

Implications of Quality Loss Function in Unified Methodology – LTB Case with Target

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ABSTRACT

Another methodology has been proposed by Sharma and Ragsdell to bring about similarity among the three cases smaller-the-better, nominal-the-best, and larger-the-better by introducing a term called the “target-mean ratio” and proposing a unified formula for quality loss. The new methodology has some implications that need to be addressed. This paper attempts to study the implications and effects of the new methodology on the field of quality engineering. This paper presents an implied classification of LTB characteristics according to Taguchi on the basis of a target value at infinity and also discusses the classification of LTB characteristics based on the new methodology. A new concept of “Complementary Characteristic” is also suggested. It is suggested that whether a given LTB characteristic or its complementary characteristic is considered for one and the same case, the quality loss must be equal for both the characteristics. It is then shown mathematically that with the use of the new methodology any LTB characteristic and its complementary characteristic have the same mean-squared deviation (*MSD*) or quality loss when the target is set according to $\alpha = 2$, or at 100%, or at any other value, without a loss of generality and consistency. In this way, it is argued that the common methodology is better and more suitable for the purpose of computing quality loss. It is also studied and deliberated upon as to what should be the most appropriate value of ‘ α ’, the “target-mean ratio”.

INTRODUCTION

Two types of performance characteristics have been discussed in the literature on quality engineering [1], [2], [3]. It is said that the characteristics having the maximum theoretically possible target of 100% are not larger-the-better (LTB) characteristics [2]. Also, it is described that the characteristics having infinity as the target value do

not really have a target value and are LTB characteristics.

Sharma and Ragsdell discussed that it is plausible to have a target value of infinity and, therefore, not really assign a value to it [4]. It has been further argued that when the target is known as in the case of the efficiency, or yield, or percent non-defective, then they can also be LTB characteristics. It has also been discussed that when the target of 100% efficiency is extremely difficult or impossible to achieve, then the target should not be considered as infinity. According to Sagan [5], infinity is larger than what we can imagine. Because of the assumption of infinity as the target, some of the characteristics are not presently permitted to be LTB characteristics. Subsequent derivation of formulae also supports the theory that there are some characteristics which do not fall under the category of LTB characteristics.

A little consideration shows that in Taguchi's loss function, the approximated loss is based on two reasons [6]: the variation (denoted by standard deviation) of performance about some mean, and the mean performance away from the target, also called bias by El-Haik [7]. (denoted by the distance by which the mean performance is away from the target). In Taguchi's approach the cases of smaller-the-better (STB) and nominal-the-best (NTB), both these causes have been clearly shown to affect the mean-squared deviation, *MSD*, and in turn the quality loss. In contrast, in Taguchi's existing approach for LTB characteristics, it is not clearly shown as to how variation of the performance affects *MSD* or quality loss.

However, Maghsoodloo showed that the *MSD* for LTB cases can be approximated by Equation (1) and Equation (1a) [8]. The *MSD* (or quality loss) given for the STB and NTB cases is exact, while it is only approximate for the LTB case.

$$MSD = \frac{1}{\bar{y}^2} \left[1 + \frac{3\sigma^2}{\bar{y}^2} \right] \quad (1)$$

Where $\bar{y}$ = mean performance, and σ = standard deviation of performance. A better approximation can be given by the following equation:

$$MSD = \frac{1}{\bar{y}^2} \left[1 + \frac{3\sigma^2}{\bar{y}^2} - \frac{4\hat{\mu}_3}{\bar{y}^3} + \frac{5\hat{\mu}_4}{\bar{y}^4} \right] \quad (1a)$$

Where the third central moment,

$$\hat{\mu}_3 = \frac{1}{n} \sum (y_i - \bar{y})^3$$

And the fourth central moment,

$$\hat{\mu}_4 = \frac{1}{n} \sum (y_i - \bar{y})^4$$

Where, in turn, y_i represents the performance value of each product in the lot. Equations (1) and (1a) for the LTB case clearly show how variation of the performance affects the *MSD* or quality loss.

In practice, any performance value infinitely large is not targeted. One view can be that the infinity as target is notional. However, there is no need for the target to be infinity notionally as well. In many cases, performance characteristics higher than a certain value would add little value to the quality of the product. For example, additional tensile strength of paper for general purposes would be of little help, what is required is wear-resistance, and durability. In the LTB case, it is assumed that the larger the value of a parameter, the better it is. Ideally, if the performance value approaches infinity, then the loss approaches zero. This is an ideal case, and thus does not match reality. The target should not be infinity because that is unachievable. Therefore, the new formulation for the quality loss function for the LTB case has been developed.

Taguchi's loss function is given in Equation (4). Fowlkes and Creveling [1], Taguchi, Chowdhury, and Wu [9], Venkateswaren [10], and Sharma and Ragsdell [4] may be referred to for a detailed discussion on the derivation.

If y is some value of a given parameter, and m is the target, then the loss function $L(y)$ is given as follows:

$$L(y) = k(y - m)^2 \quad (4)$$

In Figure 1 Δ_0 is defined as the point of intolerance, i.e., the deviation from the target that causes an average customer to take action. It is assumed that the corresponding dollar loss caused due to a defective component is A_0 . A_0 is also defined as the cost of corrective action. When the deviation of performance from the target of a product is Δ_0 and the corresponding loss is A_0 , then for NTB and STB, k is defined in Equation 5.

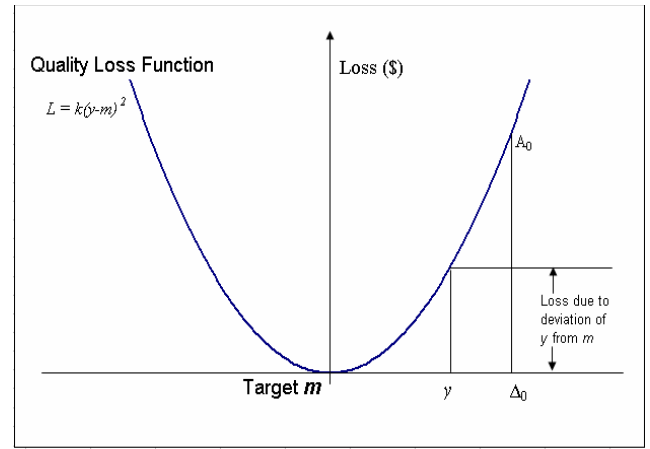


Figure 1. Taguchi Loss Function

$$k = \frac{A_0}{\Delta_0^2} \quad (5)$$

Therefore, for NTB and STB the Taguchi's loss function takes the form as in Equation (6).

$$L(y) = \frac{A_0}{\Delta_0^2} (y - m)^2 \quad (6)$$

For LTB, the quality loss function is defined as in Equation (7).

$$L(y) = A_0 \Delta_0^2 \left(\frac{1}{y_i} \right)^2 \quad (7)$$

The term $(y - m)^2$ is called the mean-squared deviation (*MSD*). For a group of n products, if the performance readings are $y_1 = y_1, y_2, y_3, \dots, y_n$, then the *MSD*, for the group of n products can be derived as shown in Equation (8).

$$MSD = \sigma^2 + (\bar{y} - m)^2 \quad (8)$$

A ratio, α , called the target-mean ratio is introduced. The advantage of defining the target equal to α multiplied by the mean performance is that as the performance improves, the target also changes. Therefore, the target is not constant for a long time; rather it might change after some time. It may remain unchanged for a short period of time.

By setting α = target / mean performance, Equation (9) is obtained.

$$\alpha = \frac{m}{\bar{y}} \text{ or } m = \alpha \cdot \bar{y} \quad (9)$$

$$MSD = \sigma^2 + \bar{y}^2 (\alpha - 1)^2$$

This equation is used interchangeably to encompass all three cases.

Smaller-The-Better

As shown in Figure 2, in the case of STB, the target is zero. Therefore, by setting, $m = 0$ in Equation (9), Equation (10) is obtained.

$$\alpha = 0;$$

$$MSD = \sigma^2 + \bar{y}^2 \quad (10)$$

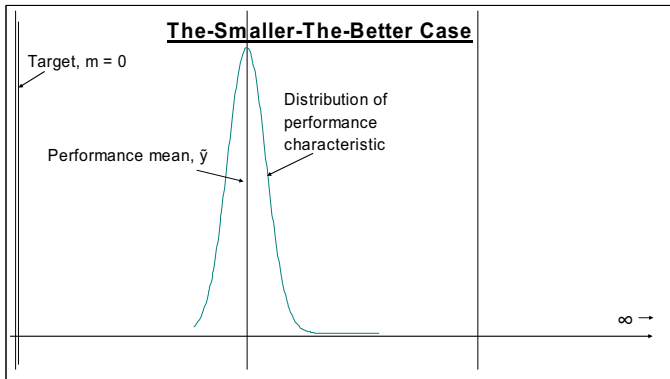


Figure 2. *Smaller-the-Better Case*

Nominal-The-Best – Performance on Target

By setting, $m = \bar{y}$ for the NTB approach Equation (11) is obtained.

$$\alpha = 1;$$

$$MSD = \sigma^2 \quad (11)$$

Nominal-The-Best – Performance Not on Target

When the performance is not on target, then either Equation (12) or (13) can be used. Please refer to Figure 3.

$$MSD = \sigma^2 + (\bar{y} - m)^2 \quad (12)$$

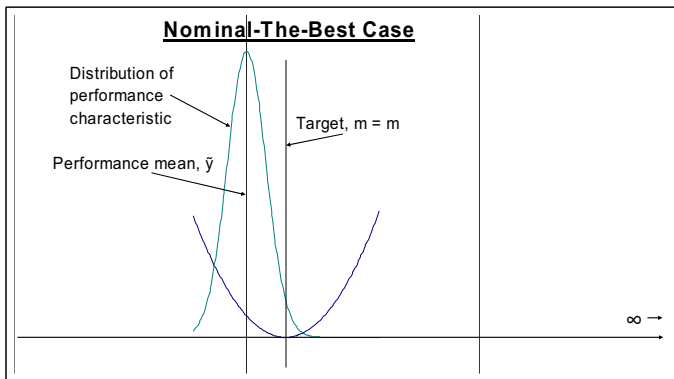


Figure 3. *Nominal-the-Best Case*

By setting, $m = \alpha \cdot \bar{y}$, Equation (14) is obtained.

$$MSD = \sigma^2 + \bar{y}^2 (\alpha - 1)^2 \quad (13)$$

Larger-The-Better

As shown in Figure 4, in the case of LTB, α needs to be significantly greater than unity but need not be a very large number or infinity.

$$MSD = \sigma^2 + \bar{y}^2 (\alpha - 1)^2 \quad (14)$$

If $\alpha > 1$, say, $\alpha = 1.5$

$$MSD = \sigma^2 + 0.25 \bar{y}^2 \quad (15)$$

This is the LTB case with the target 1.5 times the mean performance.

If $\alpha > 1$, say, $\alpha = 2$

$$MSD = \sigma^2 + \bar{y}^2 \quad (16)$$

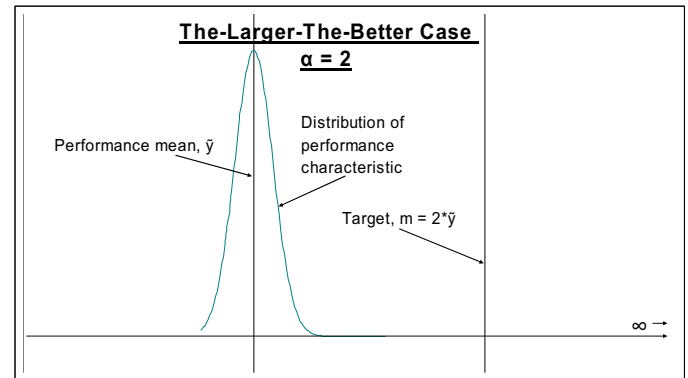


Figure 4. *Larger-the-Better Case*

This is the LTB case with the target double the mean performance. This particular situation in LTB is equivalent to the STB case [4] because the quality losses for both are equal. The only difference is that the target is not zero but is equidistant from the mean on the other side. Also, the target is not assumed to be infinity or very large, though the case is LTB. The advantage of using this approach is that one can achieve comparable results in all three cases.

Having introduced the new methodology, it is now important to discuss the implications of this methodology on quality engineering. First, a classification of LTB characteristics is presented. Then, a new concept of “Complementary Characteristic” is suggested and analyzed with regard to each type of LTB characteristic. Finally, what should be the most appropriate value of ‘ α ’, the “target-mean ratio,” is studied and discussed.

CLASSIFICATION OF LTB CHARACTERISTICS

This section presents a method of classifying LTB performance parameters on the basis of the target value. This classification helps in understanding the organization of the LTB type of characteristics and their relationship with the target value. A classification of LTB characteristics based on Taguchi's methodology as well as according to new methodology is given following a short description of coefficient of performance.

Coefficient of performance (COP) should be reviewed first in order to appreciate its position in the classification