

A Theoretical Model for the Elastic-Plastic Behaviour of Spot Welded Joints

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ABSTRACT

An analytical procedure for the evaluation of the elastic-plastic behavior of spot welded joints is presented. The procedure is based on a new theoretical model of spot weld region: a circular plate having variable thickness with a central rigid nugget. The closed-form solution allows to describe the displacement of a rigid nugget when an axial orthogonal load is applied on the plate while plasticity and moderately large deflections are present. The goal is to reach a reliable spot weld region model which can be used as the basis to develop a spot weld element in FE analysis even when plasticity and large deflections are in effect. The analytical results obtained by using the new general relations precisely match those obtained modelling spot weld area by FEA.

INTRODUCTION

The interest on structures jointed by spot welds has been recently spreading especially in automotive and railways applications, where structures may contain several thousands of spot welds. These spot welds are subjected to complex multiaxial loads both under service or crash conditions. The stress field close to the spot region is quite complex, mostly due to geometrical irregularities and several local effects at the edge of the spot weld. Moreover, structural behavior and local stiffness evaluation of spot welds show some complexity, due to the difficulties in accurate modelling, using finite element models, the region close to each spot weld, featuring local high stress/strain, associated with very high stress/strain gradients. Furthermore, it is necessary to use models that involve only a few degrees of freedom, since real structures usually contain several spot welds: modelling each of them with accurate and complex FE models would require a major computational effort.

The studies about junction behavior are mainly focused on problems of fatigue cracks or of plastic collapse. Fatigue life estimation has been investigated using different approaches. Generally, a major effort has been made so as to obtain stresses at the spot welds – using finite element models – theoretical approach or both of the above. Often, fracture mechanics has been adopted to evaluate stress intensity factors in natural crack or notch along the nugget circumference, using finite element models in linear elastic conditions [1], in elastic-plastic conditions [2] or analytical solutions for stress intensity factor [3, 4]. Structural stress approach has been introduced to predict fatigue life using stress intensity factor [5, 6]. It should also be noted that in these cases fatigue behavior models are often only applicable and valid to reference joints of simple shape. Otherwise the use of a conventional stress parameter has been proposed demonstrating its effectiveness to predict spot weld fatigue life [7, 8, 9, 10]. In this case, simple models can lead to a good estimation of the forces interchanged between the spot weld and the rest of the structure, and these loads can be related to a conventional stress parameter at the spot weld by simple [7, 8] or complex theoretical models [9, 10].

The interest on the spot weld elastic-plastic behavior is focused on the crash phenomena of structures or low cycle fatigue analysis. In order to obtain a progressive failure of the spot welds, it is desirable to have an optimal failure mechanism instead of others. The request is to maintain a sufficient rigidity for the whole of the energy absorption process in order to avoid a sudden fall in the capacity to absorb the impact loads. All automobile manufacturers who use these assembly techniques know perfectly well that provided the structural behavior of a weld spot is optimal, it will fail with a mechanism commonly referred to as 'nugget pullout failure'. There are many criterions available in

literature proposing ways to choose the spot weld joint characteristics (spot diameter, sheet thickness), in order to obtain the desired failure mechanism of spot weld [11, 12]. Moreover, failure mechanisms of spot welds have been studied using experimental observations and numerical analyses. For example Lin et al. [13, 14] developed a failure criterion for spot welds under combined loads based on a perfectly plastic limit load approach. In this approach, the weld nugget was assumed as a rigid inclusion. The failure mechanism of lap shear specimens has also been characterized from Lin et al. [15] using experimental observations, 2-D finite element analyses and two-dimensional elasticity theories. An experimental characterization of mechanical behavior and failure of spot weld joints under mixed mode has been conducted in [16]. Langrand and Combescure [17] studied non-linear and failure behavior of spot welds using an equivalent joint model and an experimental procedure for model characterization. The equivalent model allows to predict the elastic-plastic behaviour of spot weld up to the load peak response.

On the other hand, at this time no definitive approach has been outlined to tell the behavior of a complex structure featuring several spot welds stressed by different load typologies. It is particularly hard to accurately determine how the deformation of the sheets transfers the loads on the single spot while another one is failing, or what the mechanism of failure will be like.

Such a complex problem cannot be resolved using experimental criterions, but it must be faced by using FE solutions. Several approaches to the problem were proposed in the past, which introduced various degrees of simplification replacing the spot weld with a single beam element [16, 18]. This simulation approach represents at the moment the only way to cope with simulating the global behavior of structures with several spot welds, but at the same time losing on precision with regard to intermediate states of deformation. On the basis of the above, the stress field knowledge, next to spot welds, requires an accurate modelling of the structure, using analytical or numerical models. However, it is essential, when modelling real structures with several spot welds, to make use of drastic simplifications, which have a considerable impact on results.

To overcome such limitations, a method based on a theoretical solution of the region close to the spot weld has been proposed by Salvini et al. [10] and by Vivio et al. [19]. The Spot Weld Element (SWE) approach proposed in these papers represents an efficient way of modelling the spot weld in FE solutions, using a complete stiffness characterization of spot weld region - condensed on the nodes that surround the spot - using a closed-form solution of a reference theoretical model (a circular plate with a central rigid inclusion).

The model features a set of equivalent radial beam, with six degrees of freedom per node, lying on metal sheet,

and a central beam element (main link) representing the spot weld nugget and connecting the two metal sheet. Each node of main link is connected to the chosen shell nodes of the two plates by radial beams showing a rigid part and a flexible part. The stiffness characteristics of these beams are calculated from the closed-form solution of the analytical model of the spot weld area. The closed-form solutions, under the linear elastic hypothesis, comprise the combination of various applied loads (orthogonal opening loads, bending loads, shear loads). Every radial beam is associated with an angular sector of the circular plate and the beam is designed to reproduce its stiffness. The spot element introduces a very small number of degrees of freedom and the improvements in accuracy of results, under the elastic hypothesis, are apparent when a comparison with other modelling techniques is performed [19, 20].

In SWE approach all plastic deformation of the region surrounding the spot nugget can be embedded onto the deformable main link (spot nugget) that ties the two sets of beams. To this purpose, it is necessary to analytically evaluate the change in stiffness of the spot area due to large deflections and plastic deformations. There are two main needs for such an extension of SWE definition: i) the possibility to introduce more reliable modelling of spot welds in structures that are studied from a crashworthiness point of view – indeed, precision in the stiffness description of the region close to the spot weld connection is necessary, when addressing the progressive crashed configuration that is pursued by the impacted structure; ii) determining the redistributed load on spot welds, when an exceptional load is experienced; such loads may introduce local plastic deformations, with consequent change in local stiffness, which will in turn cause redistribution on loads transferred by each spot weld; it would be essential to manage this factor when estimating fatigue life of the jointed structure. Nevertheless, although the linear elastic behavior of the circular plate is completely defined irrespective of the load kind, there are no analytical solutions to the same problem under loads that exceed the elastic limit. Skrzypek and Hetnarski [21] proposed an analytical approach to the problem concerning the circular plate stressed over the elastic limit for small displacements. In spite of the fact that the limitations introduced by the hypothesis of Kirchhoff-Love lead to a simplified solution that is far from the real case, the complex system of differential equations presented by Skrzypek and Hetnarski [21] has no solution in closed-form. Faced with the impossibility of obtaining a solution to the problem in an analytical way, due to the high non-linearity of the equation system, several numerical approaches were proposed to resolve it [22, 23]. The most cumbersome aspects are represented by the definition of the plastic flow through the plate thickness and the implementation of membrane effects occurring when introducing large deflections and removing the Kirchhoff-Love hypothesis. In this paper, a new analytical model is presented in order to describe the plastic behavior of spot welded joints when an opening force acts on the spot, this can

mainly influences the overall deformation of spot weld in an elastic-plastic case. The proposed model provides a first contribution to extend the advantages of SWE approach to inelastic local behavior with moderately large deflections, by means of the analytical characterization (in terms of stiffness) of the spot weld region stiffness. The reference model here introduced is very similar to the one used for SWE definition in linear elastic hypothesis (a circular plate with a central rigid inclusion). In SWE approach the effects of large deflections and plastic deformation of the region surrounding the spot nugget can be embedded and condensed onto the deformable main link (spot nugget) that ties the two sets of beams. The analytical results have been validated by comparing with those obtained by FEA.

EQUIVALENT MODEL AND THEORETICAL SUPPORT

The stiffness evaluation of the spot weld region is computed by means of the analytic solutions of a bi-dimensional model (shown in Fig. 1). The finite plate presents a central rigid inclusion having radius r_{int} , which is clamped to the plate. The circular plate is hinged (radial and orthogonal displacement are constrained) and not clamped on the outer radius. The choice of outer hinging is necessary, otherwise secondary plasticity would also occur in correspondence of the external constraints; this secondary plasticity can affect results significance in terms of contributing to elastic-plastic deformations on variation in spot region stiffness.

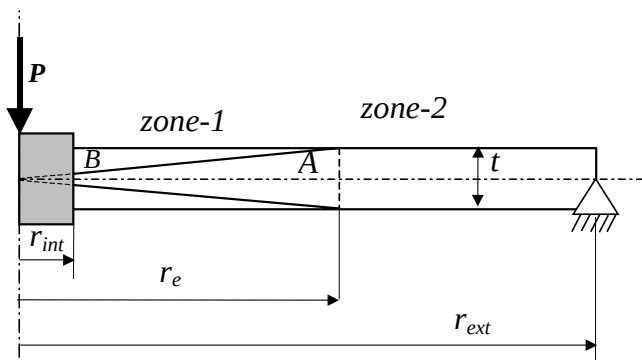


Figure 1. Actual plate section.

In order to define the plastic contribution to final deformation in small and large deflections cases respectively, the model has been developed analytically using well known stress functions valid for small displacements. Subsequently, an innovative formulation allowing to obtain the membrane additions in terms of stresses and strains has been introduced. In this last case it is included the strain of the middle plane of the plate, when the deflections are no longer small in comparison with the thickness of the plate but are still small as compared with the other dimensions (except for its thickness).

The focus of interest is on the response in terms of rigidity of the plate when an opening load P is applied on the spot nugget. In the proposed model, an isotropic and homogeneous elastic-perfectly plastic material has been chosen, and Von Mises yield criterion has been adopted. In a subsequent paper the influence on spot region stiffness of the yield strain and work hardening behavior of material will be examined. The effects of yield stress variation and hardening of material can be embedded onto the deformable main link (spot nugget) of the SWE [24]. Moreover it is necessary to highlight the importance of heat-affected zones and possible residual stresses in the mechanical behavior of spot welds. However, such factors are very changeable and difficult to quantify in real cases. In the proposed framework these effects are not included; in a SWE approach, that is the final outcome of this approach, the contribution of these effects can be added subsequently.

The stress distribution analysis of the plate clearly shows two zones with different behaviors over the elastic-plastic limit. In fact, the plastic flow begins near the rigid disc, spreading toward the outer radius: so there is a zone entirely stressed under the elastic limit (zone-2 in Fig.1) and a second one where part of the material has reached the yield condition (zone-1 in Fig. 1). Because of the distribution of plasticized material of the plate, the section of the zone-1 can be assumed geometrically divided in two further parts (A and B), as shown in Fig. 1. The main variable in the problem is how the plastic flow spreads as the applied load increases, until the load increases. Within a generic radius in zone-1, the thickness of plasticized material increases along with the load. This thickness-radius relationship is here assumed as linear, as FE investigation suggests [25]. On the contrary, the evaluation of border radius r_e variation along with the applied load, between zone-1 and zone-2, is quite more complex.

The theory of bending plates having constant thickness can be used to represent the outer part of the plate that is stressed under the elastic limit (zone-2). In the zone-1, it is possible to clearly distinguish two different material behaviors. On the one hand there is the zone A (as indicated in Fig. 1), where the material is still stressed under the elastic limit; whereas in zone B deformation is already plastic. On the division line between zones A and B, the equivalent stress must be equal to the yield one. The behavior in zone A can be described using the linearly variable thickness law introduced by Conway [26]. Zone B is more difficult to study with an analytical approach.

Since the interest is focused on the rigidity of the plate (the global stiffness of the plate), and the plastic zone offers no resistance, an equivalent model can be defined, shown in Fig. 2; in this model the plasticized part of plate (zone B) is not represented because of its nearly null addition to the plate's rigidity.