction.

$$\dot{x}|_{bit} = \frac{Q}{A_b} F_n \approx \dot{x}|_{collar} \quad 2$$

Where Q is the volume removal rate per unit force normal to the bit, F_n , and A_b is the assumed cutting area of the bit. Note that equation 2 represents a very simple bit/rock interaction model. In the literature the rock/bit interaction models are generally much more complex [14,15] and take into account the type and geometry of the bit as well as the formation rock type. These rock/bit interactions are generally used in conjunction with similarly complex sub-models for other parts of the system, and hence from a computational matching point of view it makes sense to use this level of complexity for the rock/bit interaction. However, for the fundamental well propagation model developed here where every aspect of the system is deliberately simplified equation 2 captures the fundamental behaviour of the bit and is computationally matched to the rest of the model. The product of the bit normal propagation rate, $(\partial x / \partial t)$, and the path curvature, $\partial \alpha / \partial x$, at the datum (the 1st stabiliser on the drill string) will give the rate of change of beam deflection, and it is assumed the angular rate of rotation of the inertial datum relative to a global reference frame.

$$\alpha + \alpha_o = \int \frac{\partial \alpha}{\partial x} \frac{\partial x}{\partial t} dt \quad 3$$

This integration gives the transformation angle (α_o is the indefinite integral constant corresponding to the initial transformation angle) enabling the bit normal and tangential machining rate vectors to be transformed into the well propagation rates in the global fixed coordinate system.

$$\begin{bmatrix} x \\ y \end{bmatrix}_g = T \begin{bmatrix} x \\ y \end{bmatrix}_l \quad 4$$

Where T is a 2x2 matrix of trigonometric functions of α transforming between the local datum inertial reference frame and the globally fixed reference frame (for the 3D case T becomes a 3x3 matrix of trigonometric functions of the two transformation angles α and γ). The previous discussion summarises the essentials of the simplified approach used in creating the directional drilling tool well propagation model. What follows is a description of how this approach can be expressed as a set of coupled ordinary differential equations and then linearised into state space format. From the system matrix of the state space formulation some fundamental observations can be made about the stability of well bore propagation in general. This process is discussed in a general way for 2D and 3D derivations and then a specific tool

mechanism is used to derive a 2D well propagation model. The three basic governing equations for a 2D well bore trajectory model can be stated as follows:

$$\dot{x} = QF_n / A_b \quad 5$$

$$\dot{y} = \beta QF_t / A_b \quad 6$$

$$\dot{\alpha} = \dot{x} \cdot \frac{M}{EI} \quad 7$$

Where β in equation 6 is the anisotropy factor, which relates the volume removal rate per unit force normal to the bit to the lateral machining rate of the bit. The moment from the steering mechanism reacted by the drill string can be expressed as a function of the tangential and normal reactions on the bit, F_n and F_t respectively, hence:

$$\dot{\alpha} = \dot{x} \cdot \frac{1}{EI} [C_\alpha^1 F_n + C_\alpha^2 F_t] \quad 8$$

Where C_α^1 and C_α^2 are coefficients of the tool specific function relating the moment M to the normal and tangential forces. Therefore, it can be seen that equations 5, 6 and 8 are coupled through the normal and tangential bit reactions F_n and F_t . Therefore the bit reaction forces need to be made state variables of the system, implying that for a directional drilling tool 2D trajectory model there are five state variables, these being the inertial datum propagation parallel to the collar axis, x , the lateral inertial datum propagation tangential to the collar axis, y , the rotation of the datum reference frame relative to the global reference frame, α , and the normal and tangential bit reaction forces F_n and F_t respectively. The bit reaction forces F_n and F_t are functions of the weight on bit, actuator force, rotation angle between the tool and global reference frames and the bit lateral displacement, hence:

$$F_n = f(F_w, F_a, \alpha, y) \quad 9$$

$$F_t = f(F_w, F_a, \alpha, y) \quad 10$$

Noting that F_w , F_a and α are themselves functions of time then equations 9 and 10 can be partially differentiated with respect to time giving the last two of the five coupled ordinary differential equations describing the 2D directional drilling well propagation response, hence:

$$\frac{\partial x}{\partial t} = QF_n / A_b \quad 11$$

$$\frac{\partial y}{\partial t} = \beta QF_t / A_b \quad 12$$

$$\frac{\partial \alpha}{\partial t} = \frac{\partial x}{\partial t} \cdot \frac{1}{EI} [C_\alpha^1 F_n + C_\alpha^2 F_t] \quad 13$$

$$\frac{\partial F_n}{\partial t} = \frac{\partial F_n}{\partial F_w} \frac{\partial F_w}{\partial t} + \frac{\partial F_n}{\partial F_a} \frac{\partial F_a}{\partial t} + \frac{\partial F_n}{\partial \alpha} \frac{\partial \alpha}{\partial t} + \frac{\partial F_n}{\partial y} \frac{\partial y}{\partial t} \quad 14$$

$$\frac{\partial F_t}{\partial t} = \frac{\partial F_t}{\partial F_w} \frac{\partial F_w}{\partial t} + \frac{\partial F_t}{\partial F_a} \frac{\partial F_a}{\partial t} + \frac{\partial F_t}{\partial \alpha} \frac{\partial \alpha}{\partial t} + \frac{\partial F_t}{\partial y} \frac{\partial y}{\partial t} \quad 15$$

Equations 11 to 15 represent a non-linear coupled set of ordinary differential equations. In order to put these equations in state space form they must be linearised. It can be seen that in equation 13 one of the non-linearities is that there are two terms involving products of the state variables, namely x and F_n , and x and F_t respectively. In order to remove this non-linearity the linearising assumption can be made that the ROP, $\dot{x}$, is constant, i.e. $\dot{x} \rightarrow U$. From equation 11 it can be seen that this assumption also implies that the normal bit reaction force, F_n , is constant also and is given by the expression $F_n \rightarrow A_b U / Q$. Therefore, with the assumption of constant rate of penetration equations 11 and 14 can be uncoupled from the others and a reduced system of three coupled ordinary differential equations is obtained, hence:

$$\frac{\partial y}{\partial t} = \beta QF_t / A_b \quad 16$$

$$\frac{\partial \alpha}{\partial t} = \frac{U}{EI} \left[\frac{A_b U C_\alpha^1}{Q} + C_\alpha^2 F_t \right] \quad 17$$

$$\frac{\partial F_t}{\partial t} = \frac{\partial F_t}{\partial F_w} \frac{\partial F_w}{\partial t} + \frac{\partial F_t}{\partial F_a} \frac{\partial F_a}{\partial t} + \frac{\partial F_t}{\partial \alpha} \frac{\partial \alpha}{\partial t} + \frac{\partial F_t}{\partial y} \frac{\partial y}{\partial t} \quad 18$$

Equation 18 is still a problem as it contains derivatives of the state variables on the right hand side of the equation of the same order as those on the left hand side. To get round this problem it is possible to discretise equations 16 to 18 as follows. The continuous form of a state space realisation of a system is defined as:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad 19$$

The equivalent discrete state space realisation is of the form:

$$x(n+1) = \Phi x(n) + \Lambda u(n) \quad 20$$

It can be seen that equation 20 is recursive in that the next state vector values $x(n+1)$ depend on the previous sample state vector values and inputs $x(n)$ and $u(n)$ respectively. The state space system and output matrices in equation 20 are denoted by the symbols Φ and Λ respectively to differentiate them from their equivalent continuous versions in equation 19. With reference to figure 2, equation 18 can be put in recursive discretised form by taking a first order forward finite difference of the partial derivatives with respect to time using the linearising operating point $t(0)$:

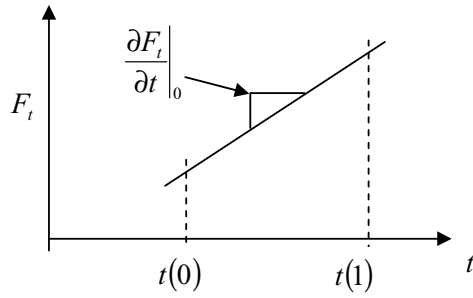


Figure 2: finite difference discretisation

Hence, the partial derivatives $\partial F_w/\partial t$, $\partial F_a/\partial t$, $\partial \alpha/\partial t$ and $\partial y/\partial t$ in equation 18 can be substituted for using forward finite difference approximations to give:

$$\frac{F_t^1 - F_t^0}{\Delta t} = \frac{\partial F_t}{\partial F_w} \left[\frac{F_w^1 - F_w^0}{\Delta t} \right] + \frac{\partial F_t}{\partial F_a} \left[\frac{F_a^1 - F_a^0}{\Delta t} \right] + \frac{\partial F_t}{\partial \alpha} \left[\frac{\alpha^1 - \alpha^0}{\Delta t} \right] + \frac{\partial F_t}{\partial y} \left[\frac{y^1 - y^0}{\Delta t} \right] \quad 21$$

Cancelling the common $1/\Delta t$ factor and equating terms:

$$F_t^1 = \frac{\partial F_t}{\partial F_w} \bigg|_0 F_w^1 + \frac{\partial F_t}{\partial F_a} \bigg|_0 F_a^1 + \frac{\partial F_t}{\partial \alpha} \bigg|_0 \alpha^1 + \frac{\partial F_t}{\partial y} \bigg|_0 y^1 \quad 22$$

$$F_t^0 = \frac{\partial F_t}{\partial F_w} \bigg|_0 F_w^0 + \frac{\partial F_t}{\partial F_a} \bigg|_0 F_a^0 + \frac{\partial F_t}{\partial \alpha} \bigg|_0 \alpha^0 + \frac{\partial F_t}{\partial y} \bigg|_0 y^0 \quad 23$$

Therefore equation 21 can be written in a recursive discretised form as follows:

$$F_t(n+1) = \frac{\partial F_t}{\partial F_w} \bigg|_0 F_w(n) + \frac{\partial F_t}{\partial F_a} \bigg|_0 F_a(n) + \frac{\partial F_t}{\partial \alpha} \bigg|_0 \alpha(n) + \frac{\partial F_t}{\partial y} \bigg|_0 y(n) \quad 24$$

Similarly equations 16 and 17 can be written in recursive discretised form enabling the system to be written as a reduced set of three coupled linearised ordinary differential equations as follows:

$$y(n+1) = \beta Q F_t(n) \quad 25$$

$$\alpha(n+1) = \frac{U}{EI} \left[\frac{A_b U C_\alpha^1}{Q} + C_\alpha^2 F_t(n) \right] \quad 26$$

$$F_t(n+1) = C_{F_t}^1 F_w(n) + C_{F_t}^2 F_a(n) + C_{F_t}^3 \alpha(n) + C_{F_t}^4 y(n) \quad 27$$

where: $C_{F_t}^1 = \frac{\partial F_t}{\partial F_w} \bigg|_0$ $C_{F_t}^2 = \frac{\partial F_t}{\partial F_a} \bigg|_0$ $C_{F_t}^3 = \frac{\partial F_t}{\partial \alpha} \bigg|_0$
 $C_{F_t}^4 = \frac{\partial F_t}{\partial y} \bigg|_0$

Equations 25, 26 and 27 can be put into discrete state space matrix format as follows:

$$\begin{bmatrix} y \\ \alpha \\ F_t \end{bmatrix}_{n+1} = \begin{bmatrix} 0 & 0 & \beta Q / A_b \\ 0 & 0 & U C_\alpha^2 / EI \\ C_{F_t}^4 & C_{F_t}^3 & 0 \end{bmatrix} \begin{bmatrix} y \\ \alpha \\ F_t \end{bmatrix}_n + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & C_\alpha^1 / EI \\ C_{F_t}^1 & C_{F_t}^2 & 0 \end{bmatrix} \begin{bmatrix} F_w \\ F_a \\ A_b U^2 / Q \end{bmatrix}_n \quad 28$$

Where $A_b U^2 / Q$ is constant but has been included in the input vector u . Equation 28 is now in the standard recursive state space form as per equation 20 where the state vector is given by $x \rightarrow \{y, \alpha, F_t\}$ and the input vector is given by $u \rightarrow \{F_w, F_a, A_b U^2 / Q\}$. The coefficients $C_\alpha^1, C_\alpha^2, C_{F_t}^1$ and $C_{F_t}^2$ can be obtained from consideration of the specific steering mechanism. The coefficients $C_{F_t}^3$ and $C_{F_t}^4$ can be evaluated by noting $F_t \rightarrow f(M, F_w, F_a)$ and that in turn $M \rightarrow M(\alpha) \& M(y)$, hence the moment curvature relationship of the drill string can be utilised to obtain expressions for $M(\alpha)$ and $M(y)$ as follows. Consider figure 3, which shows a general schematic of a steering mechanism where small angles have been assumed and the off axis deflection in the mechanism from the 1st stabiliser to the bit is treated as negligible.

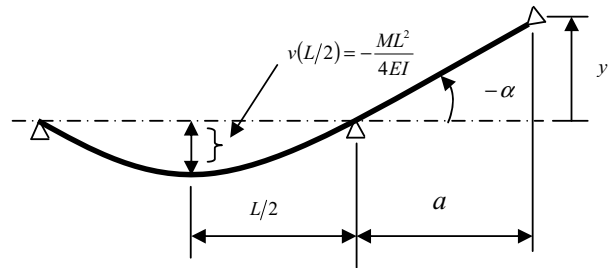


Figure 3: Schematic relating beam flexure to bit lateral deflection alignment

From consideration of figure 3 the end slope of the drill string 'beam' at the 1st stabiliser can be expressed in terms of lateral bit displacement as follows:

$$\alpha = -y/a \quad 29$$

Treating the slope of the drill string from the 1st stabiliser to the mid span of the drill string 'beam' as constant and noting the standard result for the mid span deflection of a simply supported beam subject to a uniform moment, the deflection at the mid span of the drill string beam as a function of the lateral bit displacement can be deduced as: