

Elastokinematic Analysis of Compound Crank Axle Suspensions

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ABSTRACT

In the present paper a theoretical model for the analysis of compound crank axle is presented. It is based on the set of equilibrium, constitutive and congruence equations of the axle; to this aim, the cross-member is described by its stiffness matrix, the longitudinal arms are assumed as rigid and the bearing which connects the axle to the body as ideal. For this configuration, a closed form solution for the elastokinematic analysis is obtained in the small displacements field, for the pure roll condition. This can be summarized by the knowledge of the position, in a 3D space, of the instantaneous axis of rotation of each wheel; it is shown how the instantaneous axis of rotation is determined by both the axle geometry and the stiffness properties of the cross member. An iterative procedure has been also developed for the large displacement analysis. Results, in terms of camber and toe angle alteration, are discussed in comparison with those obtained by finite element analyses.

INTRODUCTION

In the present paper a theoretical model for the analysis of compound crank axle is presented. This kind of suspension is widely employed in front wheel drive vehicles in European countries, since it is characterized by low production costs and, nevertheless, a fairly good kinematic behavior (e.g. see [1-3]).

It is characterized by two longitudinal arms, which are fixed to the vehicle frame through bearings and welded to a deformable cross member (Fig. 1), which produces a sort of interconnection between them. The longitudinal arms generally carry the seats for springs and dampers and also are rigidly connected to the wheel carriers.

Even if the cross member can have a similar effect, there is generally an anti-roll bar to set the desired roll stiffness.

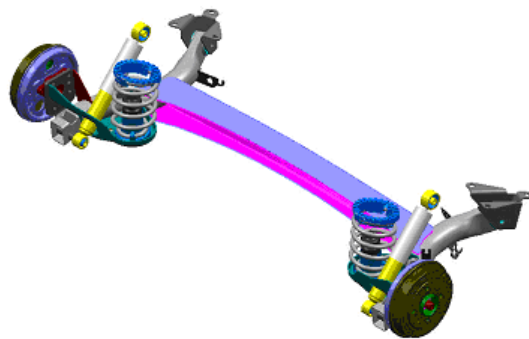


Figure 1. Compound crank axle suspension.

In the design phase of a new suspension it is desirable to fulfill desired kinematic prerequisites, such as the camber and toe angle variations, as a function of wheel travel, for symmetric and anti-symmetric wheel travel; this task is made difficult, due to the small volume available for the suspension systems (links, springs, dampers etc.). For those reasons it would be desirable, from the designer point of view, to know how the main design parameters, such as the geometry of the suspension and the stiffness of the cross member affect such properties. On the contrary, the customary way followed for the design of this kind of suspensions is based on a large number of non linear finite element analysis.

A theoretical model is presented in this work, for the analysis of the kinematic properties of this suspension for a pure roll condition; indeed, the longitudinal arms

have been considered as rigid and the bearings as ideal, the cross member has been described by its stiffness matrix and, so far, the influence of the elastic elements have been not taken into account. The contribution of the bearing compliance and spring loads represents the object of future work.

The model is based on the set of equilibrium, congruence and constitutive equations, written for both arms; indeed, if for the small displacement field the problem is anti-symmetric and it is sufficient to consider just one of the longitudinal arms, in case of large displacements the solution loses any symmetry and it is necessary to consider the full set of equations written for both arms. The governing equations have been implemented in a computer program and the results compared with those obtained by FE analyses.

In addition, the closed form solution for the kinematic analysis has been obtained for the small displacement field (linear solution). This solution is summarized by the position in a 3D space of the instantaneous axis of rotation of each of the longitudinal arms. Due to the presence of a spherical joint, for each of the longitudinal arms, such axis belongs to the family of straight lines passing through the center of the bearing, which join that arm to the vehicle chassis.

The so obtained closed form solutions results particularly interesting since it allows to know the influence of the main design parameters on the kinematic properties of this kind of suspension (toe and camber angle variations) and also allows to determine the roll center of the axle. In addition, the obtained solution seems to represent an original result with respect to the technical literature available for this kind of suspension; indeed, it is generally assumed that the instantaneous axis of rotation of the longitudinal arm is determined by the center of the bearing connecting the arm to the chassis, as mentioned above, and by the geometric center of the middle cross section of the twistable member [4]; in other cases [1, 2], it is assumed that such axis is determined by the center of the bearing connecting the arm to the chassis and the shear center of the middle cross section of the twistable member. As it will be shown, those are approximate results and don't represent the general solution.

MODEL FORMULATION

A typical geometry of the axle is shown in Fig.2; the actual geometry has been simplified as shown in Fig.3: P and Q represent the centers of the bearings which connect the axle to the chassis, B and D the centers of the end sections of the cross member and C and F the wheel centers.

The following hypotheses have been assumed:

- perfectly rigid longitudinal arms;

- ideal bearings in P and Q (with no compliance and without friction);
- straight cross member with uniform cross section;
- zero camber and toe static angles.

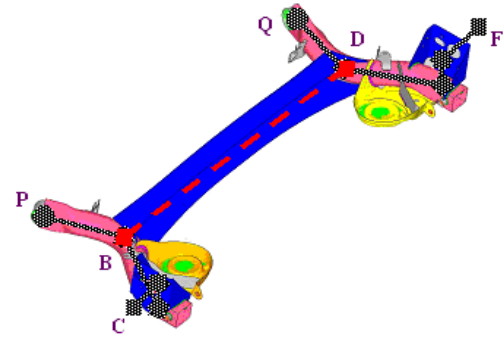


Figure 2. Schematisation of the axle.

It is worth noting that, even if the third and fourth hypotheses don't represent real cases, they have not been considered as important issues for the aims of this work, which were to develop a numerical model for the analysis of the axle and to determine the instantaneous axis of rotation of the longitudinal arms (i.e. of the wheels) in the static configuration; for instance, the third hypothesis has allowed, at this stage of the work, to use the stiffness matrix of a beam element in order to take account of the constitutive properties of the cross member.

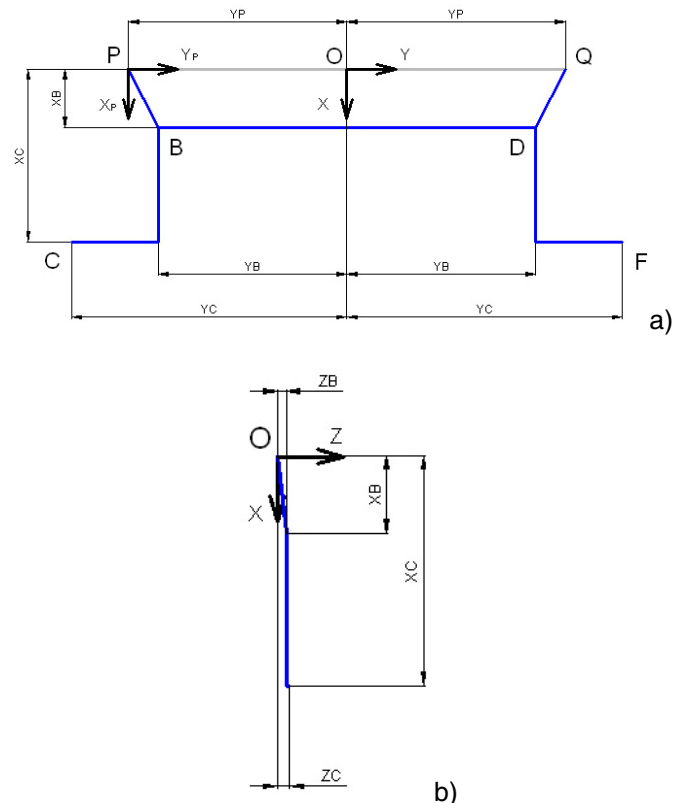


Figure 3. Model of the compound crank axle: a) in plane view, b) side view.

Equal and opposite vertical displacement are assumed at the wheel centers C and F (pure roll) as loading condition.

The following set of equations have been considered for the model: congruence equations, relating the position and orientation of a generic cross section of the longitudinal arms to the characteristic angles of that arm; constitutive equations, which give the forces exerted by the cross member at B and D as a function of the relative positions and rotations of the end cross sections; equilibrium equations, which express the equilibrium condition for both the longitudinal arms subjected to the actions of the cross member, to the vertical forces applied to the wheel centers and to the reactions of the bearings.

CONGRUENCE EQUATIONS – With reference to Fig.3, the following reference systems are considered: an absolute reference system with origin in O, having the X axis directed toward the rear of the vehicle, the Y axis directed toward the right side and the Z axis directed upward along the vertical direction; a reference system useful for the left and right arms, whose origins are located in P and Q and whose unit vectors are parallel to those of the previously defined reference system; the reference system having origin in P is shown in Fig.3.

With reference to the left arm, the following elementary rotations are then defined:

- a rotation ε_1 of the left arm about the Z_P axis; such rotation can be conveniently described by the elementary rotation matrix $\mathbf{R}_1(\varepsilon_1, Z_P)$; it should be noted that, if the right hand rule is used for rotations, for the left arm this angle has opposite sign with respect to the convention used for the toe angle;
- a rotation γ_1 of the longitudinal left arm around the X_P axis, defined by X_P after the first rotation; this rotation is described by the elementary rotation matrix $\mathbf{R}_2(\gamma_1, X_P)$; for the left arm this rotation represent the camber angle, with proper sign;
- a rotation β_1 of the longitudinal left arm around the Y_P axis, defined by Y_P after the previous two rotations; such rotation, called spin, is described by the elementary rotation matrix $\mathbf{R}_3(\beta_1, Y_P)$.

Considering the longitudinal arms as rigid, the spatial position $\mathbf{r}'$ of a given point fixed on the left arm, is given by:

$$\mathbf{r}' = \mathbf{R}_1 \mathbf{r} \quad (1)$$

in which

$$\mathbf{R}_1 = \mathbf{R}_1(\varepsilon_1, Z_P) \mathbf{R}_2(\gamma_1, X_P) \mathbf{R}_3(\beta_1, Y_P) \quad (2)$$

is the rotation matrix resulting from the three elementary rotation previously defined and the vector $\mathbf{r}$ represents the coordinates of that point, expressed in the reference system, fixed on the arm with origin in P. The $\mathbf{R}_1$ matrix, defined for the left arm, has the following expression:

$$\mathbf{R}_1 = \begin{bmatrix} \cos \varepsilon_1 \cos \beta_1 - \sin \varepsilon_1 \sin \gamma_1 \sin \beta_1 & -\sin \varepsilon_1 \cos \gamma_1 & \cos \varepsilon_1 \sin \beta_1 + \sin \varepsilon_1 \sin \gamma_1 \sin \beta_1 \\ \sin \varepsilon_1 \cos \beta_1 + \cos \varepsilon_1 \sin \gamma_1 \sin \beta_1 & \cos \varepsilon_1 \cos \gamma_1 & \sin \varepsilon_1 \sin \beta_1 - \cos \varepsilon_1 \sin \gamma_1 \sin \beta_1 \\ -\cos \gamma_1 \sin \beta_1 & \sin \gamma_1 & \cos \gamma_1 \cos \beta_1 \end{bmatrix} \quad (3)$$

A similar rotation matrix can be obtained for the right arm. By this way it is possible to express the position of a given point located on one of the arms as a function of the elementary rotations. In particular, the positions of the wheel centers C and F and those of the end sections B and D of the cross member, expressed in the (X_P, Y_P, Z_P) and (X_Q, Y_Q, Z_Q) reference systems, are given by equation (1), after substituting to vector $\mathbf{r}$ the coordinates of those points in the relative reference systems.

Among the congruence equations, there are two scalar equations which represent the vertical component of the displacement of the wheel centers, which are denoted by $u_{Z,l}$ and $u_{Z,r}$ respectively. Such equations are directly obtained by writing equation (1) for points C and F. After some calculations, the following algebraic relationships are obtained for the left and right arm respectively:

$$u_{Z,l} = (-\cos \gamma_l \sin \beta_l) X_C - \sin \gamma_l (Y_P - Y_C) + (\cos \gamma_l \cos \beta_l - 1) Z_C \quad (4a)$$

$$u_{Z,r} = -u_{Z,l} = (-\cos \gamma_r \sin \beta_r) X_F - \sin \gamma_r (Y_Q - Y_F) + (\cos \gamma_r \cos \beta_r - 1) Z_F \quad (4b)$$

CONSTITUTIVE EQUATIONS - The cross member was modeled by its stiffness matrix and thus the following relationship holds for general forces $\mathbf{T}$ and displacements $\mathbf{U}$, whose components are expressed in the global reference system:

$$\mathbf{T} = \mathbf{K} \mathbf{U} \quad (5)$$

In which $\mathbf{K}$ can be obtained as a function of the stiffness matrix of the cross member $\hat{\mathbf{K}}$ expressed in the local reference system of the cross member itself. The following relationship holds (e.g. see [5, 6]):

$$\mathbf{K} = \mathbf{R}_{OT}^T \hat{\mathbf{K}} \mathbf{R}_{OT} \quad (6)$$

in which $\mathbf{R}_{OT}$ is the rotation matrix which transforms the global coordinates into those expressed in the local