

Analysis of Tensioner Induced Coupling in Serpentine Belt Drive Systems

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ABSTRACT

A primary concern in the design of serpentine belt drive systems is resonant strand vibrations induced from engine excitation. Two analysis approaches to investigate the system vibrational response have been reported in the literature. The first, denoted as the "decoupled analysis" approach, employs longitudinal belt stiffness and takes into account only pulley rotation and tensioner displacement as system degrees of freedom. Transverse belt vibration (normal to belt travel) on all belt strands is decoupled from the analysis. The second, denoted as the "coupled analysis" approach, combines transverse tensioner strand belt motion with pulley rotation and tensioner displacement. Transverse belt vibration on strands between fixed pulleys remains decoupled from the system.

This paper provides apparently the first cross comparison of these two analysis techniques on three distinct serpentine belt system configurations reported in the literature. Two of the belt drive systems in this study involve numerical models only, and a third published work involves experimental results was used to confirm the accuracy of both analysis techniques. It was observed that the coupled and decoupled analyses did not agree well for low order modes, and only the coupled formulation was able to predict experimentally determined mode shapes. Both analyses indicated that natural frequencies associated with rotational modes were insensitive to crankshaft speed. Natural frequencies associated with transverse belt span modes as computed with the coupled analysis were less sensitive to changes in crankshaft speeds.

INTRODUCTION

An engine front-end subsystem utilizing a single V-ribbed belt is known as a serpentine belt drive due to the long and elaborate path that the belt must follow. The advantages of using a serpentine belt drive setup over multiple V-belts include compactness, ease of belt replacement, length of belt life, and the need to employ only a single tensioner mechanism. The function of the belt tensioner is to maintain constant tractive tension throughout the entire belt drive system in the presence

of belt wear, assembly variation, and deviation in belt length due to changes in accessory torques, belt speed, and belt temperature. The simplest type of tensioner is comprised of an idler pulley pinned to a rigid moment arm which pivots about a fixed point where a coil spring provides the tensioning load.

A primary concern in the design of serpentine belt drive systems are resonant strand vibrations induced from engine excitation. Two analysis approaches to investigate the system vibrational response have been reported in the literature. The first, denoted as "decoupled analysis", employs longitudinal belt stiffness and takes into account only pulley rotation and tensioner displacement as system degrees of freedom. Transverse belt vibration (normal to belt travel) on all belt strands is decoupled from the analysis. The second, denoted as the "coupled analysis", combines transverse tensioner strand belt motion with pulley rotation and tensioner displacement. Transverse belt vibration on strands between fixed pulleys remains decoupled from the system.

The basis for the work that has been done in the field of serpentine belt drives is research on the vibration characteristics of axially moving material. Mote [1] studied the vibrational characteristics of band saws and found that the band saw natural frequencies depend upon the relative motion of the band pulley axis. Modeling and analyzing serpentine belt drive systems with a dynamic tensioner was first accomplished by Ulsoy et al. [2] in which they used a mathematical model to examine the transverse vibration and stability of coupled belt-tensioner systems. Gasper and Hawker [3] and Hawker [4] developed a system of governing equations for a serpentine belt drive system and introduced a solution technique that resulted in the eigenvalues and eigenvectors of the complete system which was validated by experimentation. Hwang et al. [5] derived a nonlinear model that governs the longitudinal response of the belt spans in correlation with the rotational response of the crankshaft and accessory pulleys. Solution of the equilibrium equations leads to the tension-speed relationship for the serpentine belt drive system. The overall equations of motion are then linearized about the equilibrium state allowing the

rotational mode characteristics to be obtained from the associated eigenvalue problem. Kraver et al. [6] extended the modal vibration analysis to include a viscous belt span and coulomb tensioner arm damping. In order to validate the model, their results were compared against Hwang et al. [5] and found to be in good agreement. Using dry friction damping within their model, as opposed to the more commonly used viscous damping, Leamy and Perkins [7] were able to capture the primary and secondary resonances within the belt drive system.

All of the works mentioned previously assume that the linear response of belt drives is composed of the superposition of independent transverse and longitudinal modes. However, as shown by Beikmann et al. [8-11], there exists a linear coupling between the transverse and rotational modes in the spans adjacent to the automatic tensioner. This linear coupling is created by the rotational degree of freedom of the tensioner arm. In addition, there exists a nonlinear coupling mechanism between rotational and transverse modes arising from the finite stretching of the belt. This coupling can become greatly magnified under conditions leading to internal or autoparametric resonance. Beikmann also introduced a tensioner support constant η which is an indicator of the system's ability to maintain tractive tension (despite load and speed variations), and an indicator of the stability of the reference equilibrium state. Zhang and Zu [12, 13] employed the three pulley model developed by Beikmann to extend the understanding of serpentine belt drive behavior.

Belt bending stiffness, which is assumed to be negligible in the previous works, was studied extensively by Wasfy and Leamy [14] and Kong and Parker [15, 16]. In these works, belt bending stiffness is introduced into the models which require additional techniques to solve for the vibration characteristics.

Parker [17] formulated an efficient method for calculating the eigensolutions and dynamic responses of coupled serpentine belt drive systems. The speed of solution is drastically reduced and the numerical problems that hinder other published methods are eliminated. However, by coupling the rotational and transverse motions in the spans adjacent to the tensioner, the system of equations becomes significantly more complex than modeling the motions as uncoupled. As a result, the solution techniques required to solve these models demand considerably more computational power.

This paper provides apparently the first cross-comparison of decoupled and coupled analysis techniques on three different serpentine belt systems. The decoupled and coupled analysis approaches are taken from the representative studies of Hwang et al. [5] and Parker [17], respectively. These particular papers were chosen based on the fact that each captures the essential features of the assumptions inherent in each analysis approach, and each paper provides a distinct

serpentine belt drive sample application suitable for cross-comparison. Experimental results from a third serpentine belt drive system reported by Beikmann [8] will be used to confirm the accuracy of both analysis techniques. A parametric engine speed study will be presented to assess the impact of each analysis method on the predicted natural frequencies and associated mode shapes of each belt drive system.

SYSTEM GEOMETRY

Figure 1 shows the geometry of the serpentine belt drive system employed in both decoupled and coupled analysis approaches. The system is comprised of n pulleys with radii R_j , $j = 1, n$, connected by a single serpentine belt represented by n belt strands between adjacent pulleys. The rotational axes of the $n-1$ pulleys are fixed, while the n th tensioner pulley is attached to a movable spring-loaded tensioner arm. Pulley 1 is attached to the engine crankshaft rotating at fixed angular velocity ω_1 , and the belt is thus driven by the crankshaft at a fixed linear speed V .

The absolute rotation of the j th pulley is defined as

$$\Theta_j(t) = \omega_j t + \theta_j(t) \quad (1)$$

where pulley angular velocity ω_j is given by

$$\omega_j = V / R_j \quad (2)$$

with

$$V = R_1 \omega_1 \quad (3)$$

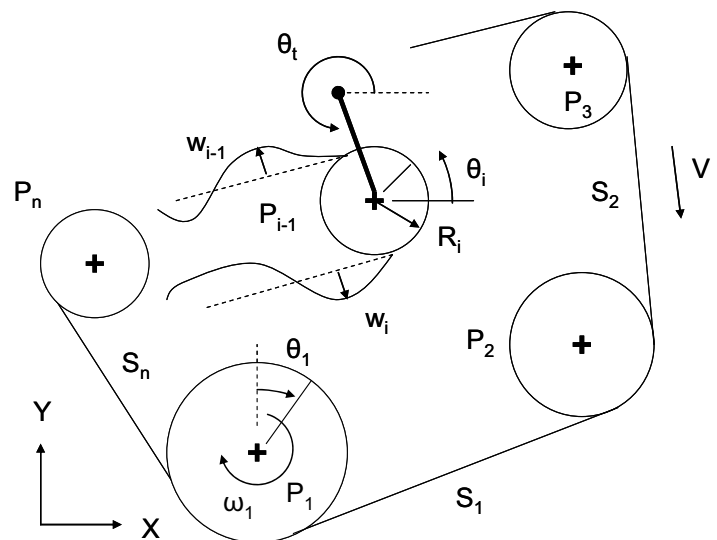


Fig. 1 Belt drive system geometry

Assuming crankshaft angular velocity ω_1 is constant, phase angle $\theta_j(t)$ represents the angular motion of the j th pulley as viewed by an observer rotating at the respective (constant) angular velocity of the j th pulley.

In other words, $\theta_j(t)$ represents the motion of the j th pulley as observed by a stroboscope tuned to the respective frequency ω_j . Transverse belt displacement on the i th belt strand is represented by a function $W_i(x_i, t)$, $i = 1, n$, where x_i is the spatial coordinate as measured between the two connected pulleys. Assuming crankshaft angular velocity ω_1 is constant, $W_i(x_i, t)$ represents the transverse motion of the i th belt stand moving with an axial velocity of V .

Under conditions of fixed crankshaft angular velocity ω_1 and no external disturbances, the belt drive system assumes an equilibrium configuration $\theta_e \equiv [\theta_2 \ \theta_3 \ \dots \ \theta_n \ \theta_t]^T$, $W_i(x_i, t) = 0$, where phase angles take on constant values and strand transverse motions are zero. Small disturbances from equilibrium arise primarily from crankshaft and camshaft torsional excitations, and this paper examines the natural frequencies and associated mode shapes of the linearized belt drive system about a specified equilibrium configuration.

DECOUPLED FORMULATION

The theoretical formulation for the decoupled analysis of the serpentine belt drive system is taken directly from Hwang et al. [5] and is outlined here in condensed form for completeness. The assumptions used in developing the governing equations include:

- The belt does not slip on the pulleys.
- The belt is uniform, perfectly flexible, and stretches in a quasi-static manner.
- Transverse belt response on all strands are decoupled from longitudinal belt response.
- The crankshaft motion and any torque inputs from accessories are prescribed (either zero or determined from experiments).
- The tensioner executes small motions about some steady state position. Moreover the tensioner mechanism is designed to be dissipative and is the dominant source of dissipation. This dissipation is assumed to be linear viscous damping, and dissipation in the belt and fixed pulleys is assumed to be negligibly small.

The linearized equations of motion for the decoupled system take on the form

$$\mathbf{M} \mathcal{D}^2 \mathbf{z} + \mathbf{C} \mathcal{D} \mathbf{z} + \mathbf{K} \mathbf{z} = \mathbf{0} \quad (4)$$

where the degrees of freedom $\mathbf{z} \equiv [\epsilon_2 \ \dots \ \epsilon_n \ \epsilon_t]^T$ represent the set of small pulley phase motion and tensioner arm rotation about an equilibrium configuration, with (time) differential operator $\mathcal{D} \equiv d/dt$. Mass, damping, and stiffness matrices $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ depend primarily upon strand stiffness, component polar moment of inertia, tensioner rotational stiffness, and

equilibrium strand tensions, all of which depend on crank angular velocity ω_1 .

The equilibrium system configuration θ_e and system vibrational response given by equation (4) are both relative to a reference tension (constant on all strands) obtained by setting $V = 0$.

Defining $\mathbf{z} = \mathbf{Z} \exp(i\lambda t)$, the natural frequencies λ and mode shapes $\mathbf{Z}$ of the system can be found in the usual manner from the eigenvalue problem

$$[\lambda^2 \mathbf{M} - i\lambda \mathbf{C} + \mathbf{K}] \mathbf{Z} = \mathbf{0} \quad (5)$$

The $m = 1, \infty$ natural frequencies associated with *all* belt strand transverse vibration (including those attached to the tensioner) are assumed completely decoupled from equation (5) and are given by Abrate [18] as

$$\lambda_{\text{belt}} = (m\pi/L_j)[(P_j - \eta\rho V^2)/\rho]^{1/2} \times [1 - (1-\eta)V^2/c^2](1 + \eta V^2/C^2)^{-1/2} \quad (6)$$

where, for the j th strand, L_j is the belt strand length, ρ is belt strand density, and P_j is the equilibrium strand tension at engine speed ω_1 . Parameter η depends upon the relative stiffness of the pulley supports and axial belt stiffness, and $c = (P_j / \rho)^{1/2}$.

COUPLED FORMULATION

The theoretical formulation for the coupled analysis of the serpentine belt drive system is taken directly from Parker [17] and is outlined here in condensed form for completeness. Key assumptions preserved in this coupled formulation include:

- The belt properties and belt speed are uniform.
- Belt bending stiffness is negligible.
- Damping is not modeled.
- Belt-pulley wedging and belt slip at the belt-pulley interfaces are not considered.

Referring to Figure 1, the transverse belt strand displacements $W_{i-1}(x, t)$, $W_i(x, t)$ attached to the tensioner pulley are now assumed coupled to pulley rotations and tensioner pulley translation. Transverse strand vibration is governed by the partial differential equation

$$\rho (\partial^2 W_j / \partial t^2) - 2\rho V (\partial^2 W_j / \partial x \partial t) - (P_j - \rho V^2) (\partial^2 W_j / \partial x^2) = 0 \quad j = i-1, i \quad (7)$$

Boundary conditions on each strand require zero belt transverse displacement at points attached to the fixed pulleys and displacements which are constrained to move with the center motion of the tensioner pulley.