

TRU and ATRU Analysis by Matrix Methods

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ABSTRACT

A new method is proposed for the analysis and design of multipulse three-phase rectifiers such as 12- and 18-pulse rectifiers. The method is based on describing the voltage transformation of the multiphase transformer by a $3 \times N$ matrix when N is the number of secondary outputs of the transformer. A key advantage of the method is that the same matrix can be used to describe the current transformation through the transformer as well, avoiding the lengthy and error-prone algebra required in the existing approaches to determine the current responses of multipulse rectifiers. The transformation matrix can be further used to determine harmonic performance of the rectifier, to calculate the rectifier small-signal input impedance, as well as to identify potential triplen harmonic problems. It provides a new, general and effective mathematical tool for the analysis and design of multipulse rectifiers that are widely used in large industrial drives and aerospace systems.

INTRODUCTION

Multipulse rectifiers, such as 12- and 18-pulse rectifiers, are widely used as the front-end converters for large industrial drives [1, 2] as well as airborne variable-speed motor controllers [3, 4]. The harmonic cancellation effects achieved by the phase shifting transformer enable these converters to meet harmonic current limits imposed by power quality standards such as IEEE 519-1992 and DO-160E. In airborne applications, these converters are commonly referred to as multipulse Transformer-Rectifier Units (TRU) and Autotransformer-Rectifier Units (ATRU) depending on whether the phase shifting transformer is an isolation transformer or an autotransformer. Although theoretically the phase shifting transformer can be designed to generate any $3N$ -phase output ($N = 2, 3, \dots$) from balanced three-phase inputs, practical applications are dominated by 12-pulse (six-phase) and 18-pulse (nine-phase) designs because of the increasing complexity of transformer design. Of the two, 18-pulse rectifiers are particularly popular in airborne applications.

In addition to the phase shifting transformer, multipulse rectifiers also differ in their design by the configuration of the rectifier bridge. One configuration is to divide the $3N$ secondary phases into N three-phase groups, and to

feed a three-phase rectifier bridge from each group. The N rectifier bridge dc outputs are connected in parallel[†] through an Inter-Phase Transformer (IPT) to achieve pulse multiplication effects and the associated harmonic cancellation benefits. Use of the IPT is necessary in this case in order to ensure six-pulse operation of each three-phase rectifier bridge as well as to achieve equal current sharing among them. The other configuration is to connect together the outputs of all rectifier bridges directly without using an IPT. This effectively forms a $3N$ -phase rectifier where each phase conducts for $360^\circ/(3N)$ instead of 120° in each half line cycle. Both configurations are practically used and have pros and cons. One particular issue with the configuration not using an IPT is the existence of triplen harmonics in the secondary phase currents, which require a circulation path to be provided by multiple transformer windings, typically in the form of a closed-delta [1, 5].

Operation of multipulse rectifiers is complex due to both the complex transformer design and the highly nonlinear nature of the rectifier bridge. The multiphase transformer lends itself well to description by phasors as far as voltage transformation between primary and secondary windings is concerned. Indeed, a common method to describe multipulse rectifiers is to draw each transformer winding such that its length is proportional to its number of turns and its orientation is the same as the phasor of the voltage induced in the winding when the transformer primary is supplied from balanced three-phase inputs [1]. Such a winding diagram can be used to explain the generation of multiphase outputs as well as to determine the number of turns for each winding. However, such a diagram is of very little use when describing the current relationship, as it applies only to the fundamental components of the current. Adapting the diagram to harmonic currents, which are the primary concern in multipulse rectifier design, is tedious and inefficient because of the presence of many harmonics in the secondary currents and the complex phase shifts created by the transformer and the rectifier bridge at different harmonic frequencies. For this reason, analysis of current transformation is usually a separate step in existing work on multipulse rectifier analysis, involving repeated application of KCL to various nodes in the transformer winding diagram and tedious algebra. The analysis process is ad hoc and

[†]. Series connection is also possible if different secondary three-phase windings are isolated from each other.

prone to mistakes. For these reasons, many works simply resorted to numerical simulation to determine the current relationship in multipulse rectifiers, which lacks the insight an analytical approach provides and is difficult to use in design or in new circuit development.

Motivated by the recent resurgence of interests in multipulse rectifiers for airborne applications, we have been developing new analytical tools and methods for the analysis and design of multipulse TRUs and ATRUs. The airborne applications also raised some new issues which have been ignored or not sufficiently addressed in industrial drive applications, such as input current distortion under unbalanced and/or distorted lines, common-mode voltages and currents, as well as dynamic interactions with the rest of the system. A general approach to describing multiphase transformers is required in order to enable unified treatment of different multipulse rectifier topologies for such diverse analyses, which is the subject of this paper.

The method proposed in this paper describes voltage transformation through a multiphase transformer by a matrix instead of a phasor diagram. The dual relationship between voltage and current transformation through the transformer enables the use of the same matrix to describe the current transformation, completely avoiding the need for a separate analysis of the current relationship. The transformation matrix can be further used to determine harmonic performance of the rectifier, to calculate the rectifier small-signal input impedance, as well as to identify potential triplen harmonic problems.

PHASOR-BASED ANALYSIS

Fig. 1a) shows a simple phase-shifting transformer. The dashed lines represent the magnetic core of the transformer. There are two windings on each limb of the core: a long winding with n_1 turns and a short winding with n_2 turns. Voltages v_{a1} , v_{b1} , and v_{c1} are balanced three-phase input voltages that are phase-shifted from each other by 120° , and voltages v_{a2} , v_{b2} , and v_{c2} are the output voltages that have desired phase shift from the inputs. Fig. 1b) shows a phasor diagram that illustrates the formation of the output voltages and the creation of phase shift by the transformer. Voltages Δv_a , Δv_b , and

Δv_c represent the voltages of the short winding of phase a, b, and c, respectively. The desired phase shift between input and output is denoted as α .

The conventional approach to the analysis and design of phase-shifting transformer is to merge the winding diagram in Fig. 1a) and the phasor diagram in Fig. 1b) into one diagram shown in Fig. 1c) [1]. In the combined winding-phasor diagram, each winding is drawn parallel to its corresponding voltage phasor in the phasor diagram, with its length made proportional to the number of turns of the winding. The windings are connected in the same way as they are in the original winding diagram, and the position of each terminal of the winding graph represents the voltage that can be obtained from windings connected to that terminal.

Such winding-phasor diagram provide a convenient graphical means for determining the winding turns in order to realize certain phase shift. For the example shown in Fig. 1, the following relationship is obtained between the turns of the short and the long winding by applying the law of sine to the small triangle shown in Fig. 1c):

$$\frac{n_2}{\sin \alpha} = \frac{n_1}{\sin(120^\circ - \alpha)} \quad (1)$$

If the desired phase shift is $\alpha = 30^\circ$, the required winding turns ratio can be determined from (1) as follows:

$$\frac{n_2}{n_1} = \frac{\sin \alpha}{\sin(120^\circ - \alpha)} = \frac{\sin 30^\circ}{\sin 90^\circ} = \frac{1}{2}$$

The phase-shifting transformer in a multipulse rectifier serves two related functions: 1) generation of multiphase secondary voltages with proper phase shift from balanced three-phase inputs applied to the primary, and 2) elimination of certain harmonic components in the primary input currents. Although the winding-phasor diagram can be used to analyze the first function, it doesn't provide any direct information about the relationship between the primary and secondary currents of the transformer. In the existing approach, such current

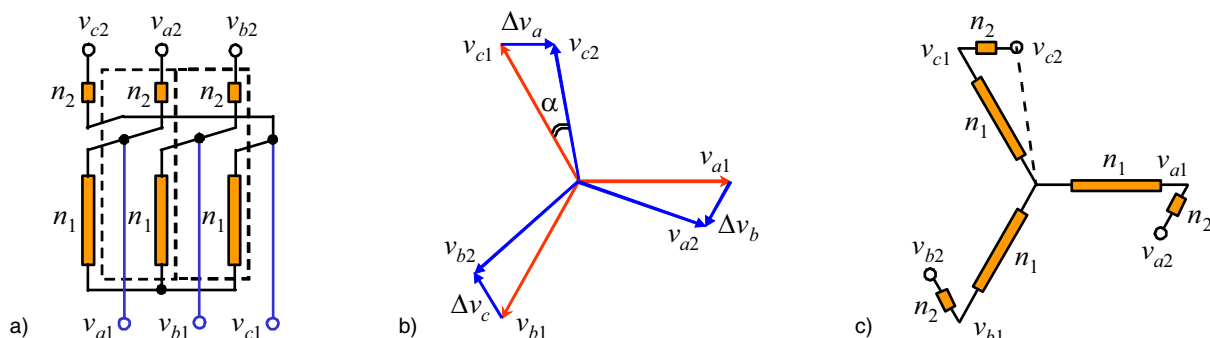


Fig. 1. Phasor-based representation of a phase-shifting transformer. a) Actual winding arrangement around the core, b) voltage phasor diagram, c) windings rearranged according to the voltage phasor each winding produces.

relationship is determined in an additional analysis by repeatedly applying KCL to each node in the winding diagram. Additionally constraints for the currents are obtained by assuming amp-turn balance for each limb of the transformer core. This is a tedious process and has been the most difficult part in the understanding and analysis of multipulse rectifiers.

As an example, consider the “differential-delta” autotransformer [1] depicted in Fig. 2. The transformer is designed to create nine-phase secondary voltages with 40° phase shift. Three different windings are used in this design: six short windings connected to the vertices of the delta, six medium windings extending outwards from the delta, and three long windings that together with the short windings form the delta. Assume the turns ratio of the medium and the long winding to that of the short winding is n_1 and n_2 , respectively. The numerically numbered terminals (1, 2, ..., 9) indicate the nine-phase secondary outputs, while the three alphabetically numbered terminals (a, b, c) indicate three-phase primary inputs. Note that the secondary phases 1, 4, and 7 share the same terminals as the primary phases a, b, and c. Denote the phase voltages by $\{v_a, v_b, v_c\}$ for the primary and $\{v_1, v_2, \dots, v_9\}$ for the secondary, and the corresponding phase currents, which are not shown in the figure, by $\{i_a, i_b, i_c\}$ and $\{i_1, i_2, \dots, i_9\}$ respectively. The reference directions for the current are such that the primary currents flow into the transformer while the secondary currents flow out of the transformer. Additionally, denote the currents through the three longest windings as i_A, i_B , and i_C , respectively, as shown in the figure.

To find the relationship between the primary and the secondary, we consider first amp-turn balance for each limb of the core, which requires that the total amp-turn of windings pointing at the same direction is equal to zero[†]:

$$\begin{aligned} n_2 i_A + (2i_A - i_2 + i_3) + n_1(i_8 - i_6) &= 0 \\ n_2 i_B + (2i_B - i_5 + i_6) + n_1(i_2 - i_9) &= 0 \\ n_2 i_C + (2i_C - i_8 + i_9) + n_1(i_5 - i_3) &= 0 \end{aligned}$$

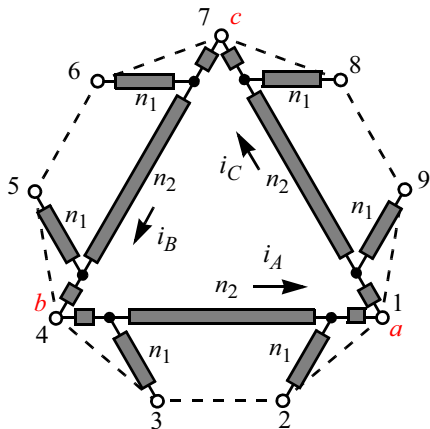


Fig. 2. A nine-phase “differential-delta” phase-shifting autotransformer.

[†]. This is equivalent to assuming that the core is ideal and has infinite permeability so no net amp-turn is required to produce the flux.

From these equations, currents i_A, i_B , and i_C can be solved:

$$\begin{aligned} i_A &= \frac{i_2 - i_3 + n_1(i_6 - i_8)}{n_2 + 2} \\ i_B &= \frac{i_5 - i_6 + n_1(i_9 - i_2)}{n_2 + 2} \\ i_C &= \frac{i_8 - i_9 + n_1(i_3 - i_5)}{n_2 + 2} \end{aligned} \quad (2)$$

Additionally, the following equations are obtained by applying KCL to each of the vertices of the large triangle:

$$\begin{aligned} i_a &= i_1 + i_2 + i_9 - i_A + i_C \\ i_b &= i_3 + i_4 + i_5 + i_A - i_B \\ i_c &= i_6 + i_7 + i_8 + i_B - i_C \end{aligned} \quad (3)$$

Therefore, the phase a input current can be expressed as a function of the secondary currents as

$$i_a = i_1 + i_2 + i_9 + \frac{i_3 - i_2 + i_8 - i_9 + n_1(i_3 - i_5 - i_6 + i_8)}{n_2 + 2} \quad (4)$$

The normalized number of turns n_1 and n_2 can be determined from the winding diagram and the 40° phase shift requirement for the secondary voltages:

$$n_1 = 2 \cos 20^\circ \approx 1.8794$$

$$n_2 = \frac{\sqrt{3} \sin 70^\circ \sin 120^\circ}{\sin 40^\circ \sin 20^\circ} - 2 \approx 4.4115$$

With these numerical values, i_a can be simplified to

$$i_a \approx i_1 + 0.844(i_2 + i_9) + 0.449(i_3 + i_8) - 0.293(i_5 + i_6)$$

Currents i_b and i_c can be determined in a similar way. As can be seen, this process is ad hoc and requires tedious algebra. There also seems to be no connection to the voltage relationship obtained from the winding-phasor diagram, although both are part of the functions of the phase-shifting transformer.

MATRIX REPRESENTATION

Definition of the proposed transformation matrix is best explained by an example. Consider again the 9-phase “differential-delta” autotransformer depicted in Fig. 2. With the notations and current reference directions introduced in the previous section and by assuming ideal operation of the transformer (i.e. ignoring the magnetizing current and leakage inductance), a matrix can be developed as follows to describe the transformer operation. First, note that $v_1 = v_a$, $v_4 = v_b$, and $v_7 = v_c$. The remaining six secondary phases are connected to one of these three phases through a medium and a short winding, such that their voltages can be expressed as a combination of the two winding voltages and the corresponding vertex voltage. For