

ALE based Finite Element Method for Characteristic Simulation of Hydraulically Damped Rubber Mount

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ABSTRACT

Characteristics analysis of hydraulically damped rubber mount (HDM) for vibration isolation in automotive powertrain is very important at the design and development stage of HDM. Fluid-structure interaction (FSI) between rubber parts and fluid flow in chambers in HDM is critical to the frequency-and amplitude-dependent characteristics of HDM. In this paper, an arbitrary-Lagrangian-Eulerian (ALE) based coupling finite element (FE) method is developed to solve fully coupling problem of FSI in HDM. A typical kind of HDM in a vehicle powertrain mounting system composed of rubber spring, two fluid chambers, fluid track and decoupler is selected to investigate performance prediction of HDM. Dynamic characteristic analysis under typical working condition are calculated and compared with experiment results. The agreeable comparison results verify the effectiveness of the presented FSI formulation and characteristic prediction approach of HDM.

INTRODUCTION

Hydraulically damped rubber mounts (HDM) have been widely equipped in vehicle powertrain mounting system (PMS) and plays an important role in automotive NVH (Noise, Vibration and Harshness) control. Its frequency-and amplitude-dependent dynamic characteristics can more effectively attenuate vibrations transmitting between powertrain and body/chassis and reduce the interior noise of automotive compartment than conventional rubber mount. The challenges in characteristic prediction of HDM are large deformation of rubber part and strong coupling analysis of fluid-structure interaction (FSI) between rubber parts and fluid flow in chambers. In HDM, complex FSI between fluid-rubber spring, fluid-uncoupler membrane and fluid-lower diaphragm are essential to its nonlinear dynamic characteristic of subsystem consisting of track fluid and volumetric elastic characteristic of upper chamber. Many researches studied the numerical prediction method of HDM on the basis of lumped-parameter model, in which rubber parts and fluid are modeled independently ignoring the fluid-rubber interactions and rubber hyperelasticity [1-4].

Finite element (FE) method of FSI has been studied in computational fluid and structure mechanics fields since 1970's. Interactions between fluid and structure, in which the fluid pressure acts on structure and deforms the structure, and the structure deformation inversely changes the fluid field, which results in the movable fluid-structure interface and fluid deformation. FEM of FSI is suitable to track movable and large deformation of FSI interface. However, mesh distortion in fluid field causes computational instability. After 1990s, great progress has been made in Arbitrary-Lagrangian-Eulerian (ALE) mesh control technology, stable algorithm for fluid FE computation, numerical method for coupled FE formulation of FSI and mesh method on fluid-structure interface. ALE mesh technology can effectively control mesh distortion and avoid computational failure in large deformation of fluid field. ALE coordinate takes advantages of both Lagrangian and Eulerian coordinates, that is, the advantages of accurately track material movement and no convective term in Lagrangian coordinate and advantages of no mesh distortion in Eulerian coordinate. After Noh [5] proposed ALE method, Hirt [6] and Pracht [7] used it in finite differential method, and then researches such as Belytschko and Kennedy [8], Hughes [9], Donea [10], Huerta and W. K. Liu [11,12] developed ALE in FE analysis of large deformation structure and fluid. Up to now, ALE coordinate description has been widely applied in FEA of fluid, structure and FSI problems with large deformation, movable boundary [11,12], incompressible hyperelasticity rubber material [13] and elastic-plastic deform simulation [14,15] and so on.

In this paper, ALE based FE formulation for barotropic fluid is set up and coupled with rubber solid mixed FE formulation to solve fully coupling problem of FSI. Movement of the internal mesh of fluid domain can be controlled to reduce element distortion and to avoid computation failure. Strong coupling FE formulation of FSI is established by combining FE formulations of rubber and fluid fields according to boundary condition on fluid-rubber interface. Numerical computation of the combined FE formulation can obtain fluid velocity/pressure and rubber displacement/pressure simultaneously. HDM working process simulation and characteristic prediction are studied based on this strong coupling FE method. Dynamic characteristic under typical working condition of a kind of HDM in a vehicle powertrain mounting system is calculated and compared with experiment result, which verifies the effectiveness of the presented modeling and characteristic prediction approach of HDM.

ALE BASED FINITE ELEMENT METHOD FOR FLUID-STRUCTURE INTERACTION

FLUID DYNAMIC EQUATIONS UNDER ALE COORDINATE DESCRIPTION - An arbitrary function under Lagrangian, Eulerian coordinates ALE is described as $f(X,t)$, $f(x,t)$ and $f(\chi,t)$,

$$f(X,t) = f(x,t) = f(\chi,t) \quad (1)$$

The derivative of this function in ALE coordinate can be expressed as,

$$\frac{Df}{Dt} = \frac{\partial f(\chi,t)}{\partial t} \Big|_{\chi} + \frac{\partial f(\chi,t)}{\partial \chi_i} \cdot \frac{\partial \chi_i}{\partial t} = \frac{\partial f(\chi,t)}{\partial t} \Big|_{\chi} + w_i \cdot \frac{\partial f(\chi,t)}{\partial \chi_i} \quad (2)$$

When fluid is assumed to be viscous, isothermal and barotropic with $\partial p / \partial \rho = B / \rho$, the continuity equation in ALE obtained can be expressed as [16]

$$\frac{1}{B} \frac{\partial p}{\partial t} \Big|_{\chi} + \frac{1}{B} c_i \frac{\partial p}{\partial \chi_i} + \frac{\partial v_i}{\partial \chi_i} = 0 \quad (3)$$

The momentum equation of fluid in ALE coordinate can be expressed as

$$\rho \frac{\partial v_i}{\partial t} \Big|_{\chi} + \rho c_j \frac{\partial v_i}{\partial \chi_j} = \rho g_i + \frac{\partial \sigma_{ij}}{\partial \chi_j} \quad (4)$$

where g_i is gravity accelerate. When the constitutive law of Newton fluid is expressed as $\sigma = -pI + 2\mu(D - \frac{1}{3}I \text{div}(v))$, where σ is Cauchy stress,

$$D_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial \chi_j} + \frac{\partial v_j}{\partial \chi_i} \right), \quad \mu \text{ the dynamic viscosity, and}$$

fluid is incompressible, $\text{div}(v)=0$, the constitutive law becomes

$$\sigma = -pI + 2\mu D \quad (5)$$

Then, fluid Navier-Stokes of equation (4) can be expressed as

$$\rho \frac{\partial v_i}{\partial t} \Big|_{\chi} + \rho c_j \frac{\partial v_i}{\partial \chi_j} = \rho g_i - \frac{\partial p}{\partial \chi_i} + \frac{\partial}{\partial \chi_j} \left\{ 2\mu \left(D_{ij} - \frac{1}{3} \frac{\partial v_i}{\partial \chi_j} \delta_{ij} \right) \right\} \quad (6)$$

FE METHOD FOR FLUID-RUBBER INTERACTION

Fluid FE equation - Galerkin FE formulations of fluid continuity equation (4) and Navier-Stokes equation (6) can be described as

$$\sum_e \int_{R_x^e} \left[\delta p \frac{1}{B} \frac{\partial p}{\partial t} \Big|_{\chi} \right] dR_x + \sum_e \int_{R_x^e} \left[\delta p \frac{1}{B} c_i \frac{\partial p}{\partial \chi_i} \right] dR_x + \sum_e \int_{R_x^e} \left[\delta p \frac{1}{B} c_i \frac{\partial v_i}{\partial \chi_i} \right] dR_x = 0 \quad (7)$$

$$\sum_e \int_{R_x^e} \left[\delta v_i \rho \frac{\partial v_i}{\partial t} \Big|_{\chi} \right] dR_x + \sum_e \int_{R_x^e} \left[\delta v_i \rho c_j \frac{\partial v_i}{\partial \chi_j} \right] dR_x - \sum_e \int_{R_x^e} \left[\delta v_i \rho g_i \right] dR_x - \sum_e \int_{R_x^e} \left[\delta v_i \frac{\partial \sigma_{ij}}{\partial \chi_j} \right] dR_x = 0 \quad (8)$$

where δp and δv_i are the differentials of p and v_i , respectively; R_x is the spatial region occupied by material particles at time t .

Finite element equation of continuity equation is obtained by discretion of Equations (7),

$$M^P \cdot \dot{p} + L^P \cdot p + G^T \cdot v = 0 \quad (9)$$

where $\dot{p}$ is fluid pressure derivative in ALE coordinate,

$$M^P = \sum_e M^{Pe}, \quad M^{Pe} = [M_{IJ}^P],$$

$$M_{IJ}^P = \int_{R_x^e} \frac{1}{B} N_a^P N_b^P dR_x, \quad L^P = \sum_e L^{Pe}, \quad L^{Pe} = [L_{IJ}^P],$$

$$L_{IJ}^P = \int_{R_x^e} \frac{1}{B} N_a^P c_k \frac{\partial N_b^P}{\partial x_k} dR_x, \quad G = \sum_e G^e, \quad G^e = [G_{MJ}],$$

$$G_{MJ} = \int_{R_x^e} N_b^P \frac{\partial N_c}{\partial x_m} dR_x, \quad \begin{cases} I = a \\ J = b \\ M = (c-1) \cdot NSD + m \end{cases} \quad \text{and}$$

$$\begin{cases} 1 \leq a, b \leq NEPN \\ 1 \leq c \leq NEN \\ 1 \leq k, m \leq NSD \end{cases}.$$

N_a^P and N_a are shape functions of fluid pressure and velocity respectively, M^P and M are the pressure and velocity terms of mass matrix respectively, L^P and L are the convective terms of pressure and velocity respectively, G is the dissipative matrix, NEPN is the number of pressure nodes, NEN is the number of velocity nodes, and NSD is the dimensional number of coordinate.

FE equation of momentum equation is obtained by discretion of equation (8),

$$M \cdot \dot{v} + L \cdot v + K_\mu \cdot v - G \cdot p = F \quad (10)$$

where $\dot{v}$ is the fluid acceleration in ALE coordinate, K_μ is the viscous matrix, $M = \sum_e M^e$, $M^e = [M_{IJ}^e]$,

$$M_{IJ}^e = \int_{R_x^e} \delta_{ij} \rho N_a N_b dR_x, \quad L = \sum_e L^e, \quad L^e = [L_{IJ}^e],$$

$$L_{IJ}^e = \int_{R_x^e} \delta_{ij} \rho N_a c_m \frac{\partial N_b}{\partial x_m} dR_x, \quad F = \sum_e F^e, \quad F^e = [F_I^e],$$

$$F_I^e = \int_{R_x^e} \rho N_b g_i dR_x, \quad K_\mu = \int_{R_x^e} B^T \cdot D \cdot B dR_x,$$

$$B = [B_1 \cdots B_a \cdots B_{NEN}], \quad B_a^T = \begin{bmatrix} \frac{\partial N_a}{\partial x_1} & \frac{\partial N_a}{\partial x_2} & 0 & 0 & 0 & \frac{\partial N_a}{\partial x_3} \\ 0 & \frac{\partial N_a}{\partial x_1} & \frac{\partial N_a}{\partial x_2} & 0 & \frac{\partial N_a}{\partial x_3} & 0 \\ 0 & 0 & 0 & \frac{\partial N_a}{\partial x_3} & \frac{\partial N_a}{\partial x_2} & \frac{\partial N_a}{\partial x_1} \end{bmatrix},$$

$$D = \begin{bmatrix} 2\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}, \quad \begin{cases} I = (a-1) \cdot NSD + i \\ J = (b-1) \cdot NSD + j \end{cases},$$

and $\begin{cases} 1 \leq i, j, m \leq NSD \\ 1 \leq a, b \leq NEN \end{cases}.$

Fluid FE equation is obtained by combining Equation (9) and (10),

$$M_f \dot{\theta} + C_f \theta = F_f \quad (11)$$

where $M_f = \begin{bmatrix} M^P & 0 \\ 0 & M \end{bmatrix}$, $C_f = \begin{bmatrix} L^P & G^T \\ -G & L + K_\mu \end{bmatrix}$, $\theta = \begin{bmatrix} p \\ v \end{bmatrix}$, and $F_f = \begin{bmatrix} 0 \\ F \end{bmatrix}$.

FE equation of hyperelasticity rubber - The constitutive law of rubber is generally expressed with hyperelasticity as

$$\sigma = -pI + 2\left(\frac{\partial U}{\partial I_1} + I_1 \frac{\partial U}{\partial I_2}\right)B - 2\frac{\partial U}{\partial I_2}B^2 \quad (12)$$

where U is strain-energy function, σ Cauchy stress, I_i ($i=1,2,3$) the principal invariants, B the left Cauchy-Green deformation tensor, I the unit matrix, p the hydrostatic pressure arising from the incompressibility constraint. Various forms of strain energy function used to describe different kinds of rubber. Polynomial strain energy function is a usual kind as shown in the following,

$$U = \sum_{i+j=1}^N C_{ij} (I_1 - 3)^i (I_2 - 3)^j + \sum_i \frac{1}{D_i} (J_{el} - 1)^{2i} \quad (13)$$

where the initial shear and bulk modulus are $\mu_0 = 2(C_{10} + C_{01})$ and $k_0 = \frac{2}{D_1}$. J_{el} is the elastic

volume strain. D_i determines the compressibility of the material and is determined from pure volumetric compression test data. I_1 and I_2 are the first and second derivatoric strain invariants, $I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$ and $I_2 = \lambda_1^{-2} + \lambda_2^{-2} + \lambda_3^{-2}$.

Mixed formulation FE equation is obtained by spatial discretization of the hyperelastic solid,

$$M_s \cdot \ddot{d}_s + K_s \cdot d_s = F_s \quad (14)$$

where analyzed variable is $d_s = \begin{bmatrix} u_s \\ p_s \end{bmatrix}$, exciting force $F_s = \begin{bmatrix} F_u \\ 0 \end{bmatrix}$ and F_u is the external force on hyperelasticity

