

Development of a Variable Deceleration Rate Approach to Rollover Crash Reconstruction

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ABSTRACT

The goal of this paper is to advance rollover crash reconstruction techniques beyond the assumption typically made that a rolling vehicle decelerates at a constant rate. The paper presents and applies a planar vehicle-to-ground impact model to explore the manner in which a vehicle's deceleration rate would be expected to vary over the course of a rollover. Based on this analysis, several possible variable deceleration rate profile shapes are then suggested for rollover crash reconstruction. Then, two rollover crash tests are analyzed to determine the extent to which these suggested variable deceleration rate profiles can be expected to yield accurate reconstructions of the translational and angular velocity histories for actual rollovers. Overall, each of the suggested variable deceleration rate profiles represented a significant improvement over using a constant deceleration rate.

INTRODUCTION

Calculations carried out to determine a rolling vehicle's¹ over-the-ground (OTG) speed have typically assumed that the vehicle decelerated at a constant rate. The technical literature related to rollover reconstruction has reported average deceleration rates for rolling vehicles that vary between 0.36 and 0.65g [3, 5, 16, 17, 18, 19, 21, 22, 25, 31]. Until recently, though, none of the literature reporting these deceleration rates has discussed how they vary over the course of the rollover.

¹ Throughout this paper, the phrase "rolling vehicle" should be taken to refer to a vehicle in the midst of a rollover, not a vehicle that is rolling on its wheels.

It is intuitive that the deceleration rate of a rolling vehicle varies over the distance of the roll. Empirical data has supported this intuition. Recently, several researchers have reported specific crash tests and real-world rollovers in which the deceleration rate was higher during the early portions of the rollover than during the later portions [1, 9, 15]. For instance, Reference 1 reported analysis of a real-world rollover that was captured on video. This analysis showed that the vehicle's deceleration rate during the rollover was non-constant, that it was highest early in the roll, and that it varied between 0.6 and 0.2 g over the course of the roll. Though not explicitly discussing the variability of the deceleration rate over the roll distance, other authors have noted that wheel-to-ground impacts have the potential of producing higher deceleration rates than roof-to-ground impacts. Orłowski observed that "tire/wheel impacts are capable of causing higher and more sustained deceleration of the vehicle than the roof structure would for a comparable impact...As a result, tire/wheel impacts cause a higher change in velocity..." [24].

In Reference 9, Carter examined the accuracy of using a constant deceleration rate for rollover reconstruction by comparing crash test data to reconstructed results. For the two rollover crash tests that he examined, Carter found that the vehicles' deceleration rates were not constant over the roll distance and that they were higher during the early portions of the rolls than during the later. Thus, when using a constant deceleration approach, the deceleration rates for the early portions of the rolls were underestimated and those during the later portions were overestimated. For the two cases examined by Carter,

the constant deceleration approach overestimated the vehicles' OTG speeds at most discrete points along the roll distances.

In rollover reconstruction, estimates of a vehicle's roll velocity history are obtained by determining roll positions at discrete points along the roll path and then estimating the time that elapsed while the vehicle traversed the distance between these positions. Since the deceleration rate influences these time estimates, it will also influence the roll velocities that are obtained. Thus, using the constant deceleration approach can also introduce errors into estimates of the roll velocity history. Carter [9] found that the accuracy of the constant deceleration approach depended on the number of roll positions that could be established during the roll phase, but in general, the constant deceleration approach tended to overestimate the peak roll velocities.

It is clear from Carter's research that the accuracy of reconstructed translational and roll velocity histories for rollovers could be improved if reconstructionists were able to prescribe physically realistic variations in a rolling vehicle's deceleration rate. By developing an understanding of how a rolling vehicle's instantaneous deceleration rate varies over the course of the roll distance, analysts could achieve greater accuracy in calculating instantaneous translational and angular velocities at discrete points along the roll path. The accuracy of these instantaneous velocities becomes relevant and significant when one begins to consider a rollover on an event-by-event basis, considering each ground impact and airborne phase as a discrete event [26, 27, 28], and when one uses mathematical models to analyze occupant dynamics [2, 12, 13, 14, 23].

To date, there has been virtually no published research that has explored how a rolling vehicle's instantaneous deceleration rate will vary over the roll distance. Some authors have made empirical observations related to these topics based on particular crash tests or real-world rollovers [1, 9, 15, 19]. It should be observed, though, that a coherent and logically justified approach to treating a rollover as a multi-phase event cannot be developed without some theoretical underpinning that would allow one to generalize from the results of specific crash tests to conclusions about what is true of rollovers, in general, and what would likely be true in a particular rollover that does not exactly resemble the available crash test data.

The authors can envision several potentially viable analytical approaches for estimating the variability of the deceleration rate of a rolling vehicle over the course of the roll distance. The first of these, we will term the "discrete event approach". In this approach, one would first determine the discrete distances over which the rolling vehicle was in contact with the ground and those distances over which it was airborne. Then, the analyst would either use empirical data or an impact model,

such as the one treated in this paper, to determine a deceleration rate for each ground contact. Simulation could also be a potentially useful tool for this purpose. The deceleration rate for each airborne phase would be set to zero. This would result in a deceleration rate history that would, conceptually, look something like that in Figure 1.²

In the graph of Figure 1, we have plotted distance on the horizontal axis and the deceleration rate on the vertical axis. This graph depicts a rollover with four discrete ground contact phases having deceleration rates f_1, f_2, f_3 and f_4 .

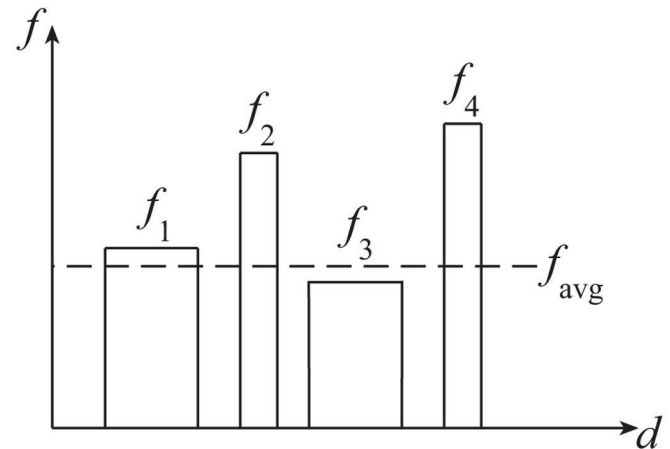


Figure 1

This discrete event approach appears to be what Carter had in mind in his comments in the discussion of Reference 9: "...it was apparent that there are phases of the roll event where the vehicle is in contact with the ground and decelerating and there are phases where the vehicle is in the air and maintaining a more-or-less constant OTG speed. In the field, given enough evidence, it may be possible to define, within reason, both phases throughout the roll sequence. Then, using a multiphasic approach, it may be possible to develop a more realistic distance history of OTG speed. Further, it may be possible to more accurately define the time intervals during the roll sequence, and thereby generate roll rate histories that more closely match the actual event."

It is true that, given sufficient evidence, a skilled reconstructionist could define distances over which a vehicle is in contact with the ground and distances over which it is airborne. However, in general, a discrete events approach would still present substantial challenges. First, there will be many cases when the documented evidence is insufficient to delineate each ground contact and airborne phase. Not only that, once the ground contact regions were identified, the analyst would need some means of linking the impact conditions

² Figure 1 depicts constant deceleration rates for each impact. Using simulation, one could potentially prescribe a variable deceleration rate for each discrete impact.

to the deceleration rate for that impact. This process would likely be rather time consuming and would involve analytical impact modeling or simulation. Further, this process would be iterative since the deceleration rate for a ground impact would depend on the vehicle's OTG speed, the very quantity that the analyst is attempting to calculate. These difficulties could be resolved with further research and this approach is likely to receive attention in the upcoming literature of rollover accident reconstruction.

A second approach, which given the current state of rollover reconstruction techniques seems more viable than the first, will be termed the "discrete regions approach". In this approach, one would not need to precisely parse out each discrete event that occurs during the rollover (though one might have other reasons for doing so). Instead, the analyst would vary the deceleration rate in a sequence of regions, as depicted in Figure 2. This graph depicts a rollover consisting of three regions, each having a different average deceleration rate. One or more of the regions could also be assigned a non-constant deceleration rate – say for instance, Region 2 could be assigned a linearly decreasing deceleration rate that links a high deceleration rate in Region 1 to a low deceleration rate in Region 3. Or, perhaps, Region 1 could be assigned a constant deceleration rate and a linearly decreasing deceleration rate could then traverse both Regions 2 and 3.

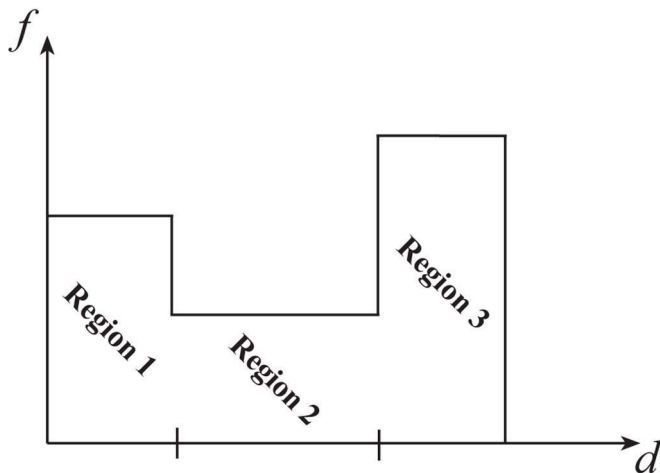


Figure 2

To use this approach, one would need to decide what criteria to use to identify and separate the regions. Much of this paper will focus on a rationale for identifying regions between which a rolling vehicle's deceleration rate would vary. We will show that a vehicle's roll velocity history reveals such regions in which the vehicle's deceleration rate would vary and provides the necessary information for delineating these regions in a manner that has physical meaning.

A third approach, which we'll term the "continuously variable approach", would involve specifying the

variability of the deceleration rate over the course of a rollover with a continuous function. The simplest assumption for this approach would be to assume the deceleration rate decreases linearly with the roll distance as shown in Figure 3. With this assumption, the analyst would either specify the initial and final deceleration rates or the initial deceleration rate and the rate at which it decreases. This approach would be simple to implement and it will be explored in this paper along side the discrete regions approach. This approach is essentially a modified version of the second approach, in which the region-by-region variation in the vehicle's deceleration rate is accomplished with a continuous function.

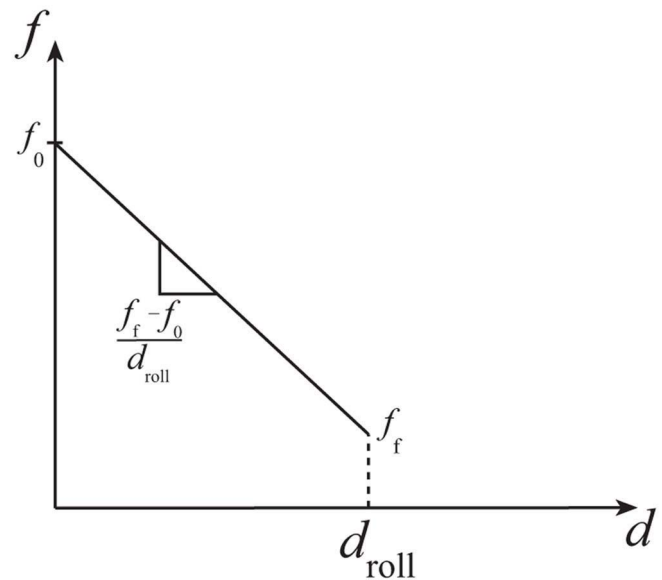


Figure 3

PREVIEW OF CONCLUSIONS

Though empirical data will be discussed extensively in this paper, a large portion of the discussion is slanted towards developing a coherent theory that would provide the basis for a variable deceleration rate approach to rollover accident reconstruction. In developing this theory, we draw heavily on planar impact mechanics, which is introduced most systematically in Reference 4. For readers unacquainted with planar impact mechanics, Reference 4 will provide useful background discussion, particularly related to the critical impulse ratio concept that is used extensively in this paper.

At any rate, in the spirit of getting to the point, we offer the following preview of the conclusions at which this paper ultimately arrives.

1. A typical high-speed rollover can be divided into the following three regions related to the vehicle's roll velocity history: (1) a region during which the vehicle's roll velocity builds up to its peak region; (2) a region in which the vehicle's roll velocity plateaus, with the vehicle

experiencing either small increases or decreases in roll velocity; (3) and a region in which the vehicle's roll velocity decreases more significantly until the roll motion of the vehicle terminates. Some rollovers lack a clearly defined region in which the roll velocity plateaus (Region 2) [9, 10], but these can be seen as special cases of the more general three-region behavior of the typical roll velocity history.

2. These three regions of the roll velocity history reflect changes in the physics of a rollover that occur as the vehicle's OTG speed decreases. Region 1 is generally associated with high OTG speeds, Region 2 with moderate OTG speeds, and Region 3 with low OTG speeds.

3. The underlying physics that causes this three-region behavior in the roll velocity history will also manifest itself in the rate at which the vehicle decelerates over the course of the rollover. Assuming a constant available surface friction coefficient, a rolling vehicle's deceleration rate will generally be highest in Region 1 and lowest in Region 3. In other words, a rolling vehicle's deceleration rate will generally decrease as it progresses through the three regions.

4. Variations in the available surface friction can cause variations in this general trend of a deceleration rate that decreases as the vehicle progresses through the roll. Variations in the available surface friction can be caused by changes in the properties of the surface on which the vehicle is rolling (i.e., moving from asphalt to dirt) or by changes in the way the vehicle engages the surface on which it is rolling (i.e., a rim digging into the surface will have higher "friction" associated with it than a roof rolling smoothly over the same surface).

5. The coefficient of friction between the rolling vehicle and the surface on which it is rolling will influence the deceleration rate of the vehicle. The structural properties of that surface and the manner in which the vehicle engages that surface will influence that coefficient of friction [19].

6. Implementing a variable deceleration rate approach to rollover reconstruction would involve the following steps:

- a. Spatially reconstruct the motion of the vehicle based on physical evidence deposited at the crash scene and on the crash vehicle.
- b. Use a constant deceleration rate to generate an initial estimate of the OTG speed versus distance and roll velocity versus distance curves.
- c. Identify the three (or two) regions of the roll velocity distance history in terms of the roll distance.

- d. Generate a variable deceleration rate profile that will yield the same average deceleration rate with which the initial estimate curves were generated. Multiple shapes, which are discussed in this paper, are available for this profile (e.g., a step function with a different constant deceleration rate for each zone or a continuous function that yields a smooth variation in the deceleration rate).

The variable deceleration rate profile generated in this step should be setup consistent with the basic principles laid out in this paper, but also consistent with the specific details of the particular case being reconstructed. For instance, this paper will show that for a surface of constant friction the vehicle's OTG deceleration rate will tend to decrease as the rollover progresses. However, if the vehicle rolls across multiple surfaces, this trend may not be realized. The real-world rollover analyzed in Reference 1 had a higher deceleration rate in the later stage of the rollover once it had rolled onto a grass and dirt surface than it did in the middle stage of the rollover when it was still rolling on asphalt.

- e. Having generated a variable deceleration rate profile, recalculate the speed versus distance and roll velocity versus distance curves using the variable deceleration rate profile.

ANALYZING VEHICLE-TO-GROUND IMPACTS

A rollover crash consists of a series of vehicle-to-ground impacts separated by periods of airborne motion. The specific conditions of the impacts - including the initial velocities, the orientation and geometry of the impact, and the vehicle's inertial and structural properties - will determine the forces to which the vehicle is subjected. These forces will, in turn, determine the specific motion that the vehicle exhibits. Thus, the rate at which a rolling vehicle decelerates along the ground will be determined by the accumulation of the ground plane forces that are applied to the vehicle during the rollover. Any factor that influences the ground plane forces would also be expected to influence the deceleration rate that the vehicle experiences during a particular vehicle-to-ground impact and, potentially, over the course of the entire rollover.

Several previous papers [26, 27, 28] have reported analysis with a planar, impulse-momentum, vehicle-to-ground impact model based on the idealized vehicle-to-ground impact shown in Figure 4. This model can provide a theoretical context in which to explore those factors that will influence the ground plane forces applied to a vehicle during a rollover, and thus, those factors that will influence its deceleration rate.

The vehicle in Figure 4 is depicted in an inverted orientation with the driver's side roof impacting the

ground. The vehicle has velocity both along and into the ground and a roll velocity that contributes to the speed with which the roof impacts the ground. As a result of this impact, the vehicle is subjected to an impact force that consists of both vertical and ground surface components. The geometry of the impact is defined by the impact radius, which is the distance from the vehicle center-of-mass (CoM) to the point at which the impact force is applied, and the impact angle, which is the angle between the ground plane and the impact radius. Though depicted as a roof-to-ground impact in Figure 4, the equations of this impact model are equally applicable to other types of vehicle-to-ground impacts, such as wheel-to-ground impacts.

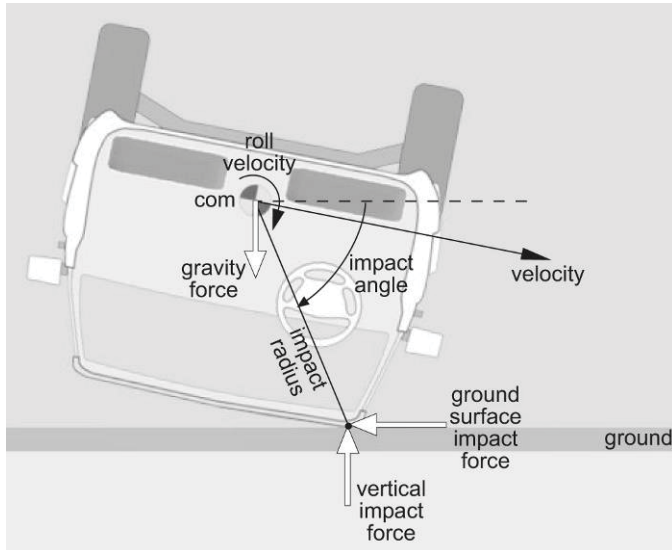


Figure 4

Figure 5 again depicts this idealized impact between a vehicle and the ground. In this figure, the descriptive labels of the previous figures have been replaced with the symbols which are used in the impact model equations. The impact angle and impact radius are designated with the symbols ϕ and r . The velocity vector is designated with the letter v and the vehicle's roll velocity is designated ω_r . During the depicted impact, the vehicle is subjected to both upward and ground surface impact force components, F_{vertical} and F_{ground} , and the gravity force, which is the vehicle's weight. In general, F_{ground} can act in either the positive (left) or negative (right) direction.

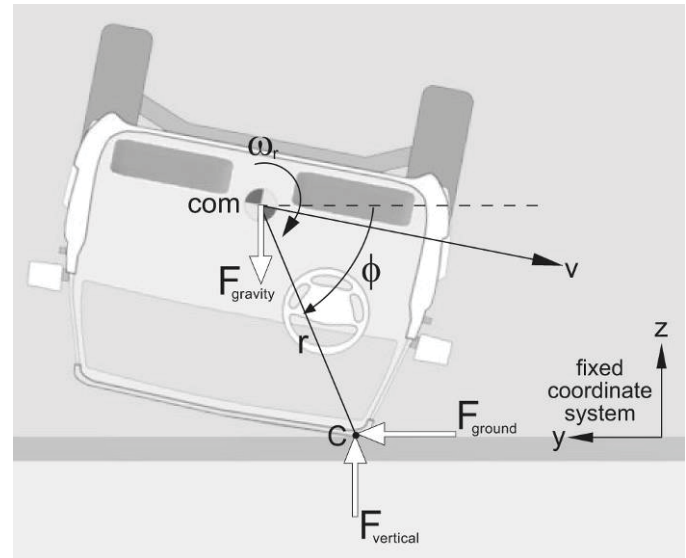


Figure 5

In developing the equations for this impact model, the following assumptions were invoked:

1. The impact was assumed to occur entirely in a single plane, and thus, velocity changes along the vehicle's longitudinal axis are neglected, as are changes in pitch and yaw velocity.
2. The impact model equations recognize no change in the position of the vehicle through the impact. This is not to say that the model cannot be used to analyze a vehicle-to-ground impact during which significant vehicle rotation occurs. When analyzing such an impact with this model, though, the analyst would need to select a vehicle position and orientation that would be most representative of the vehicle's position and orientation through the impact. Reference 27 discusses this issue.
3. The impact force was assumed to be concentrated at a single point. This point is assumed to lie a fixed distance (r) from the vehicle's center-of-mass. Mathematically, this is a rigid body assumption. Nonetheless, to some degree, effects of vehicle deformation can be included in analysis with this model. To incorporate deformation effects, the analyst would define an impact radius and impact angle that reflect the deformed shape of the vehicle. The effect of deformation on the impact angle and radius was discussed briefly in Reference 28.
4. It was assumed that no moment arises at the contact point.
5. Any effects of ground plane restitution have been neglected. In other words, the ground surface impact force was assumed to be a frictional, or retarding, force that depends on relative velocity at the contact point for its development. It is assumed that there is no structural restitution that could potentially cause a velocity reversal in the contact region [4, 27].

Restitution normal to the ground plane is included in the model.

Application of the principle of impulse and momentum for the idealized impact of Figure 5 results in the following equations, which yield the vehicle's upward and ground plane CoM velocity changes (ΔV_z and ΔV_y) and the vehicle's change in roll velocity ($\Delta \omega_r$). The derivation of these equations is provided in Appendix A of Reference 28.

$$\Delta V_z = (1+e) \cdot v_{zc,i} \cdot \left\{ \frac{k_r^2}{r^2 \cdot c\phi \cdot (\mu \cdot s\phi - c\phi) - k_r^2} \right\} \quad (1)$$

$$-g \cdot \Delta t \cdot \left\{ \frac{r^2 \cdot c\phi \cdot (\mu \cdot s\phi - c\phi)}{r^2 \cdot c\phi \cdot (\mu \cdot s\phi - c\phi) - k_r^2} \right\}$$

$$\Delta V_y = \mu \cdot (\Delta V_z + g \cdot \Delta t) \quad (2)$$

$$\Delta \omega_r = (\Delta V_z + g \cdot \Delta t) \cdot \frac{r \cdot (\mu \cdot s\phi - c\phi)}{k_r^2} \quad (3)$$

In these equations, $v_{zc,i}$ is the vertical velocity of the vehicle at Point C immediately preceding the ground contact, k_r is the vehicle's radius of gyration for the roll axis, g is the gravitational constant, Δt is the duration of the impact, and the letters s and c designate the sine and cosine. Although the collision force has been assumed to be transferred without any movement of the vehicle, accounting for the effect of the gravity impulse has required inclusion of the impact duration.

The initial vertical velocity at Point C, $v_{zc,i}$, is given by the following equation:

$$v_{zc,i} = v_{z,i} - r \cdot c\phi \cdot \omega_{r,i} \quad (4)$$

Equations (1) through (3) also include the coefficient of restitution, e , and the impulse ratio, μ . The coefficient of restitution is the negative ratio of the post-impact to the pre-impact vertical velocity at Point C. The impulse ratio is the ratio of the ground plane collision impulse to the vertical direction collision impulse. In many instances, the impulse ratio can be thought of as a coulomb friction value [29], though its application is not limited to this interpretation [5, 6, 7, 8, 20, 28].

In addition to the effects of friction between the ground and the vehicle body, the ground plane impulse will also include the effects of forces generated by snagging between the vehicle and the ground or furrowing of the vehicle into the ground. The "available friction" would be set at a value that reflects such snagging or furrowing when it occurs. For example, a coefficient of friction of 0.5 may reasonably represent the available friction when a vehicle body is sliding on asphalt. But, if a wheel rim gouges into the asphalt, the "available friction" coefficient may need to be set significantly higher than

0.5 to represent the ground plane force arising from the mechanical engagement between the rim and the asphalt [29, 30]. The same would likely be true for some portion of the vehicle gouging or furrowing into soil. The "available friction" for such engagement would likely be much higher than for the vehicle body simply sliding on the surface of the soil [11].

Within the impact model, the sign of the impulse ratio governs the direction in which the ground plane collision force acts. A positive impulse ratio produces a ground plane force that acts in the positive direction and a negative impulse ratio results in a ground plane force that acts in the negative direction. The direction of the ground plane impact force, in turn, determines whether the vehicle will experience a positive or negative ground plane velocity change and whether the ground surface impact force will tend to increase or decrease the roll velocity.

Physically, the ground plane impulse will act in a direction opposing the velocity of the vehicle in its region of contact with the ground (Figures 6a and 6b). Thus, the impulse ratio can be further specified with the following equation:

$$\mu = -\mu_0 \cdot \text{sign}(v_{yc,i}) \quad (5)$$

In this equations, μ_0 is the nominal value of the impulse ratio that defines the magnitude of the ground plane impulse relative to the vertical impulse and $v_{yc,i}$ is the ground plane velocity of the vehicle at Point C.

This velocity is given by the following equation:

$$v_{yc,i} = v_{y,i} + r \cdot s\phi \cdot \omega_{r,i} \quad (6)$$

Thus, for a particular vehicle-to-ground impact, the signs and relative magnitude of the vehicle's CoM ground plane velocity ($v_{y,i}$) and the ground plane component of its rotational perimeter velocity ($r \cdot s\phi \cdot \omega_{r,i}$) will determine the sign of $v_{yc,i}$ and, thus, the sign of μ .

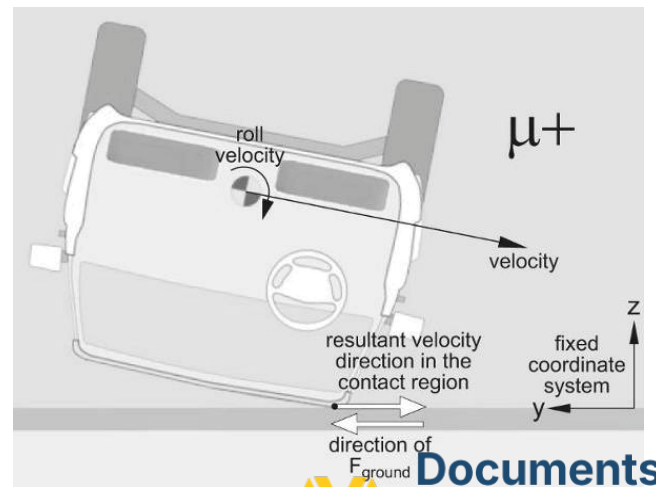


Figure 6a

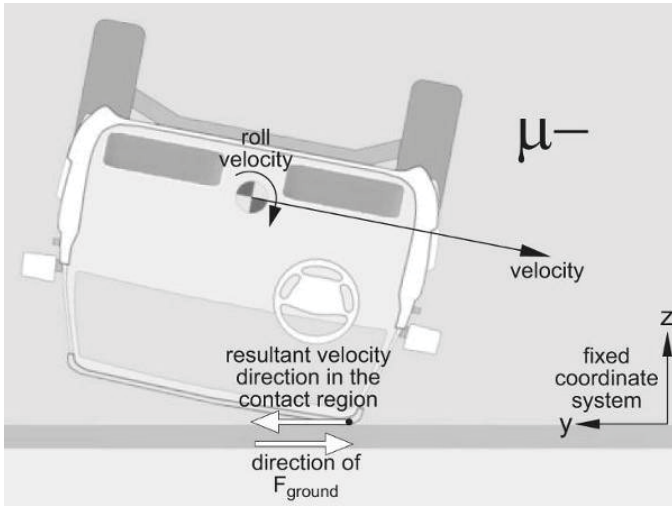


Figure 6b

For any particular vehicle-to-ground impact, there is a critical value of the impulse ratio, μ_c , which will cause relative motion between the vehicle and the ground to cease in the contact region. The impulse ratio will not take on a value that exceeds this critical impulse ratio. Therefore, the magnitude of the available friction coefficient relative to the magnitude of the critical impulse ratio for a particular impact will have physical significance.

The available friction coefficient represents the magnitude of friction force that can be recruited during the impact. The critical impulse ratio represents the magnitude of friction force that must be recruited in order for relative motion to cease along the ground surface in the contact region between the vehicle and the ground. When the critical impulse ratio is greater than the available friction coefficient, then all of the available friction will be recruited during the impact, but that friction will be insufficient to cause sliding to cease in the contact region. When the available friction coefficient exceeds the critical impulse ratio, only a portion of the available friction will be recruited and sliding will cease in the contact region. In such cases, the value of the impulse ratio for the impact model should be set at the value of the critical impulse ratio, not the available friction coefficient. This is because recruitment of the available friction depends on relative velocity being present between the ground and the vehicle body. Once this relative motion ceases, no additional friction can be recruited. These concepts can be summarized as follows:

$$\text{If } \mu_c > \mu_{\text{available}}, \text{ then } \mu = \mu_{\text{available}}.$$

$$\text{If } \mu_c < \mu_{\text{available}}, \text{ then } \mu = \mu_c.$$

The critical impulse ratio for any particular vehicle-to-ground impact is the value of the impulse ratio that results in the final velocity at the Point C being zero, as follows:

$$v_{yc,f} = v_{y,f} + r \cdot s\phi \cdot \omega_{r,f} = 0 \quad (7)$$

To determine how the critical impulse ratio is influenced by the impact conditions and vehicle properties, the authors derived an analytical expression for the critical impulse ratio, which is given by Equation (8) below. The derivation of Equation (8) is detailed in Appendix A.

$$\mu_c = \frac{[(1+e) \cdot v_{zc,i} - g\Delta t] \cdot (r^2 \cdot s\phi \cdot c\phi) + v_{yc,i} \cdot (k_r^2 + r^2 \cdot c^2\phi)}{[(1+e) \cdot v_{zc,i} - g\Delta t] \cdot (k_r^2 + r^2 \cdot s^2\phi) + v_{yc,i} \cdot (r^2 \cdot s\phi \cdot c\phi)} \quad (8)$$

As this equation shows, the critical impulse ratio depends on the following factors: (1) the vehicle's initial velocity conditions for the impact; (2) the coefficient of restitution; (3) the impact duration; (4) and the impact configuration and geometry (which can include the deformed shape of the vehicle). The authors used Equation (8) to calculate critical impulse ratios for a number of impact scenarios. Some of these scenarios are shown graphically in Figure 7.

In this graph, the vehicle's OTG speed is plotted on the horizontal axis and the critical impulse ratio on the vertical axis. The impact scenarios depicted in this graph are for a vehicle CoM downward speed of 2 mph, a roll velocity of 300 degrees per second, a radius of gyration for the roll axis of 2.04 feet, an impact radius of 3.0 feet, a coefficient of restitution of 0, and an impact duration of 300 milliseconds. The motivation for selecting these values is discussed in Reference 28. The graph contains curves for impact angles of 60, 70, 80 and 90 degrees.

This graph demonstrates that for a fixed roll velocity and impact configuration, the critical impulse ratio will decrease as the vehicle's OTG speed decreases. For the sake of illustration, consider a situation in which the available friction coefficient can be assumed to be 0.5. If this is the case, then Figure 8 shows the value which the impulse ratio would take on for each of the impact scenarios of Figure 7.

For all of the impact scenarios occurring at OTG speeds above 25 mph, the critical impulse ratio exceeds the available friction coefficient. Thus, for these scenarios, the impulse ratio takes on a value of 0.5. As the OTG speed drops below 25 mph, impact scenarios begin to arise in which the critical impulse ratios are less than the available friction. In these cases, the impulse ratio takes on the value of the critical impulse ratio.

As it turns out, these concepts surrounding the critical impulse ratio are central to explaining certain aspects of the dynamics of high-speed rollover crashes. In the sections that follow, we will describe the typical shape of the roll velocity history for a vehicle during a high-speed rollover and will show, first, that the behavior of the critical impulse ratio provides a physical explanation for this shape, and second, that there is a direct link between the shape of the roll velocity history for the vehicle and the OTG deceleration history for the vehicle. In fact, we will contend that the roll velocity history for

the vehicle provides the best means for a reconstructionist to setup realistic variations in the

vehicle's deceleration rate over the course of the roll distance.

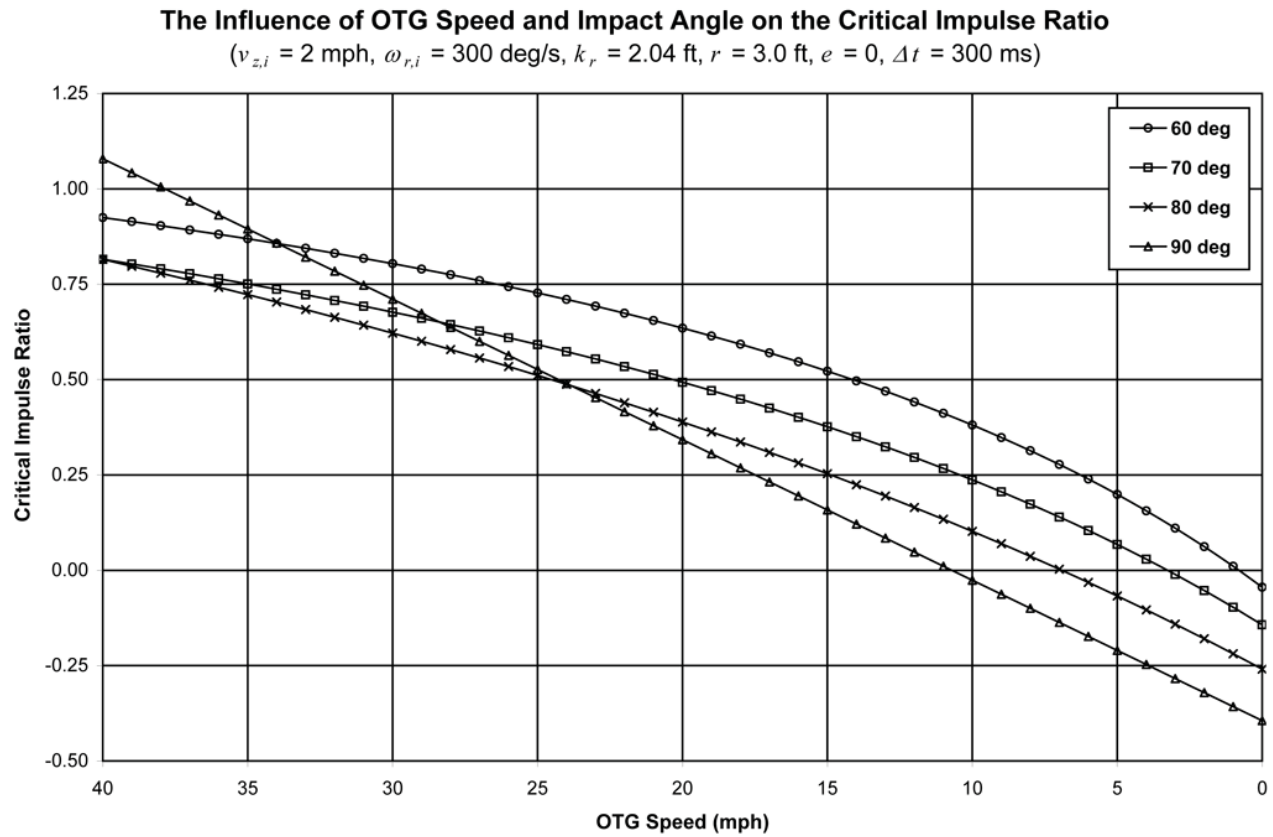


Figure 7