

Optimal Robust Design Optimization with Application to a Piezoelectric Brake

Massimiliano Gobbi, Paolo Guarneri and Gianpiero Mastinu

Politecnico di Milano

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ABSTRACT

A robust optimization approach has been applied to the design of a piezoelectric brake. The force generated by the piezoelectric actuator is transmitted to the pad shoe through a lever. The optimal design of the lever is crucial for obtaining the desired performance of the brake. Increasing the stiffness and reducing the mass of the lever is the key problem for such kind of mechatronic brake. A trade off between mass and stiffness of the lever must be obtained. Multi-objective programming (MOP) has been applied in order to achieve the best compromise. In addition to MOP, the optimal robust design method has been applied to perform the optimal design not only by considering the performance of the system (the stiffness and mass of the lever) but also by taking into account the robustness (the sensitivity to the uncertain system parameters).

This approach has allowed the proper design of the mechanism for the piezoelectric brake by taking into account different cross sections and materials.

INTRODUCTION

The application of piezoelectric actuators (PAs) for braking systems has been proposed in the literature [1], [2]. These actuators can generate large forces and have frequency response suitable for ABS control. They also do not need power supply in order to keep the force constant due to the capacitive load (obviously a voltage must be applied). PAs work on the piezoelectric phenomenon and they generate displacement, the force generation is obtained by coupling the actuators to a reaction structure. The force generated depends on the stiffness of the reaction structure. The maximum displacement of the PAs is about 100 μm and a mechanism (a lever) is needed to transmit the force to the pad shoe. The mechanism must have high stiffness in order to avoid large compliance which can reduce the brake efficiency. It is also important to reduce the mass of the mechanism. A compromise solution must take into account both the stiffness and the mass of the lever. The design of such a complex system is performed by applying robust optimization techniques [3], [4]. The proper theory, which has been adopted to perform the present optimization, is based on multi-objective

programming [5]. The design solution (defined by a set design variables) is selected among the Pareto-optimal solutions which represent the set of the best compromise solutions. This approach is defined in a deterministic framework but actual engineering problems must be solved in a stochastic framework. A mechanical system is, in fact, subject to variations and uncertainties related to the operating conditions, manufacturing process, material properties. Usually the robustness is mostly related to the variations on the design variables and not only to external disturbances. So a problem solved by means of a deterministic approach can be not robust (i.e. the performance is worse than expected, a constraint violation may occur, ...). The method adopted in this paper is based on [7] where the optimization process is integrated with a probabilistic approach which allows the designer to solve two important tasks, the choice of the best compromise solution and the evaluation of the tolerable level of risk of the design failure (performance deterioration or constraints violation). The robust optimization problem is related to the design of the lever which transmits the PA force to the pad shoe. In particular, the problem is the definition of the optimal cross section shape of the two lever arms. The lever is loaded at one end by the PA and at the other by the shoe pad reaction (the normal force acting on the disc surface). The loading condition depends on the PA load-displacement curve which is considered subject to stochastic variations. Also the lever material properties are uncertain. The objective functions to be optimized are the mass (to be minimized) and the force transmitted to the pad shoe (to be maximized). The constraints are the maximum stress, the lateral buckling and the local buckling.

DESCRIPTION OF THE PIEZOELECTRIC BRAKE

The PA force is transmitted to the pad shoe by a lever, see Fig. 1 a). Due to the small displacements of the PA, the deflections of the lever arms reduce the maximum force acting on the disc. The design problem is the definition of the optimal shape of the lever in order to maximize the transmitted forces (i.e. the braking force) and to minimize the lever mass.

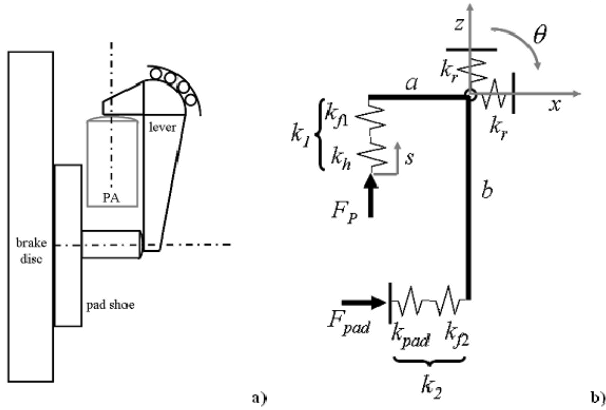


Fig. 1, a) Piezoelectric brake with lever. b) Mechanical model of the brake.

The model used to compute the force transmitted F_{pad} is shown in Fig. 1 b). The model parameters are reported in Tab. 1. The two lever arms are loaded at the free ends by the forces F_P and F_{pad} and are modelled as two cantilever beams, whose bending stiffness k_{r1} e k_{r2} can be easily computed (see Fig. 2).

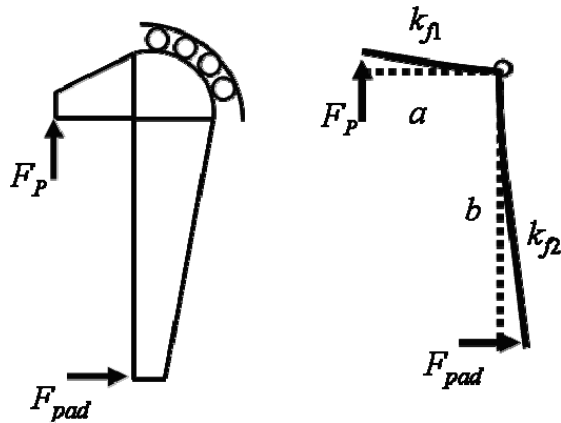


Fig. 2, Model of the lever.

Tab. 1
Mean values and standard deviation of the model parameters [6].

Parameter	Mean value	Standard deviation
Pad stiffness, k_{pad} [N/m]	$255 \cdot 10^6$	-
Contact hertzian stiffness, k_h [N/m]	10^9	-
Radial stiffness, k_r [N/m]	$1.6 \cdot 10^9$	-
PA blocking force, F_m [kN]	76	-
PA maximum displacement, s_m [μ m]	120	5% of mean value
Yield stress, R_S [MPa]	300	5

If the standard deviation is not present, the parameter is deterministic.

The lever is hinged to the brake caliper via rolling elements which have some compliance (the small

strains due to hertzian contact pressures adsorb the PA stroke). The radial stiffness k_r has been estimated [8] and included into the model. Also the contact deformation between the PA and the lever arm have been considered and the hertzian contact stiffness k_h has been properly computed [9]. By writing the equilibrium equations, the three degrees of freedom x , z and θ (see Fig. 1 b)) have been written as function of the external force F_P (the PA force)

$$\begin{cases} x = F_P \frac{a}{b} \frac{1}{k_r} \\ z = \frac{F_P}{k_r} \\ \theta = F_P \frac{a}{b^2} \frac{k_r + k_2}{k_r k_2} \end{cases} \quad (1)$$

The PA force F_P as a function of the PA stroke s is

$$F_P = k_1(s - z - a\theta) \quad (2)$$

By substituting z and θ of (1) in (2), the stroke s is given by

$$\begin{aligned} s &= \frac{F_P}{k_1} + z + a\theta = \\ &= F_P \left(\frac{1}{k_1} + \frac{1}{k_r} + \frac{a^2}{b^2} \frac{k_r + k_2}{k_r k_2} \right) = F_P C \end{aligned} \quad (3)$$

This equation describes the relation between the stroke and the force. This relation must match the force-stroke characteristic curve of the PA. This curve is described by the linear equation (see [10])

$$F_P = F_m \left(1 - \frac{s}{s_m} \right) = F_m - k_p s \quad (4)$$

where F_m is the blocking force (i.e. the maximum force that the PA can generate), s_m is the maximum stroke (i.e. the maximum PA displacement) and $k_p = F_m/s_m$ is the PA stiffness. By substituting the stroke s given by (3) in (4), the maximum force generated by the PA coupled to the mechanical brake system can easily be obtained

$$F_P = \frac{F_m}{1 + Ck_p} \quad (5)$$

Graphically the maximum force generated can be obtained by intersecting the PA force-stroke curve (4) with the force-stroke relation of the brake system (3) as shown in Fig. 3. It can be noted that by increasing the

system stiffness (i.e. reducing C) the maximum generated force increases.

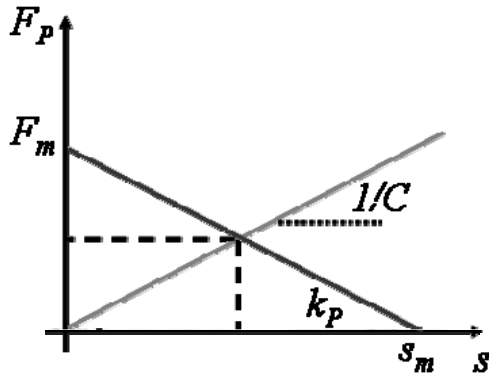


Fig. 3, The two stroke-force curves (black=PA, grey=brake).

Finally the maximum force transmitted to the pad shoe is given by

$$F_{pad} = F_P \frac{a}{b} = \frac{F_m}{1 + Ck_p} \frac{a}{b} \quad (6)$$

The maximum force F_{pad} which can be generated by the piezoelectric brake is a function (through the C term) of the lever arms geometry.

The optimization is performed by considering the lines of action of F_P and F_{pad} as known. The PA force magnitude is considered stochastic because the maximum displacement s_m is given with a certain level of tolerance [10]. The loads applied to the lever are stochastic and also the yield stress of the lever material is considered as stochastic.

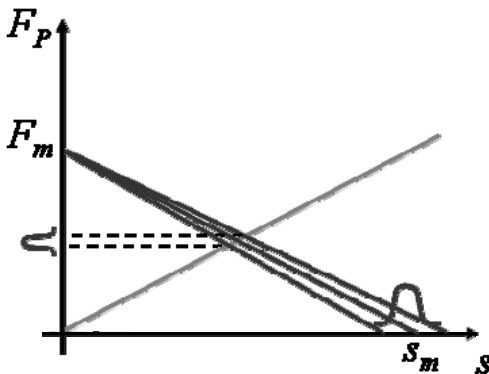


Fig. 4, Effect of the stochastic variation of the parameter s_m on the generated force.

MULTI-OBJECTIVE OPTIMIZATION

The multi-objective *deterministic* optimization problem is formulated as follows [5]:

$$\begin{aligned} \min_{x \in X} (f_1(x), f_2(x), \dots, f_m(x)) \quad & X \subset \mathbb{R}^n \text{ subject to} \\ g_j(x) \leq 0 \quad & j = 1, \dots, q \end{aligned} \quad (7)$$

where f_i are the m objective functions, x is the vector of n design variables whose domain is X and g_j are the q inequality constraints. The solution which minimizes the vector of the objective functions is not unique if the objective functions are conflicting. The definition of the optimal solution for this type of problems has been proposed by Pareto [5]. A design solution x^* is Pareto-optimal if does not exist any design solution x such that

$$\begin{aligned} f_i(x) &\leq f_i(x^*) \quad i = 1, \dots, m \\ \exists k \quad &f_k(x) < f_k(x^*) \end{aligned} \quad (8)$$

The definition of Pareto optimality leads to a *set* of optimal solutions (called Pareto-optimal set) which represent the best compromise solutions. The designer must select a solution belonging to this set by defining properly the design variables.

In a *probabilistic* framework, the problem is described by a mathematical model which takes into account the random variations of some quantities. These stochastic quantities can be parameters (which will be denoted by p) or design variables. The model parameters are quantities whose values are not selected by the designer who instead selects the expected (mean) values of the design variables. The variations of the parameters and the design variables are propagated to the objective and constraint functions which assume a stochastic nature, too. The solution can be found by translating the stochastic problem into an equivalent deterministic problem. The formulation adopted in this paper is the so called K_β formulation (β -efficient problem) [11]

$$\begin{aligned} \text{Given the probabilities } \beta_1, \beta_2, \dots, \beta_m \\ \min_{x \in X} (u_1(x, p), u_2(x, p), \dots, u_m(x, p)) \text{ such that} \\ \text{prob}[f_i(x, p) \leq u_i(x, p)] \geq \beta_i \quad i = 1, \dots, m \end{aligned} \quad (9)$$

The probabilities are fixed and the values u_i (that guarantee that the risk is less than β_i) are minimised. There is a probability $(1-\beta_i)$ that the performance of the Pareto-optimal solutions are worse than expected. If the objective functions can be described by means of normal distributions, the problem can be written as (K_α formulation, β -efficient problem with normal distribution)