

Determining Vehicle Steering and Braking from Yaw Mark Striations

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ABSTRACT

This paper presents equations that relate the orientation and spacing of yaw mark striations to the vehicle braking and steering levels present at the time the striations were deposited. These equations, thus, provide a link between physical evidence deposited on a roadway during a crash (the tire mark striations) and actions taken by the driver during that crash (steering and braking inputs). This paper also presents physical yaw tests during which striated yaw marks were deposited. Analysis of these tests is conducted to address the degree to which the presented equations can be used to determine a driver's actual steering and braking inputs. As a result of this testing and analysis, it was concluded that striated tire marks can offer a meaningful glimpse into the steering and braking behavior of the driver of a yawing vehicle. It was also found that consideration of yaw striations allows for the reconstruction of a vehicle's post-impact yaw motion from a single tire mark.

INTRODUCTION

Braking and steering will both affect the characteristics of the striations that are sometimes present in the tire marks that a yawing vehicle deposits. As an example, consider Figure 1, which is a photograph from testing conducted by Daily [1] that depicts a yaw mark deposited by a vehicle during a counter-clockwise yaw. The vehicle that deposited this tire mark was traveling into the page. Initially, the vehicle was unbraked, then, the brakes were applied. As a result of this changing braking level, the striations in the tire mark undergo a change in direction.

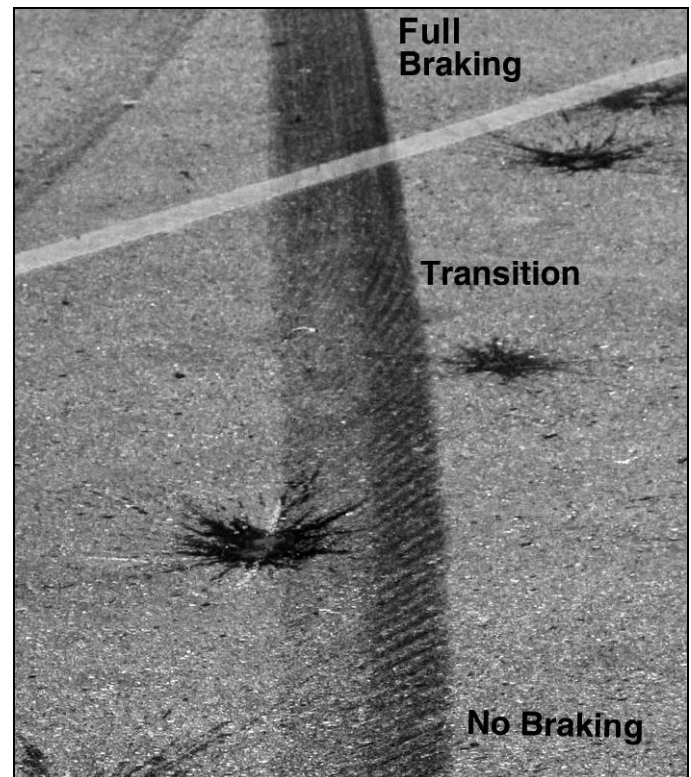


Figure 1 – A yaw mark deposited during testing performed by Daily et al.

As will be explained in greater detail later, the striations in a yaw mark indicate the travel direction of tire shoulder blocks during the yaw. Since frictional forces act to oppose the direction of motion, these striations are also indicative of the direction of the frictional force acting on these portions of the tire. In the case of increasing

braking depicted in Figure 1, the direction of the forces on the tire, and thus, the direction of the striations, transition from a direction perpendicular to the heading of the tire to a direction parallel to the velocity direction of that tire hub (tangent to the path of the tire mark). In other words, the no braking case references the angle of the tire and the full braking case references the general direction of the tiremark. Thus, the analysis of striations includes consideration of the vehicle and the tire mark evidence. In most cases, the tire mark evidence by itself is insufficient to determine the amount of braking or steering that was occurring.

Of course, even in the absence of braking, tire mark striations could make a similar transition to that shown in Figure 1. In the absence of braking, the tire forces will be oriented perpendicular to the tire heading, and thus, the striations will also be oriented perpendicular to the tire heading. In the case of steadily increasing slip angle, the striations in a tire mark would transition in a manner similar to that shown in Figure 1. The striations would become parallel with the direction of the tire mark when the slip angle reached 90 degrees. To take this a step further, it should be noted that steering will influence a tire's slip angle, and thus, will also influence the orientation of striations deposited by an unbraked tire.

The tire slip angle and braking level will both influence the characteristics of the striations in a yaw mark. If one considers only the orientation of the striations, then these various influences will be indistinguishable. However, it will be shown below that consideration of both the orientation and the spacing of the striations will allow the effects of slip angle and braking to be distinguished and parsed out.

Thus, a relationship exists between the actions of a driver and the characteristics of the tire marks their vehicle deposits during a yaw. Formalizing this relationship can enable reconstruction of a driver's actions during a crash based on physical evidence. The remainder of this paper presents and tests equations that relate the striation characteristics (direction and spacing) to both the percentage of braking slip and the slip angle of the tire at the time a yaw mark was deposited.

YAW MARKS

When a vehicle yaws, its velocity direction deviates from its heading direction. This difference can result in tire marks being deposited on the roadway, within which striation marks are often visible, as discussed above. Tire mark striations are direct physical evidence of the direction of the force that was applied to the tire at the time the mark was deposited. Without braking, the tire force is perpendicular to the tire heading, and thus, the striations produced are perpendicular to the tire heading. When a yawing vehicle locks its brakes, the resultant tire force is in line with velocity direction of the tire hub, hence the direction of the striations becomes in line with that velocity direction.

Except for in the cases of full lockup or 90-degree tire slip angle, a tire rolls and slides as it deposits striations. This sliding results in forces that deform the tire carcass laterally and load the leading shoulder of the tire (the shoulder is the area where the tread or bottom of the tire and the tire sidewall meet). An extreme case of such shoulder loading is shown in Figure 2 [2]. When distinct striations are visible in a yaw mark (Figure 1), it is the individual tread blocks on the shoulder of the tire that are responsible for the darker striations. The fact that the shoulder tread blocks create distinct striations is fundamental to the modeling presented below.

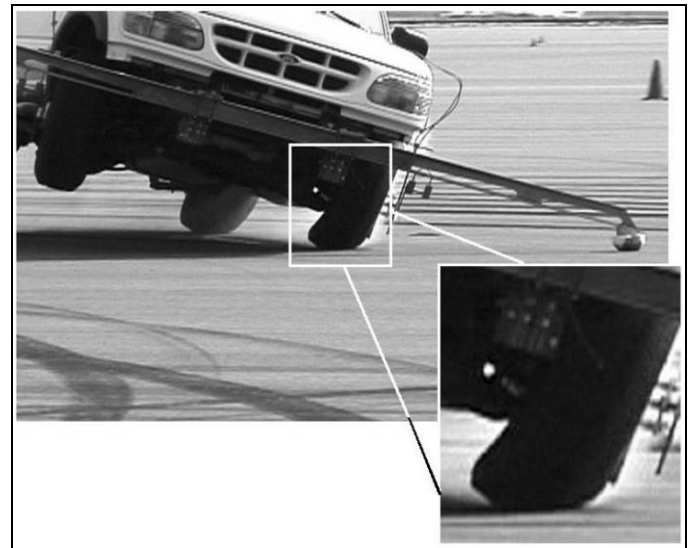


Figure 2 – Testing performed by NHTSA exhibiting tire deformation

Figure 3 is a schematic depicting the alignment of a tire relative to a yaw mark that it is depositing. In Figure 3, the contact patch, or portion of the tire in contact with the roadway surface, has been identified with a dark grey box. The tire mark is divided, with darker striations on the right being deposited by the shoulder blocks and lighter striations on the left being deposited by the tread. Such a division in the darkness of the striations from left to right is visible in the tire mark in Figure 1. In Figure 3, the striations are neither perpendicular to the tire heading or parallel to the velocity direction, indicating that partial braking is occurring. The dark striations from the shoulder blocks will be the focus of the model to be presented below. The slip angle, α , is labeled in Figure 3.¹

THEORY

In this section, mathematical equations are presented that relate braking and steering to the orientation and spacing of striations deposited on the roadway. This theory assumes that the tire is rigid, which as shown in Figure 2, is obviously not the case. Later in this paper, the implications of this rigid tire assumption are

¹ A full listing and description of the variables used in this paper can be found in Appendix A.

discussed. The equations below also don't consider the fact that both the size of the shoulder tread blocks and the spacing between them are variable around an actual tire's circumference. This is an important characteristic of tires to consider in practice and is discussed later.

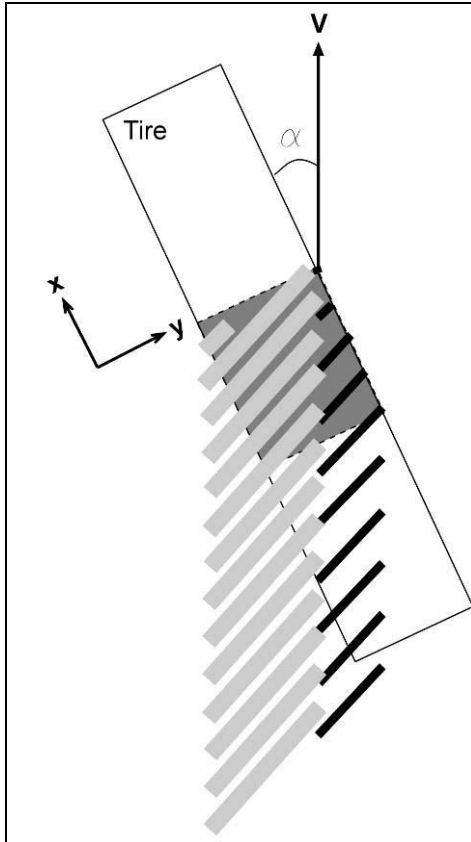


Figure 3 – Tire depositing a striated tire mark.

TIRE SLIP ANGLE – Figure 4 depicts the same yawing tire as Figure 3. It is the dark striations deposited by the shoulder of the tire that form the foundation of the modeling below, thus, these striations have been singled out in Figure 4. The coordinate system depicted in Figure 4 is consistent with the SAE tire axis system. It is right-handed, with the z-axis pointing into the page, and the x-axis aligned with the tire's heading. The angle α is the tire's slip angle, which is the angle between the tire's heading and the tire's velocity direction. The dark black lines represent the striations in the tire mark, deposited by the shoulder treads of the tire, and they are oriented an angle ψ from the tire's heading. As drawn in Figure 4, both α and ψ are positive. The angle θ is the angle between the tire's velocity direction and the direction of a striation. The striations are a distance of SD apart in the direction of the tire mark, or tire velocity direction.

In the lower portion of Figure 4, a zoomed in view of two right triangles is shown. One side of both triangles is formed by a line connecting two adjacent striations in a direction perpendicular to the striations. The hypotenuse of the larger triangle is SD . The second right triangle has a hypotenuse of length TD and is denoted by a dashed line. TD has a length equivalent to the spacing between

the tread blocks on the tire and is oriented such that it connects two adjacent striations. This alignment orients the line TD parallel to the tire heading. Both triangles are completed by lines along the striation mark, as depicted. The striation angle, θ , is shown on the triangle. Because TD is parallel to the tire heading, the angle ψ is created in the second triangle. In practice, the angle θ and the distance SD can be measured from a striated tire mark. The tread block spacing distance, TD , is a measurement taken from the tire. Measurement techniques for θ , SD and TD are discussed later.

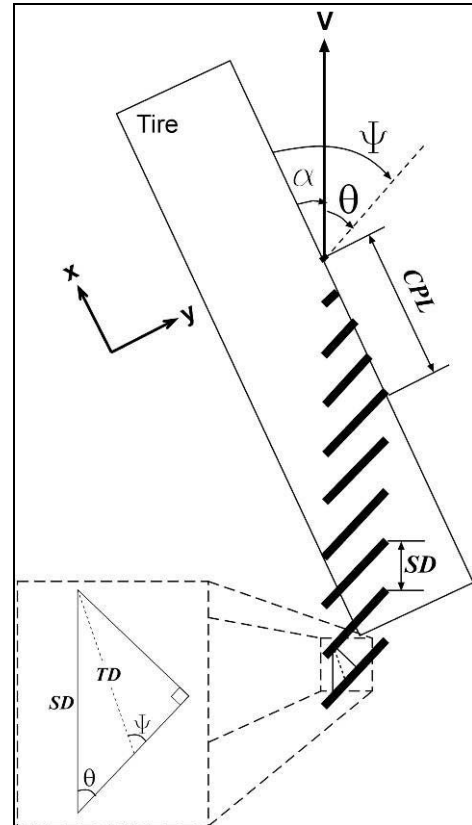


Figure 4 - Top view of a yawing tire depositing a striated tire mark.

The triangles to the lower left of Figure 4 can be geometrically related to obtain an expression for ψ , given by Equation (1) below.

$$\Psi = \sin^{-1} \left(\frac{SD \sin \theta}{TD} \right) \quad (1)$$

Using Equation (1) along with geometric relationships between ψ , α and θ , an expression for the slip angle, or the angle of the tire relative to the tire velocity, can be obtained, given below in Equation (2). All of the variables in Equation (2) can be measured from either a tire or a tire mark.

$$\alpha = \sin^{-1} \left(\frac{SD \sin \theta}{TD} \right) - \theta \quad (2)$$

When multiple tire marks are deposited on the roadway, vehicle yaw angles can be determined by aligning a vehicle with its tire marks. Using the equations above, the slip angles of each tire can also be determined. Rear tire slip angles are equal to the slip angle of the vehicle in the absence of tire misalignment or toe in/out design differences. The difference between the heading direction of the front tires and the heading direction of the vehicle is the steering angle at the front tires. The angle of the steering wheel can then be calculated through the steering ratio.

BRAKING/SLIP PERCENTAGE – Along with the orientation of the tire, the amount of braking of a yawing tire is often of interest. Braking can be discussed in terms of slip percentage, which can be calculated using some of the parameters that appear in Figure 4. As will be shown, slip percentage is dependant on fewer variables than slip angle. This is important because in some cases, slip percentage can be calculated when the slip angle cannot.

Our discussion of slip will begin with the classic definition of braking slip, which appears in several vehicle dynamics texts. For instance, Gillespie states: “Slip of the tire is defined by the ratio of slip velocity in the contact patch to forward velocity” [3]. Mathematically, this is shown in Equation (3) for the case of in-line braking.

$$S = \frac{V - \omega r}{V} \quad (3)$$

In Equation (3), V is the velocity of a wheel hub, ω and r are the rotational speed and radius of its tire, respectively. The quantity ωr will be equal to V when the tire is free rolling so the slip will be 0. Comparatively, at full wheel lockup when ω is equal to 0, slip will be equal to 1.

Equation (3) assumes a tire that has its heading and its travel direction aligned. For a yawing vehicle, it is the velocity of the tire along its heading direction that is of interest. The velocity of the tire along its heading, V_T , is the actual velocity multiplied by the cosine of the slip angle of the tire. Thus, Equation (5) gives braking slip for a tire in a yaw.

$$S = \frac{V_T - \omega r}{V_T} \quad (4)$$

$$S = \frac{V \cos \alpha - \omega r}{V \cos \alpha} \quad (5)$$

Now consider Figure 5. While the shoulder tread block of a yawing tire deposits the striation mark indicated by the bold line, it moves from Position 1 to Position 2 up the page. The two images on the right depict a side view of

the tire at Positions 1 and 2. Between Positions 1 and 2, the tire rotates β radians. While the tire travels from Position 1 to 2, it deposits a striation of length LS , depicted by the bold line in the top view image on the left. The tire mark has a width w . The tire slip angle and the striation angle are indicated by α and θ respectively. The contact patch length is also indicated on the diagram by the dashed line.

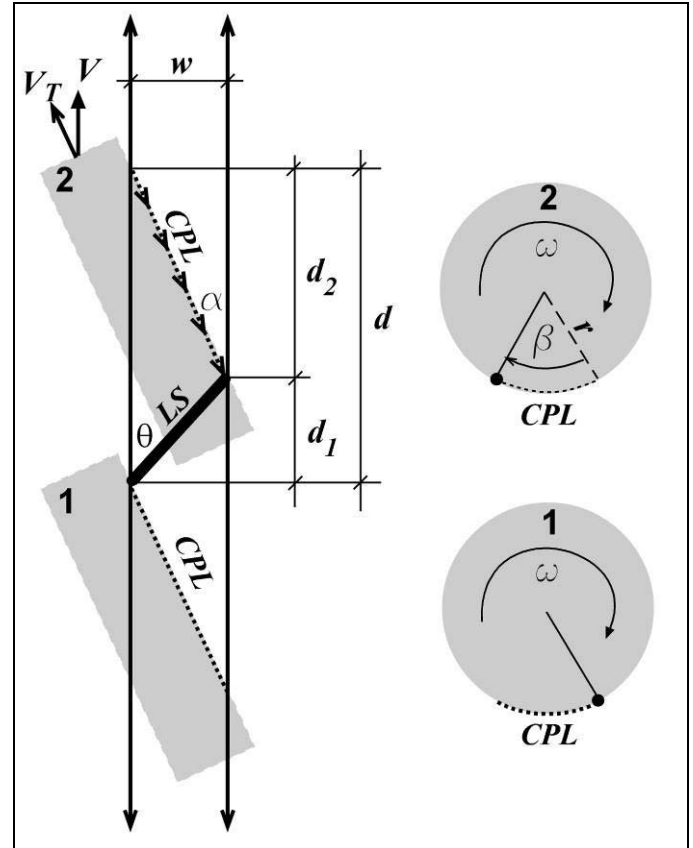


Figure 5 - Depositing of one striation mark. On the left, a top down view. On the right, a right side view.

Physically, the contact patch length, CPL , is the length of the portion of the tire that is in contact with the ground. Figure 5 depicts the contact patch length. In Position 1, a point on the circumference of the tire marked with a black dot has just come into contact with the ground at the front of the contact patch. While the striation is being deposited, the tire rotates and that point on the tire moves towards the rear of the contact patch before lifting away from the ground at Position 2. All of the parameters that appear in Equation (5), V , α , ω , and r are visible in Figure 5.

The time it takes to deposit one striation mark, t , will now be explored. Equation (5) can be rewritten as a function of this time, t , to yield Equation (6). Time is equal in all terms of Equation (6) and cancels out. Rearranging results in Equation (7).

$$S = \frac{\frac{d}{t} \cos \alpha - \frac{\beta r}{t}}{\frac{d}{t} \cos \alpha} \quad (6)$$

$$S = 1 - \frac{\beta r}{d \cos \alpha} \quad (7)$$

Two different triangles can be used to equate the tire mark width, as shown in Equation (8). The contact patch length can then be determined using Equation (9). The images on the left of Figure 5 reveal that the contact patch length is equal to βr . It should be noted that this contact patch length calculated is theoretical because only the shoulder blocks are assumed to be leaving marks on the road. In actuality, the bottom tread of the tire also leaves marks and the contact patch length is not so readily determined. For the modeling that follows, the triangles would be of different proportions if the actual contact patch length were utilized. As will be shown, all the triangle side lengths fall out of the analysis leaving the trigonometric relationships which are unaffected by the shoulder tread block assumption.

$$CPL \sin \alpha = w = LS \sin \theta \quad (8)$$

$$\beta r = CPL = \frac{LS \sin \theta}{\sin \alpha} \quad (9)$$

As is seen in Figure 5, d is equal to the summation of d_1 and d_2 which can be determined geometrically, as follows:

$$d_1 = LS \cos \theta \quad (10)$$

$$d_2 = CPL \cos \alpha = \frac{LS \sin \theta \cos \alpha}{\sin \alpha} \quad (11)$$

$$d = d_1 + d_2 = LS \cos \theta + \frac{LS \sin \theta \cos \alpha}{\sin \alpha} \quad (12)$$

Equation (9) and Equation (12) can then be substituted into Equation (7) yielding Equation (13).

$$S = 1 - \frac{\left(\frac{LS \sin \theta}{\sin \alpha} \right)}{\left(LS \cos \theta + \frac{LS \sin \theta \cos \alpha}{\sin \alpha} \right) \cos \alpha} \quad (13)$$

Equation (13) can then be simplified through the following series of equations to obtain Equation (20).

$$S = 1 - \frac{\sin \theta}{(\cos \theta \sin \alpha + \sin \theta \cos \alpha) \cos \alpha} \quad (14)$$

$$S = 1 - \frac{\sin \theta}{\sin(\theta + \alpha) \cos \alpha} \quad (15)$$

$$S = 1 - \frac{\sin \theta}{\sin \psi \cos \alpha} \quad (16)$$

$$S = 1 - \frac{\sin(\psi - \alpha)}{\sin \psi \cos \alpha} \quad (17)$$

$$S = 1 - \left(\frac{\sin \psi \cos \alpha - \cos \psi \sin \alpha}{\sin \psi \cos \alpha} \right) \quad (18)$$

$$S = 1 - 1 + \frac{\cos \psi \sin \alpha}{\sin \psi \cos \alpha} \quad (19)$$

$$S\% = \frac{\tan \alpha}{\tan(\theta + \alpha)} \times 100 \quad (20)$$

Equation (20) is multiplied by 100 giving slip percentage, $S\%$. Equation (20) is mathematically equivalent to the classic equation of slip during yaw, Equation (5). Equation (20) demonstrates that the slip percentage of a tire during the depositing of a striation mark can be calculated if the angle of that striation and the slip angle of the tire are known.

Next, consider the effects of braking on tire mark/tire geometry. Figure 6 depicts three scenarios; a free rolling tire on the left, partial braking in the center and full wheel lockup on the right. The vertical line in each scenario, which is aligned to the tire's velocity direction, has a dimensionless length of 1. From the bottom end of this vertical line, two lines are drawn, one along the heading direction of the tire, an angle α from the velocity direction, and the other along the direction of the striation, θ from the tire velocity. Two lines are then drawn from the top end of the vertical line that intersect perpendicularly with the tire heading and striation direction. In all three scenarios, the tire has the same slip angle. The striation directions, however, are different in each case, ranging from perpendicular to the tire in the no braking case, to in line with the velocity direction in the full braking case.

The ratio of the lengths of d_{actual} and $d_{freeroll}$, which are labeled in Figure 6, also define the amount of slip of the tire. As is shown in the partial braking case, the vector d_{actual} is parallel to $d_{freeroll}$ and becomes shorter in length as the angle θ decreases, which occurs as braking increases. Because d_{actual} is parallel to $d_{freeroll}$, an angle of ψ separates it from the striation direction.

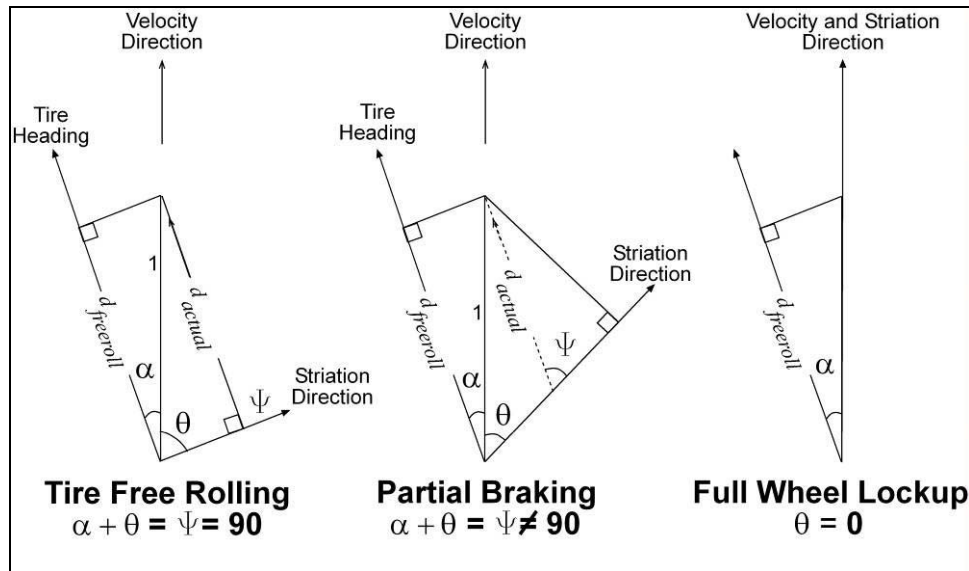


Figure 6 – Braking triangles.

Equation (21) defines the amount of slip in terms of the geometry presented in Figure 6.

$$Slip = S = 1 - \frac{d_{actual}}{d_{free_roll}} \quad (21)$$

For the free rolling case, d_{actual} and d_{free_roll} are of equal length resulting in a slip value of zero. The full lockup case, on the other hand, d_{actual} is zero resulting in a slip value of 1, or 100%. Equation (21) and Figure 6, describe mathematically the idea that striation marks are perpendicular to the tire in the absence of braking and parallel to the velocity direction in the presence of full wheel lock up. Equation (21) also offers insight into the cases where partial braking is occurring.

The triangles in the partial braking case in Figure 6 can be used to find expressions for d_{actual} and d_{free_roll} , as follows:

$$\sin \theta = d_{actual} \sin \psi \quad (22)$$

$$d_{actual} = \frac{\sin \theta}{\sin \psi} \quad (23)$$

$$d_{free_roll} = \cos \alpha \quad (24)$$

Equations (23) and (24) can be substituted into Equation (21), yielding Equation (25). Equation (25) can then be simplified to Equation (30), which again is equivalent to the classic equation for slip.

$$S = 1 - \frac{\sin \theta}{\sin \psi \cos \alpha} \quad (25)$$

$$S = 1 - \frac{\sin(\psi - \alpha)}{\sin \psi \cos \alpha} \quad (26)$$

$$S = 1 - \frac{\sin \psi \cos \alpha - \cos \psi \sin \alpha}{\sin \psi \cos \alpha} \quad (27)$$

$$S = \frac{\sin \alpha \cos \psi}{\sin \psi \cos \alpha} \quad (28)$$

$$S = \frac{\tan \alpha}{\tan \psi} \quad (29)$$

$$S\% = \frac{\tan \alpha}{\tan(\theta + \alpha)} \times 100 \quad (30)$$

STRIATION STUDY I: TIRE MARKS DEPOSITED FOLLOWING AN IMPACT

The simplest case examined during this research is that of a rear tire with no braking. As discussed in earlier sections, when no braking is present during a yaw, striations are deposited in a direction perpendicular to the heading of the tire. For most vehicles, which are steered by only the front tires, the heading direction of the rear tires is the same as the heading direction of the vehicle. Thus, in theory, the slip angle of a rear tire with no braking can be determined by simply aligning the tire in a direction perpendicular to a striation mark. Further, striations deposited by a rear tire can allow the reconstruction of an entire yaw sequence from a single tire mark with relative ease, so long as no braking was occurring during the yaw.

One scenario where there is often no braking, at least no braking input by the driver, is in a yaw induced by an impact. On July 23rd, 2008, the author conducted a