

Brake Squeal Analysis: A New Method for Predicting Limit Cycles Using the Constrained Harmonic Balance Method

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ABSTRACT

In this paper we study the nonlinear steady-state dynamic of a reduced finite element model of a brake system subjected to flutter instability. Both the pad and the disc are retained in the model and contact is enforced by a penalty method. A numeric Constrained Harmonic Balance Method (CHBM) is employed to compute the limit cycle responses. Results and computational performances are compared with those from implicit time integration. To demonstrate the effectiveness of the proposed method, the influence of modal damping for the brake system is carried out.

INTRODUCTION

Brake squeal phenomenon has been a wide issue for scientific and engineering communities for a long time. Extensive reviews on this subject have been done [1-2]. The development of computers and finite element softwares in the past decades has improved the fine understanding of brake squeal event. Moreover, it allows quicker developments with reduced costs compared to physical tests. Many configurations can be numerically checked out before doing validation tests. Brake squeal is an autonomous system due to a mode coupling effect [3-5], and stability analyses are performed by computing complex eigenvalues of the coupled system. Unstable modes are detected for sets of parameters and areas of stability versus instability are derived. However these linear studies do not allow the calculation of the corresponding nonlinear dynamical steady-state solutions (called limit cycles) when the brake system is unstable. Time integration domain is the most general and straightforward method for computing dynamical responses of any mechanical system. Unfortunately its computational cost is prohibitive when looking for balanced responses for large nonlinear autonomous systems [6]. A well known alternative is the use of harmonic balance method in the frequency domain. Computation time is affordable and parametric studies are then possible. The presented method in the paper has the originality to work with nonlinear autonomous systems. This paper is divided in five sections. In a first

part the nonlinear brake model is presented. Secondly a stability analysis is performed on the nonlinear model. The third part deals with the presentation of the new Constrained Harmonic Balance Method (CHBM). The fourth part is devoted to the comparison between the CHBM and a time marching method. Finally the last part presents the influence of modal damping both on stability analysis and on limit cycles.

THE BRAKE MODEL

For the sake of simplicity, we only consider the pad and the disc in the current study. Corresponding finite element models derived from a finite element software package are reduced with a Craig and Bampton technique [7]. Hence each condensed component consists in nine contact nodes at the disc/pad interface together with the first fifty eigenmodes of the considered body. Four extra-nodes are retained onto the backplate of the pad where piston force is applied. It is clear that using only nine contact nodes at the friction interface is probably not fair for a fine comprehensive physical description however the first goal of the study is to develop a nonlinear method and such a model is appropriate. Figure 1 shows the finite element models with the retained nodes:

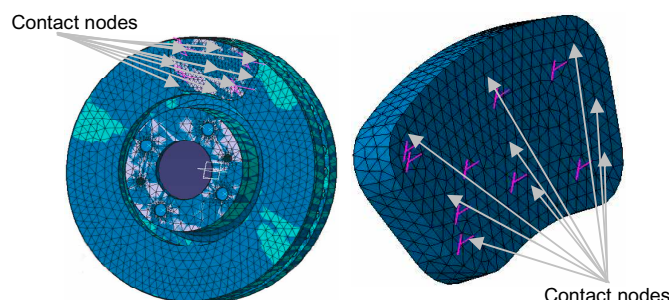


Figure 1: Finite element models of the disc and the pad

Resulting reduced model is a 158 degrees-of-freedom (dofs) system and is written as:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \mathbf{F}_{nl} = \mathbf{F}_{ext} \quad (1)$$

where $\mathbf{x}$ is the response displacement, the dot denotes the derivation respect to the time, $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the reduced mass, damping and stiffness matrices, respectively. $\mathbf{F}_{nl}$ represents the nonlinear contact and friction force vector whereas $\mathbf{F}_{ext}$ is the external force vector. Basically, two contact definitions can be used to simulate connection between bodies. The use of Lagrangian multipliers is achieved to obtain exact contact condition by adding extra equations into system (1) but computation time become larger. The penalty method is easier to implement and does not need extra equations but it is at the cost of an inexact contact condition. However the use of appropriate contact stiffnesses leads generally to acceptable results [2]. The frictional material is a mixture of many components and is about one thousand times softer than a steel disc, thus its deformation under loading is preponderant and nonlinearity occurs. Contact stiffnesses are chosen to fit the first and the third order of pad compression curves obtained from tests. Contact forces computation takes the form:

$$\mathbf{F}_{contact} = \begin{cases} k_l(x_i - x_j) + k_{nl}(x_i - x_j)^3 & \text{if } (x_i - x_j) > 0 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

with k_l and k_{nl} linear and nonlinear stiffnesses contact respectively and x_i and x_j the displacements of interface nodes i and j respectively. A permanent sliding state is considered and a constant friction coefficient is assumed. Friction forces take the following form, using the classical Coulomb's law:

$$\mathbf{F}_{friction} = \mu \mathbf{F}_{contact} \text{sign}(v_r) \quad (3)$$

with μ the friction coefficient and v_r the relative velocity between the both bodies. No tangential friction has been considered in order to simplify the model. Damping matrix $\mathbf{C}$ is built by solving the generalized eigenvalue problem of the frictionless undamped coupled system (1). An $\mathbf{M}$ orthonormal eigenvector matrix Φ is obtained and expressing equally damped modes related to physical dofs yields to:

$$\mathbf{C} = \Phi^{-T} \mathbf{D} \Phi^{-1} \quad (4)$$

where $\mathbf{D}$ is a diagonal matrix containing damping on the both modes involved in instability.

STABILITY ANALYSIS

Stability analysis is the first step for studying nonlinear systems subjected to flutter instability. It firstly consists in performing a static loading of the brake model for a given set of parameters. One has to remember that the disc has a nonzero velocity and thus friction forces occur in the static step. The computation takes the form:

$$\mathbf{K}\mathbf{x}_0 + \mathbf{F}_{nl} = \mathbf{F}_{ext} \quad (5)$$

Then, the stability of the static position $\mathbf{x}_0$ is achieved by linearizing system (1) about the static position $\mathbf{x}_0$ and using the perturbation technique. Writing:

$$\mathbf{x} = \mathbf{x}_0 + \bar{\mathbf{x}} \quad (6)$$

and substituting equation (6) into equation (1) leads to:

$$\mathbf{M}\ddot{\bar{\mathbf{x}}} + \mathbf{C}\dot{\bar{\mathbf{x}}} + \mathbf{K}(\mathbf{x}_0 + \bar{\mathbf{x}}) + \mathbf{F}_{nl}(\mathbf{x}_0 + \bar{\mathbf{x}}) \approx \mathbf{F}_{ext} \quad (7)$$

Developing $\mathbf{F}_{nl}$ as a Taylor series and only keeping the first order leads to:

$$\mathbf{F}_{nl}(\mathbf{x}_0 + \bar{\mathbf{x}}) \approx \mathbf{F}_{nl}(\mathbf{x}_0) + \mathbf{K}_{NL}^L \bar{\mathbf{x}} \quad (8)$$

$\mathbf{K}_{NL}^L$ is the linearized Jacobian matrix of $\mathbf{F}_{nl}$ at the equilibrium position and takes the form:

$$\mathbf{K}_{NL}^L = \begin{bmatrix} 0 & -\mu k_l & 0 & 0 & \mu k_l & 0 \\ 0 & k_l & 0 & 0 & -k_l & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu k_l & 0 & 0 & -\mu k_l & 0 \\ 0 & -k_l & 0 & 0 & k_l & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

$$\text{with } k_l = \left. \frac{\partial F_{nl}}{\partial x} \right|_{x_0}$$

The reader remarks that $\mathbf{K}_{NL}^L$ becomes asymmetric due to the contribution of friction forces. The resulting linearized system is written under the form:

$$\mathbf{M}\ddot{\bar{\mathbf{x}}} + \mathbf{C}\dot{\bar{\mathbf{x}}} + (\mathbf{K} + \mathbf{K}_{NL}^L)\bar{\mathbf{x}} = \mathbf{0} \quad (10)$$

One observes that the damping term due to the linearization of friction has not been taken into account in equation (10) for simplicity of the numerical model. Then an eigenvalue computation of the previous equation is carried out. Since the linearized stiffness matrix is asymmetric, computed eigenvalues are complex and result in:

$$\lambda = a + i\omega \quad (11)$$

where a is a real part corresponding to the growth rate of the amplitude and ω is a natural frequency of the corresponding mode. A negative real part indicates that the mode is stable and any perturbation onto the system will not affect its equilibrium. However, a positive real part that is equivalent to a negative damping implies an unstable mode. A perturbation about the static equilibrium will generate growing oscillations until reaching the limit cycle. Figure 2 (a-b) displays the evolution of normalized real parts and frequencies along a varying parameter, i.e. the normalized friction coefficient.

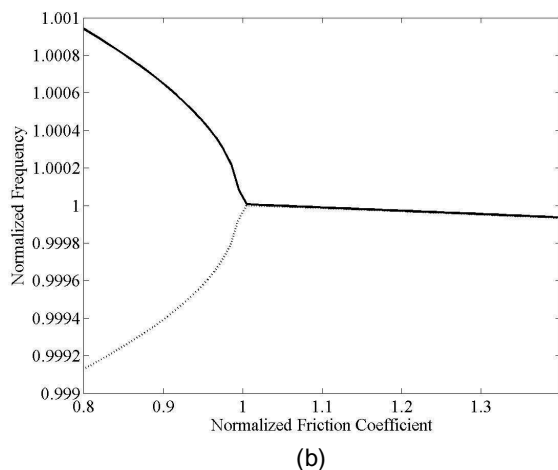
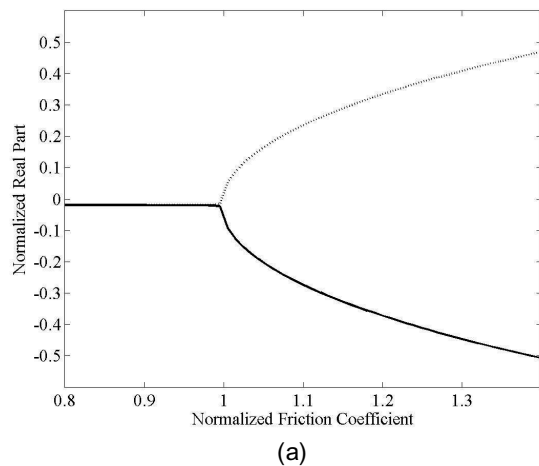


Figure 2: Evolutions of real part (a) and frequencies coalescence (b) (dotted lines : unstable modes)

Two distinct eigenfrequencies of the undamped, frictionless coupled structure become closer together while μ is increasing until lock-in at the coalescence point. They remain equal beyond this point. The coalescence is perfect since both modes are equally damped. The corresponding real parts split into two branches with one going towards the positive real part half space and becoming positive at the Hopf bifurcation point μ_0 indicating an unstable mode whereas the other one remains negative indicating a stable mode. Although the complex eigenvalues analysis gives information about the stability of the system, it is not sufficient to get the dynamic steady-state of the complete nonlinear system and using the linearized system (10) for computing limit cycles will not be accurate since it will only be valid near the stationary sliding state. In the next part of the paper, a new method that computes limit cycles of nonlinear autonomous systems will be presented and discussed.

NON LINEAR DYNAMIC ANALYSIS

THE HARMONIC BALANCE METHOD

Harmonic balance methods are computationally efficient regarding time marching methods for computing steady-state responses of nonlinear dynamic systems. They are used to solve from one dof nonlinear systems to large complex structures where flutter onset are of much interest. Extensive reviews on these methods have been done in [8]. In the following, the most general variation of the harmonic balance method is presented and particularities of our methodology that works for autonomous nonlinear systems are undertaken.

EQUATION OF MOTION

The major hypothesis when using harmonic balance methods is the periodicity of the computed response of a considered nonlinear system. Thus the approximate nonlinear periodic solution $\mathbf{x}(t)$ is replaced as:

$$\mathbf{x}(t) = \sum_{k=0}^{N_h} \mathbf{U}_k^C \cos(k\omega t) + \sum_{k=1}^{N_h} \mathbf{U}_k^S \sin(k\omega t) \quad (12)$$

where $\mathbf{U}_k^C$ and $\mathbf{U}_k^S$ are the cosine and the sine Fourier coefficient vectors respectively, ω is the natural frequency of the limit cycles and k is the harmonic number. Reader has to notice that ω is an unknown parameter since we are in presence of a self-excited system and the unstable frequency from the stability analysis slightly differs from the one of the dynamic steady-state solution. N_h is the harmonic number retained in the solution. Substituting equation (12) into the time equation (1) leads to the following residue in the time domain:

$$R_{N_h}(t) = \sum_{k=0}^{N_h} \left[\left(\mathbf{K} - (k\omega)^2 \mathbf{M} \right) \mathbf{U}_k^C + (k\omega \mathbf{C}) \mathbf{U}_k^S \right] \cos(k\omega t) + \sum_{k=0}^{N_h} \left[\left(\mathbf{K} - (k\omega)^2 \mathbf{M} \right) \mathbf{U}_k^S - (k\omega \mathbf{C}) \mathbf{U}_k^C \right] \sin(k\omega t) + \mathbf{F}_{nl}(\mathbf{U}_k^{C,S}) - \mathbf{F}_{ext} \quad (13)$$

Using a Galerkin procedure by projecting the residue onto sine and cosine orthonormal basis and gathering the terms corresponding to each harmonic, the equation of motion in the frequency domain is written as:

$$\mathbf{\Lambda} \tilde{\mathbf{U}} + \tilde{\mathbf{F}}_{nl}(\tilde{\mathbf{U}}) = \tilde{\mathbf{F}}_{ext} \quad (14)$$

with

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{K} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_1 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{\Lambda}_{N_h} \end{bmatrix} \quad (15)$$

and

$$\mathbf{\Lambda}_k = \begin{bmatrix} -(k\omega)^2 \mathbf{M} + \mathbf{K} & k\omega \mathbf{C} \\ -k\omega \mathbf{C} & -(k\omega)^2 \mathbf{M} + \mathbf{K} \end{bmatrix} \text{ for } k = [1, N_h] \quad (16)$$

$\tilde{\mathbf{U}}, \tilde{\mathbf{F}}_{nl}$ and $\tilde{\mathbf{F}}_{ext}$ are respectively the N_h first Fourier coefficient vectors of displacement, nonlinear forces and external forces. Nonlinear forces Fourier coefficients depend on $\tilde{\mathbf{U}}$ and their determination could be fastidious if doing analytically. To alleviate this problem, the alternate frequency-time method proposed by Cameron and Griffin [9] permits the computation of nonlinear forces in the time domain and then transforms them in the frequency domain by using a Fourier transform:

$$\tilde{\mathbf{U}} \xrightarrow{FFT^{-1}} \mathbf{x}(t) \rightarrow \mathbf{F}_{nl}(\mathbf{x}(t)) \xrightarrow{FFT} \tilde{\mathbf{F}}_{nl} \quad (17)$$

Finally, equation (14) can be solved with any root finding algorithm.

REDUCTION

When a nonlinear system has a significant number of dofs but only a few of them are related to nonlinear components, it is possible to reduce system (14) without loss of accuracy [10]. Linear and nonlinear dofs are reorganized such:

$$\begin{bmatrix} \mathbf{\Lambda}_{ln,ln} & \mathbf{\Lambda}_{ln,nl} \\ \mathbf{\Lambda}_{nl,ln} & \mathbf{\Lambda}_{nl,nl} \end{bmatrix} \begin{Bmatrix} \tilde{\mathbf{U}}_{ln} \\ \tilde{\mathbf{U}}_{nl} \end{Bmatrix} + \begin{Bmatrix} \mathbf{0} \\ \tilde{\mathbf{F}}_{nl} \end{Bmatrix} = \begin{Bmatrix} \tilde{\mathbf{F}}_{ext,ln} \\ \tilde{\mathbf{F}}_{ext,nl} \end{Bmatrix} \quad (18)$$

And then it is possible to solve the reduced equation of motion:

$$\mathbf{\Lambda}_{red} \tilde{\mathbf{U}}_{nl} + \tilde{\mathbf{F}}_{nl} = \tilde{\mathbf{F}}_{red} \quad (19)$$

With

$$\mathbf{\Lambda}_{red} = \mathbf{\Lambda}_{nl,nl} - \mathbf{\Lambda}_{nl,ln} \cdot \mathbf{\Lambda}_{ln,ln}^{-1} \cdot \mathbf{\Lambda}_{ln,nl} \quad (20)$$

$$\tilde{\mathbf{F}}_{red} = \tilde{\mathbf{F}}_{ext,nl} - \mathbf{\Lambda}_{nl,ln} \cdot \mathbf{\Lambda}_{ln,ln}^{-1} \cdot \tilde{\mathbf{F}}_{ext,ln} \quad (21)$$

Once equation (19) is solved, linear displacements are straightforward obtained from (22):

$$\tilde{\mathbf{U}}_{nl} = \mathbf{\Lambda}_{ln,ln} \cdot (\tilde{\mathbf{F}}_{ext,ln} - \mathbf{\Lambda}_{ln,nl} \tilde{\mathbf{U}}_{nl}) \quad (22)$$

This exact reduction diminishes the number of equation from $N \cdot (2N_h + 1)$ to $N_{nl} \cdot (2N_h + 1)$, with N_{nl} the number of nonlinear nodes and has a great interest for large models with only few nonlinear components.

THE CONSTRAINED HARMONIC BALANCE METHOD

This section is devoted to the presentation of the extra constraint equation required in the classical harmonic balance method for resolving nonlinear autonomous systems.

Equation (19) is a cost function that has a minimum when $\tilde{\mathbf{U}}_{nl}$ is a solution of the nonlinear equation system.

As explained by Moirot in his thesis [11], the unicity of the solution is lost for autonomous systems at the Hopf bifurcation point. Hence two solutions coexist for a same set of parameters and only initial conditions drive the solution. Hence, for a non disturbed system at a Hopf bifurcation point the system remains on the unstable static position whereas a little change in initial conditions or any variation of parameters will lead to growing oscillations until the dynamic steady-state solution is reached. The coexistence of two solutions involves the coexistence of two local minima in the optimization domain and the finding root algorithm may converge towards the static or the dynamic solution without any control. In fact, first performed computations always produce the trivial solution that is the static one. Hence there is a need for constraining equation (19) to compute only the solution of interest, which is the dynamic one. The constraining condition will be outlined in the following.