

Designing for Large-Displacement Stability in Aircraft Power Systems

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ABSTRACT

Due to the instabilities that may occur in power systems with regulated loads such as those used in military aircraft, ships, and terrestrial vehicles, many analysis techniques and design methodologies have been developed to ensure stable operation for expected operating conditions. However, many of these techniques are difficult to apply to complex systems and do not guarantee large-displacement stability following major disturbances such as faults, regenerative operation, large pulsed loads, and/or the loss of generating capacity. In this paper, a design paradigm is set forth guaranteeing large-displacement stability of a power system containing a significant penetration of regulated (constant-power) loads for any value of load power up to and including the steady-state rating of the source. Initial investigations are performed using an idealized model of a dc-source to determine the minimum requirements that ensure large-displacement stability. These investigations illustrate that a three-way tradeoff exists between the responsiveness of the source, its transient overload capacity, and the dc-bus capacitance. These results are then extended to a detailed design of a permanent-magnet generator/rectifier source that is large-displacement stable for bounded changes in load power. Formulating the design as an evolutionary optimization problem, the response of system may be improved without compromising stability.

INTRODUCTION

It is well known that power systems with regulated power electronic loads such as those used in ships, submarines, aircraft, and spacecraft are subject to negative-impedance instabilities [1-2]. These instabilities may be attributed to the regulation capabilities of

modern power converters. Specifically, load converters are commonly designed so that the output power is essentially unaffected by perturbations in the input voltage. As a result, the input power remains constant even if the input voltage varies, at least for perturbations within a certain range. For small perturbations, it can be readily shown that this constant-power load appears as a negative resistance within the regulation bandwidth of the converter, which has a potentially destabilizing effect on the overall power system. Consequently, considerable attention has been devoted over time to the study and mitigation of instabilities attributed to this effect.

The analysis of potential instabilities typically begins with a small-signal analysis. A key first step is the development of average-value models that can be linearized and therefore can produce linear state-space models and/or transfer functions relating the output impedance of source converters Z_S and the input admittance of load converters Y_L to the frequency of the voltage/current perturbations. For a single source connected to a single load, small-signal stability is achieved if and only if the Nyquist criterion is satisfied. This criterion is certainly satisfied if the Nyquist contour of $Z_S(\omega)Y_L(\omega)$ remains inside the unit circle for all frequencies leading to the Middlebrook criterion [3], i.e.

$$|Z_S(\omega)Y_L(\omega)| < 1; \forall \omega, \quad (1)$$

which is a sufficient but not necessary criterion for small-signal stability. Although it has been shown that this criterion introduces conservatism [1], it is readily applied and therefore widely used. Specifically, given the source impedance, it is easy to set forth a bound on the load admittance that will ensure the system is stable for small-signal disturbances. The primary argument against the Middlebrook criterion has been that it leads to a

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conservative design. Consequently, alternative stability criteria such as the gain and phase margin criterion and the opposing argument criterion are sometimes used [2]. Although these methods are less conservative than the Middlebrook criterion, they are more difficult to apply for complex systems.

One disadvantage of the Middlebrook, gain and phase margin, and opposing argument criteria is that they do not explicitly deal with uncertainties, parameter variations, and nonlinearities. An admittance/impedance based design approach that explicitly incorporates uncertainty, the wide range of operating points, and the high degree of reconfigurability found in many dc distribution systems has been presented in [1]. There are two stages to this method. The first is a stability criterion that is less conservative and less sensitive to component grouping. The main disadvantage is that it is not conducive to the formulation of design specifications. However, this disadvantage is addressed in the second stage, which allows design specifications to be established for arbitrary stability criteria. Two advantages of this two-stage process are that it is graphically interpretable, and that it can be used to explicitly address operating point and parameter variations. However, the main disadvantages are: (1) guaranteeing small-displacement stability, albeit over a range of operating conditions, does not guarantee large-displacement stability following large disturbances such as pulsed loads, faults, component failures, or simply the connection/disconnection of large loads; and (2) the procedure is substantially more difficult to apply than the Middlebrook criterion. Moreover, the number of potential source/load configurations that may be encountered in complex systems may become intractable. Each and every potential configuration must either be considered in the design phase, or an on-line strategy is needed to evaluate the stability before a new configuration is entered. Neither approach appears practicable at this time for complex systems.

The large-displacement stability of power electronic based systems was recently investigated in [4]. The underlying mathematical framework is based upon concepts developed by Lyapunov. An asymptotically stable equilibrium point has the property that the state of the system will return to the equilibrium point following small disturbances. A region of asymptotic stability (RAS) is a region for which all system trajectories starting within the RAS asymptotically return to the equilibrium point. Lyapunov theory gives conditions that ensure that a region bounded by a Lyapunov function contour is an RAS. The problem of finding the largest Lyapunov function contour that bounds an RAS is considered in [5], wherein the search for such an RAS is formulated as an optimization problem.

In this paper, the source requirements are set forth that guarantee large-displacement stability of a power system containing constant-power loads for any value of load power up to and including the steady-state power rating of the source. A relationship is established between the key source parameters and the RAS. This

relationship is used to establish a design procedure that ensures large-displacement stability for arbitrary changes in load power within the rating of the source. It is shown that a three-way tradeoff exists between the responsiveness of the source, its transient overload capacity, and the bus capacitance. This design procedure is then used to develop a detailed design of a permanent-magnet generator/rectifier source that is large-displacement stable for arbitrary changes in load power so long as the load power does not exceed the steady-state rating of the source. Finally, an optimization is performed on the detailed design to improve the system response. The results obtained from this optimization can be applied to a variety of systems.

SIMPLIFIED SYSTEM STUDY

A simplified model of a power-electronic system is shown in Fig. 1. Therein, the source is depicted as a controllable current source with upper and lower current limits of i_{lim} and $-i_{lim}$, respectively. The source is assumed to have finite bandwidth. That is, the output current i_S cannot be changed instantaneously; instead, it is related to the commanded current i_S^* by the first-order transfer function

$$i_S = \frac{1}{\tau s + 1} i_S^* \quad (2)$$

Throughout this paper, the superscript * is used to denote the commanded value of the indicated variable. There are two parameters associated with this simplified source model: the current limit i_{lim} and response time τ . The load is represented as an ideal constant-power load. That is, the load current i_L is related to the voltage as

$$i_L = \frac{P_L}{v} \quad (3)$$

where P_L is the load power, which is considered to be an input variable or constant parameter. The capacitor in Fig. 1 is needed to limit voltage transients following changes in the load power.

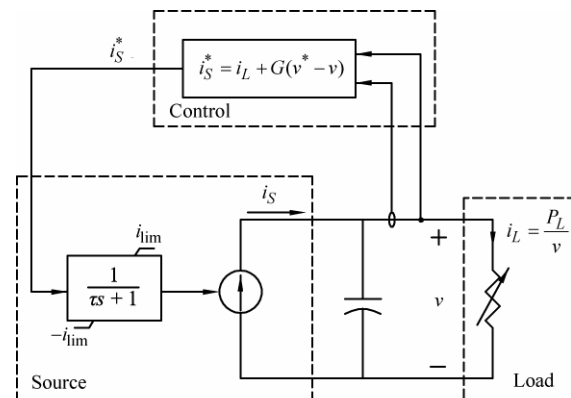


Figure 1 Simplified model of power-electronic-based system.

In order to regulate the output voltage, a feedforward/feedback control scheme is employed. In particular, the commanded source current is set equal to the measured or actual load current plus a term proportional to the voltage error. Specifically,

$$i_S^* = i_L + G(v^* - v) \quad (4)$$

where v^* is the commanded voltage and G is a feedback gain. The output voltage satisfies the differential equation

$$\frac{dv}{dt} = \frac{1}{C}(i_S - i_L) \quad (5)$$

where $v(0)$ is the initial bus voltage.

To make the results of this analysis applicable to arbitrary voltage/current/power levels, it is convenient to normalize the preceding equations by defining rated or base values as: $V_{b,dc}$ - rated dc voltage, $I_{b,dc}$ - rated dc current, $P_b = V_{b,dc}I_{b,dc}$ - rated power, and $Z_{b,dc} = V_{b,dc}/I_{b,dc}$ - base impedance. The per unit (pu) value of a variable is defined as the actual value divided by the corresponding base value, i.e.

$$v_{pu} = \frac{v}{V_{b,dc}} \quad (6)$$

$$i_{pu} = \frac{i}{I_{b,dc}} \quad (7)$$

The pu capacitance and feedback gain are defined as

$$C_{pu} = CZ_{b,dc} \quad (8)$$

$$G_{pu} = GZ_{b,dc} \quad (9)$$

It is readily shown that, with the addition of the subscript pu to all variables/parameters, equations (2)-(5) are identical when expressed in terms of per unit values/parameters. For notational compactness, however, the subscript pu will hereafter be omitted and all variables/parameters are assumed to be expressed in pu unless otherwise noted. Per unit variables/parameters are dimensionless with the exception of τ , which has units of seconds.

Next, it is convenient to define the scaled time $\tau' = \tau/C$, whereupon (5) becomes

$$\frac{dv}{d\tau'} = i_S - i_L \quad (10)$$

In scaled time, the source current satisfies the differential equation

$$\tau' \frac{di_S}{d\tau'} = -i_S + i_S^* \quad (11)$$

provided $|i_S| < i_{lim}$. The relationships among the previous equations may be depicted in block-diagram

form as shown in Fig. 2. The associated parameters and inputs are: G , τ' , i_{lim} , v^* , and P_L . It is assumed that the commanded voltage $v^* = 1$ and the feedback gain $G = 2$.

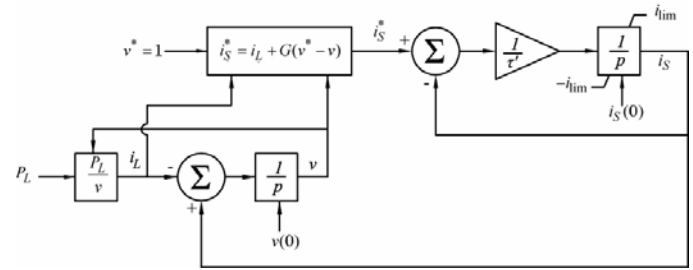


Figure 2 System time-domain block diagram.

The effects of changes to this feedback gain on the system response are addressed in a later section. It should be noted that at rated power, the steady-state source current $i_S = 1$. In general, i_{lim} must be set to a value greater than 1 to maintain stability in the presence of disturbances. The difference between i_{lim} and 1 is subsequently referred to as the transient overload capacity.

It is desirable to examine the tradeoff that exists between the responsiveness of the source (i.e., its bandwidth) and the transient overload capacity needed to maintain stability following a step change in load power from zero to rated (large-displacement stability). Therefore, in the following series of studies, it is assumed that the load power is stepped from an initial value of 0 to 1 with $i_S(0) = 0$ and $v(0) = 1$. The parameters that are varied are τ' and i_{lim} . A prototypical stable response is depicted in Fig. 3 and a prototypical unstable response is shown in Fig 4. In both cases, the horizontal axis is scaled time τ' . In the stable case, the voltage momentarily decreases immediately following the application of the load; however, the source is able to respond by supplying more current whereupon the voltage recovers and settles out to its original value of 1.0. In the unstable case, the source is not able to respond fast enough. In particular, as the voltage decreases, the load demands more current. Since the source has finite bandwidth, it is not able to respond fast enough to the increasing load current causing the bus voltage to decrease further and the load current to increase further. If the bus voltage is still decreasing when the source current limit is reached, the voltage is sure to collapse. When the voltage decreases to zero, the study is terminated since the load current theoretically becomes infinite. In practice, this would not occur since duty cycle constraints would normally limit the load current when the voltage becomes small.

Repeating the previous studies for various values of i_{lim} and determining the minimum response time needed to maintain voltage stability for the given change in load power results in the boundary between stable