

Stability Analysis of a Three-Wheeled Motorcycle

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ABSTRACT

In this work the modal analysis of a three-wheeled tilting motorcycle is presented. This new kind of vehicle has two front wheels and a single rear wheel, but is driven like a common motorcycle.

In order to study the stability of the system in straight running, two models have been developed: a simplified motorcycle model, with locked suspensions and rigid and thin tires and a more accurate model having 14 degrees of freedom, in which the stiffness and damping of suspensions and the radial stiffness of tires have been taken into account. In both models the frame has been considered as rigid and the driver was assumed to be fixed to the frame. A linear model with transient behaviour has been employed for describing the tire behaviour.

A reference model of a two wheeler with similar inertial properties has been also developed for comparison.

INTRODUCTION

This paper deals with the stability analysis of a novel motorcycle: a three wheeled tilting vehicle, having two front steering wheels. This work has been carried out in the framework of the European co-funded SIM ("Safety in Motion") project, within the 6th Framework Program; Piaggio MP3 (Fig.1), a vehicle which has recently obtained a high success in the Italian market, has been selected as the reference vehicle for the project. Such vehicle represents an effort to conjugate the advantages of a two wheeler, in terms of dynamics and mobility, with some advantages, in terms of safety, typical of automobiles.

A general description of this vehicle and the analysis of some handling maneuvers have been already discussed in previous works by the authors [1, 2]; the rear body of the vehicle is typical of motorscooter and consists of the engine suspension assembly; on the other hand, the

front architecture is based on a four link mechanism, which allows the vehicle to roll as an ordinary motorcycle.

Even if the driving feel is much like that of motorscooter, the vehicle has some distinctive features, in particular the two front wheels, which are expected to influence its dynamic behavior.



Figure 1. Piaggio MP3.

The dynamic behaviour of two wheelers is a well established subject in the technical literature (see for example [3-9]); however, there is no technical literature on the stability analysis of vehicles similar to MP3.

In order to analyze the stability of this kind of vehicle an accurate model of the vehicle has been developed, starting from technical drawings and experimental data. The model has been obtained by the Lagrange approach, i.e. by deriving the dynamic equation starting from the kinetic energy of the system. The model takes into account the compliance of the suspensions, the geometry and the radial stiffness of tires; the frame has been assumed as rigid. A much simpler model has also been set up having rigid and thin tires and locked suspensions. In addition a model of a similar (i.e. having

similar geometric and inertial properties) two-wheeler has also been obtained, in order to investigate the dynamic behavior with reference to a typical two wheeler.

VEHICLE MODEL

KINEMATIC ANALYSIS - A kinematic analysis of the vehicle architecture has been employed in order to select the Lagrange coordinates, representing the degrees of freedom of the system. With reference to Fig. 2 the following 14 coordinates have been defined: the rear wheel middle plane has been used to define the forward direction $\mathbf{i}$ and the roll angle ϕ ; the pitch angles ϑ_2 and ϑ_3 of the engine-rear suspension assembly and of the main body (i.e. sprung weight) of the scooter have been defined in the same plane; with reference to the front suspension, the steering angle (equal steering of front wheels) δ has been defined, while the strokes of the two front suspensions have been identified by the rotations ϑ_{11} and ϑ_{12} of the lever arms; the angle β represents the angular position of the transverse arms of the four linkage system (it is no longer a degree of freedom of the system, and is determined by the roll angle, if we assume rigid tires and locked suspensions); the angular position of the wheels are represented by χ_{11} , χ_{12} and χ_2 . In addition to the previous quantities, the vehicle configuration is determined by the position (x, y) in the horizontal plane, of the contact point P_2 of the rear tire, the yaw angle ψ and the distance r_2 of the rear wheel center C_2 from the horizontal plane, as measured in the middle plane of the rear wheel.

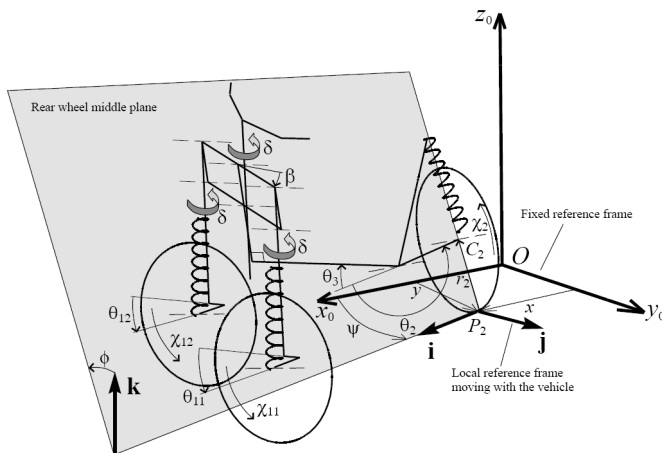


Figure 2. Model schematization and Lagrangian coordinates.

The forward direction is given by the intersection of the rear wheel middle plane with the horizontal plane (ground), while the contact point P_2 is the projection of the point C_2 on the forward direction. The unit vector $\mathbf{i}$ has been defined along the forward direction, while the unit vector $\mathbf{k}$ is normal to the horizontal plane (see Fig. 2). The unit vector $\mathbf{j}$ is obtained by $\mathbf{k} \times \mathbf{i}$; the unit vectors

$\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ represent the local reference frame moving with the motorcycle. The Lagrangian coordinates are grouped into the vector $\mathbf{q}$:

$$\mathbf{q} = [x, y, \psi, \phi, r_2, \vartheta_2, \vartheta_3, \beta, \delta, \vartheta_{11}, \vartheta_{12}, \chi_2, \chi_{11}, \chi_{12}]^T$$

The driver has been assumed as a rigid body, rigidly connected to the vehicle mainframe.

The non linear properties of the suspensions have been experimentally obtained; moreover, in order to estimate the vertical load of each wheel, the tires have been modeled as a linear spring-damper.

A simpler model of MP3 has also been developed, in which the degrees of freedom representing the suspension strokes and tire radial stiffness were not considered.

Finally, a model representing a two wheeler has also been set up, on the basis of a scheme similar to that described in Fig. 2.

TIRE MODEL - Since the interest is focused on the stability of the vehicle in straight running, a linear tire model with transient behavior has been employed. In particular, the following linear equations have been used for the generic tire:

$$\begin{aligned} \frac{l}{u} \dot{F}_y + F_y &= C_\alpha \alpha + C_\gamma \gamma, \\ F_x &= C_x s_x, \end{aligned}$$

where α , γ and s_x are respectively the slip angle, the camber angle and the longitudinal practical slip of the tires. These variables have been obtained by kinematic analysis, starting from the previously defined degrees of freedom of the system. Moreover, l is the relaxation length of the tire, C_α the cornering stiffness, C_γ the camber stiffness and C_x the braking stiffness. In order to estimate the last parameters an experimental characterization was carried out by TNO Automotive within the SIM project. Finally, F_y and F_x are respectively the lateral and longitudinal forces acting on the tire. The moment with respect to the contact point of each tire has been neglected.

EQUATIONS OF MOTION - For all the models the equation of motion of the system have been obtained by the well known Lagrange's equations:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\mathbf{q}}} \right) - \frac{\partial T}{\partial \mathbf{q}} = \mathbf{Q} \quad (1)$$