

Effects of Specimen Width and Specimen Length on Stress Intensity Factor Solutions for Spot Welds in U-Shape Specimens

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ABSTRACT

In this paper, the stress intensity factor solutions for spot welds in U-shape specimens are investigated by finite element analyses. Three-dimensional finite element models are developed for U-shape specimens to obtain accurate stress intensity factor solutions. In contrast to the existing investigations of the stress intensity factor solutions based on the finite element analyses, various ratios of the sheet thickness to the nugget radius, the half width of the central square portions to the nugget radius and the half specimen length to the half specimen width are considered in this investigation. The computational results confirm the functional dependence on the nugget radius and sheet thickness of Zhang's stress intensity factor solutions for U-shape specimens. The computational results provide a geometric function in terms of the normalized width of the central square portions and the aspect ratio of the bottom and top sheets of the U parts to Zhang's stress intensity factor solutions for U-shape specimens. Finally, based on the analytical and computational results, the dimensions of U-shape specimens are suggested.

INTRODUCTION

Resistance spot welding is widely used to join sheet metals for automotive components. The fatigue lives of spot welds have been investigated by many researchers in various types of specimens, for example, see Zhang [1]. Since the spot weld provides a natural crack or notch along the weld nugget circumference, fracture

mechanics has been adopted to investigate the fatigue lives of spot welds in various types of specimens based on the stress intensity factor solutions at the critical locations of spot welds [1-11]. Note that the stress intensity factors usually vary point by point along the circumference of spot welds in various types of specimens. In order to obtain accurate stress and strain distributions and/or stress intensity factor solutions for spot welds in various specimens, finite element analyses have been carried out by various investigators [12-16]. However, no systematic investigation of the effects of the geometric factors on the stress intensity factor solutions for U-shape specimens exists in the literature.

Figure 1 schematically shows three types of U-shape specimens, typical U-shape specimen, double-U specimen and Chrysler specimen, commonly used to investigate the strength and fatigue lives of spot welds under opening or opening/shear loading conditions. As shown in the figure, the double-U specimen is developed by Hahn et al. [17], the Chrysler specimen is developed by Lee et al. [18], and the typical U-shape specimen is given in the standard of AWS [19]. The applied opening force $F/2$ is shown as the bold arrows. The weld nugget is idealized as a circular cylinder as shown in the figure. The critical locations with the maximum mode I stress intensity factors are marked as point A and point B. The U-shape specimen has the sheet thickness t , the nugget diameter $2a$, the specimen width $2W$, the specimen length $2L$, the flange height H , and the corner radius r as shown in Figure 1. Note that $2b$ is the width of the central square portions of the bottom

and top sheets of the upper and lower U parts marked by dashed lines as shown in Figures 1(b) and 1(c). Based on the standard of AWS [19] and the research work of Lee et al. [18], two cuboidal spacers are needed to insert inside the upper and lower U parts of the specimen. The flanges of the specimen with the spacers are clamped by the fixtures/grips or bolts to apply the opening load.

In this paper, we investigate the stress intensity factor solutions for spot welds in U-shape specimens by a systematic finite element analysis. Three-dimensional finite element models based on the finite element model for two circular plates with connection [20] are used to obtain the stress intensity factor solutions for U-shape specimens. The stress intensity factor solutions as functions of the ratios t/a , b/a , L/W and H/a are investigated. The stress intensity factor solutions of our computational results are compared with some existing computational and closed-form analytical stress intensity factor solutions for U-shape specimens. Geometric functions in terms of the ratios b/a and L/W based on our computational results are suggested to the stress intensity factor solutions proposed by Zhang [9].

STRESS INTENSITY FACTOR SOLUTIONS

Zhang [9] showed the stress intensity factor K_I solutions at the critical locations (point A and point B as shown in Figure 1) for double-U specimens and Chrysler specimens. For double-U specimens, the K_I solution at the critical locations (point A and point B as shown in Figure 1) is given as

$$K_I = \frac{0.637F(L-a)\xi_D}{2\sqrt{3}at\sqrt{t}} \quad (1)$$

where F is the total resultant force, L is the half specimen length, a is the nugget radius, t is the sheet thickness and ξ_D is the geometric factor for double-U specimens as listed in Table 1. Zhang [9] indicated that Equation (1) is applicable for $3 \leq L/a \leq 7$ and $W \geq L$. For Chrysler specimens, the K_I solution at the critical locations (point A and point B as shown in Figure 1) is given as

$$K_I = \frac{0.637F(L-a)\xi_C}{2\sqrt{3}at\sqrt{t}} \quad (2)$$

where ξ_C is the geometric factor for Chrysler specimens as listed in Table 2. Zhang [9] indicated that the K_I solution for Chrysler specimens is applicable for $3 \leq W/a \leq 7$ and $L \geq W$. Note that the flanges of double-U specimens and Chrysler specimens are clamped by the spacers and fixtures during testing [17,18].

Lin and Pan [11] showed another K_I solution at the critical locations (point A and point B as shown in Figure 1) for U-shape specimens with finite size under opening conditions given as

$$K_I = \frac{\sqrt{t}}{\sqrt{3}} \left\{ \frac{-3(\tilde{M}_b - \tilde{M}_c)}{t^2 XY} [2b^2 X + 4Y(a^4 b^4 + b^8)] - \frac{3F[(a^2 - b'^2)(-1 + \nu) + 2b'^2(1 + \nu)\ln(b'/a)]}{2\pi^2[a^2(-1 + \nu) - b'^2(1 + \nu)]} - \frac{24\tilde{M}_{x'y'}}{t^2 X'} [a^4(b/\sqrt{2})^4 + (b/\sqrt{2})^8] \right\} \quad (3)$$

where b is the half width of the central square portions of bottom and top sheets of the upper and lower U parts, b' is the equivalent radius for the equivalent circular plate model and is approximate equal to $2b/\sqrt{\pi}$ using the simple area equivalence rule [11]. As mentioned earlier, $b = W$ when $L \geq W$. Here, X , Y and X' are defined as

$$X = (-1 + \nu)(a^4 + b^4)^2 - 4a^2 b^6 (1 + \nu) \quad (4a)$$

$$Y = a^2(-1 + \nu) - b^2(1 + \nu) \quad (4b)$$

$$X' = -[a^4 + (b/\sqrt{2})^4]^2(1 - \nu) - 4a^2(b/\sqrt{2})^6(1 + \nu) \quad (4c)$$

where ν is the Poisson's ratio of the sheet material. $\tilde{M}_{x'y'}$ ($= F/16$) is the uniform distributed twisting moments per unit length and $\tilde{M}_b$ ($= F(L - b)/4b$) is the additional counter bending moment per unit length due to longer bottom and top sheets of the upper and lower U parts [11]. $\tilde{M}_c$ is the counter bending moment per unit length caused by the constraint of the flanges and is given as

$$\tilde{M}_c = g_c \frac{F[a^2 - b'^2 + 2a^2 \ln(b'/a)]}{4\pi(a^2 - b'^2)} + h_c \frac{\tilde{M}_b}{2} \quad (5)$$

where g_c is a geometric factor which is a function of the flange height H , the sheet thickness t , the specimen width $2W$, and the specimen length $2L$ and h_c is another geometric parameter which is applicable when $L > W$ ($L > b$) and $\tilde{M}_b = F \times (L - b)/4b$ [11]. When the flanges of the specimens give full constraint and no constraint, the values of g_c for $L = b$ and h_c for $L > b$ are defined as 0 and 1, respectively. However, when the flanges do not give a full constraint, specific values of g_c and h_c are needed. Lin and Pan [11] indicated that the K_I solution for U-shape specimens with finite size is applicable for $L \geq W$. The details of the validation of the

K_I solution in Equations (3), (4) and (5) are reported in Lin and Pan [11].

FINITE ELEMENT ANALYSES

In order to obtain accurate stress intensity factor solutions for spot welds in U-shape specimens and to systematically examine the validity of the stress intensity factor solutions of Zhang [9] and Lin and Pan [11], three-dimensional finite element analyses are carried out. Note that no systematic investigation of the effects of t/a , b/a , L/W , H/a and r/a on the stress intensity factor solutions for U-shape specimens exists in the literature. Therefore, we emphasize our well-benchmarked finite element models used in this investigation [20]. The details to select an appropriate three-dimensional mesh for obtaining accurate stress intensity factor solutions for spot welds can be found in Wang et al. [20].

Due to symmetry, only a half U-shape specimen is considered. Figure 2(a) schematically shows a half U-shape specimen with a uniform displacement applied to the upper edge surfaces of the specimen shown as the bold arrows, a clamped boundary condition applied to the lower edge surfaces of the specimen and a roller edge boundary condition applied to the side surfaces of the flanges. The half cylinder represents the half weld nugget. The specimen has the sheet thickness t (=0.65 mm), the half specimen length L (=12.7 mm), the half specimen width W (=12.7 mm), the flange height H (=25.4 mm), the corner radius r (=0 mm) and the nugget radius a (=3.2 mm) based on the dimensions of the U-shape specimen listed in the standard of AWS [19]. Note that the corner radius r is in general determined based on the bendability of the sheet material and is usually very small [19]. Therefore, the corner radius r is taken as 0 mm for simplicity. Both the upper and lower sheets have the same thickness. A Cartesian coordinate system is also shown in the figure. As shown in Figure 2(a), a uniform displacement is applied in the $-z$ direction to the upper edge surfaces of the specimen, and the displacements in the x , y and z directions for the lower edge surfaces of the specimen are constrained to represent the clamped boundary conditions in the experiment. The displacement in the y direction of the symmetry plane, the $x-z$ plane, is constrained to represent the symmetry condition due to the loading conditions and the geometry of the specimen. The displacement in the x direction of the inner and outer side surfaces of the flanges is constrained to represent the constraint conditions due to the spacers and fixtures.

Figure 2(b) shows a mesh for a left half finite element model. Figure 2(c) shows a close-up view of the mesh near the main crack tip. Note that the main crack is modeled as a sharp crack here. The three-dimensional

finite element mesh near the weld nugget is evolved from the three-dimensional finite element mesh for two circular plates with connection as discussed in Wang et al. [20]. As shown in Figure 2(c), the mesh near the center of the weld nugget is refined to ensure reasonable aspect ratios of the three-dimensional brick elements. The three-dimensional finite element model for the half U-shape specimen has 25,240 20-node quadratic brick elements. The main crack surfaces are shown as bold lines in Figure 2(c).

In this investigation, the weld nugget and the base metal are assumed to be linear elastic isotropic materials. The Young's modulus E is taken as 200 GPa, and the Poisson's ratio ν is taken as 0.3. The commercial finite element program ABAQUS [21] is employed to perform the computations. Brick elements with quarter point nodes and collapsed nodes along the crack front are used to model the $1/\sqrt{r}$ singularity near the crack tip.

First, the distributions of the stress intensity factor solutions along the circumference of the nugget are investigated here. Figure 3 shows the normalized K_I solutions as functions of θ for the crack front along the nugget circumference based on the three-dimensional finite element computational results for the U-shape specimen with $t/a=0.2$, $b/a=3.97$, $L/W=1$, $H/a=7.94$, and $r/a=0$. Note that both U-shape specimens with and without spacers are considered. The K_I solutions are normalized by the K_I solution for the U-shape specimen with spacers at $\theta=180^\circ$, denoted by $(K_I)_{\max}$. The normalized K_I solutions based on the computational results for U-shape specimens with and without spacers and fixtures are denoted as $\bar{K}_I_Spacer$ and $\bar{K}_I_NoSpacer$, respectively. As shown in Figure 3, the general trends of $\bar{K}_I_Spacer$ and $\bar{K}_I_NoSpacer$ agree well with each other. The maxima of the $\bar{K}_I$ solutions are located at $\theta=0^\circ$ and 180° and the minima of the $\bar{K}_I$ solutions are located at $\theta=90^\circ$ and 270° . Note that the maxima of $\bar{K}_I_NoSpacer$ are larger than those of $\bar{K}_I_Spacer$ but the minima of $\bar{K}_I_NoSpacer$ are smaller than those of $\bar{K}_I_Spacer$. The distributions of the stress intensity factor solutions along the circumference of the nugget as shown in Figure 3 are similar to those in Radaj et al. [12].

Next, we investigate the effects of the boundary conditions on the right and left edge surfaces of the bottom and top sheets of the upper and lower U parts of the specimen. Two models of two square plates joint by a spot weld subjected to a uniform displacement along the right and left edge surfaces of the upper plate shown as the bold arrows and different types of boundary conditions are considered. One square plate model is

subjected to simply supported edge boundary conditions on the right and left edge surfaces of the lower plate. Another square plate model is subjected to roller edge boundary conditions on the right and left edge surfaces of the square plates. The two square plate models represent the bottom and top sheets of the upper and lower U parts. The normalized K_I solutions based on the computational results for the two square plate models subjected to roller edge and simply supported edge boundary conditions on the right and left edge surfaces are denoted as $\bar{K}_I_Roller$ and $\bar{K}_I_Simple$, respectively. The square plates have $t/a=0.2$, $W/a=3.97$, and $L/a=3.97$. As shown in Figure 3, the general trends of $\bar{K}_I_Roller$ and $\bar{K}_I_Simple$ also agree well with those of $\bar{K}_I_Spacer$ and $\bar{K}_I_NoSpacer$. The maxima of $\bar{K}_I_NoSpacer$ are smaller than those of $\bar{K}_I_Simple$ but larger than those of $\bar{K}_I_Spacer$ and $\bar{K}_I_Roller$. On the other hand, $\bar{K}_I_Roller$ virtually coincides with $\bar{K}_I_Spacer$ as shown in Figure 3. This indicates that the flanges with spacers and fixtures gives full constraint and is identical to the roller edge boundary condition. Therefore, the effects of the flange height for specimens with spacers and fixtures can be neglected.

Then we investigate the effects of the corner radius r on the K_I solutions. A U-shape specimen of $b/a=3.97$, $L/W=1$, $H/a=7.94$ and $r/a=1.25$ with spacers and fixtures is considered. Note that as $r/a=1.25$, the corner radius is 4.0 mm which is ten times larger than that of the specimen listed in the standard of AWS [19]. The normalized K_I solutions based on the computational results for the specimen with $r/a=1.25$ is denoted as $(\bar{K}_I_Spacer)_{r=4}$. As shown in Figure 3, the general trend of $(\bar{K}_I_Spacer)_{r=4}$ agrees well with that of $\bar{K}_I_Spacer$. The maxima of $(\bar{K}_I_Spacer)_{r=4}$ are nearly 10% lower than those of $\bar{K}_I_Spacer$. This result indicates that the effects of the corner radius are weak and can be neglected when the value of the corner radius closes to zero.

EFFECTS OF SHEET THICKNESS (t/a)

Here, we investigate the effects of the ratio of the sheet thickness t to the nugget radius a on the mode I stress intensity factor solutions at the critical locations (point A and point B as shown in Figure 1) for the U-shape specimens given in the standard of AWS [19] with the ratios of $b/a=3.97$, $L/W=1$, $H/a=7.94$ and $r/a=0$. Note that the flanges of the U-shape specimen are constrained by the spacers and fixtures in the following investigations. Also note that the maxima of the K_I solution occur at point A and point B as shown in Figure 1. We develop different three-dimensional finite element

models by changing the sheet thickness t with the nugget radius a being fixed.

Figure 4 shows the normalized K_I solutions at the critical locations (point A and point B as shown in Figure 1) as functions of t/a based on our three-dimensional finite element computations, the analytical solution of Zhang [9] for double-U specimens in Equation (1), and the analytical solution of Zhang [9] for Chrysler specimens in Equation (2) under the applied load F . The range of the ratio t/a is selected from 0.08 to 0.80 to represent the typical values for spot welds used in the automotive industry. The K_I solutions are normalized by Zhang's solution in Equation (1) without ξ_D . The normalized $\tilde{K}_I$ solutions based on the computational results, the analytical solution of Zhang [9] for double-U specimens in Equation (1) and the analytical solution of Zhang [9] for Chrysler specimens in Equation (2) are denoted as $\tilde{K}_I_FEM$, $\tilde{K}_I_DoubleU$, and $\tilde{K}_I_Chrysler$, respectively. Note that for a U-shape specimen with $b/a=3.97$, $L/W=1$, $H/a=7.94$ and $r/a=0$, the values of ξ_D for $\tilde{K}_I_DoubleU$ and ξ_C for $\tilde{K}_I_Chrysler$ are 1.43 and 1.48 obtained from Tables 1 and 2, respectively, by using the interpolation approach.

As shown in Figure 4, $\tilde{K}_I_FEM$ based on our three-dimensional finite element computations appears to be relatively constant for the ratios of t/a considered. The general trends of $\tilde{K}_I_DoubleU$, and $\tilde{K}_I_Chrysler$ agree well with that of $\tilde{K}_I_FEM$. As shown in the figure, $\tilde{K}_I_DoubleU$ and $\tilde{K}_I_Chrysler$ are identically 1.44 and 1.48, respectively. $\tilde{K}_I_FEM$ based on our three-dimensional finite element computations is nearly 30% and 25% higher than $\tilde{K}_I_DoubleU$ and $\tilde{K}_I_Chrysler$, respectively.

Although the computational results show a slight dependence on t/a in Figure 4, the general trend of $\tilde{K}_I_FEM$ appears to confirm the functional dependence of $\tilde{K}_I_DoubleU$ and $\tilde{K}_I_Chrysler$ on the sheet thickness t and the nugget radius a . Therefore, the analytical solutions of Zhang [9] can be used as the basis to express the K_I solution for U-shape specimens. However, for Zhang's solutions, ξ_D is used to consider the effects of L/a for double-U specimens with a specific value of W/a whereas ξ_C is used to consider the effects of W/a for Chrysler specimens with a specific value of L/a . The effects of L/a and W/a or those of L/W and b/a are not considered together for the geometric factors of either double-U specimens