

A Study on Effect of Part-loading Errors on the Accuracy of Compound Hole Axis

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ABSTRACT

Drilling compound hole is very common material processing operation in engineering industry. In this special machining process before drilling the hole the part is rotated about an auxiliary axis parallel to one of the reference axes. Accuracy of the drilled hole is paramount in several applications such as in cylinder head and some other applications of bone surgery. Inaccuracy in hole axis may result in the loss of engine efficiency in the case of Cylinder Head or unsuccessful bone surgery.

In the previous papers Murty et al. [8], [9] the algorithms developed had restricted application. In the first paper the tool was restricted to move in vertical plane and *WY*-plane was considered as reference plane. In the second paper the tool movement was unrestricted in space but the *WY*-plane was considered as reference plane and the skew error was assumed only in *WY*-plane. However, in the present paper the authors developed a general purpose algorithm for error compensation when the tool movement is unrestricted in space and the part loading errors may include translational errors in 3-Cartesian directions and rotational errors in *WY*-plane and *XY*-plane about 2 axes parallel to Cartesian axes. The authors also give trend lines which can be used to sufficient accuracy for computing corrective motions of the machine axes. This paper employs the principles of inverse kinematics wherein a faulty compound-hole angle axis in space caused by the translational and rotational errors at the part loading position is identified with an imaginary true axis in space by enforcing identity through a modified machine axes taking into effect inclination of the drill tool in space. In the absence of any specific application, this algorithm is verified on AutoCAD, a commercial CAD tool and found to be correct as shown in an example problem given at the end. For small values of part-loading errors plot between skew error in *XY*-plane and angular corrections in part rotation and spindle axis orientation show linear relationship.

Key Words: *Compound angle, Cylinder head, Cummins engine, error compensation, Valve seat, robotic manipulator, orthopedic operations.*

1 INTRODUCTION

Figure 1 shows a schematic of a part loaded with errors in a fixture. For general kind of compound-angle machining problems, the possible part loading errors are transformed into translation and rotation errors of the compound-angle hole-axis. One widely used procedure to solve them is the error compensation scheme by way of an algorithm. The correction algorithm is loaded into the CNC controller to compute and impart the required corrected movements to the machine axes. The major difficulty associated with part-loading is that such errors are very difficult to eliminate due to irregular part configuration and human intervention. The part is loaded only once in its position to the best of the operator's ability minimizing the errors. The translation errors and skew angle errors are measured by probing the locating holes at fixed positions placed with reference to the part location. The effect of these part-loading errors on accuracy of the compound-angle axis is determined to mathematical accuracy using forward and inverse kinematics and the corrected machine axes movements are formed with the help of the algorithm. The correction algorithm is then loaded into the CNC machine controller to impart corrected movements to the machine axes as a function of the measured errors and other known initial parameters. This method is preferred by many and gains popularity in that less human interruption is involved. However, it demands much more skill on the part of the operator in accurately implementing the algorithm with due regard to the sign convention and sequence of operations as explained in subsequent paragraphs.

1.1 LITERATURE REVIEW OF PREVIOUS WORK

In multi-axis machining errors are associated with the toolpath generation. Moreton and Durnford [7] developed an algorithm for a three axis-turning center (*X*, *Y* and *C*-axis) which is used for milling surfaces. An algorithm is used to compensate for kinematic and interpolation errors which are compensated in the CNC program. Their algorithm provides the tool diameter and the errors in

function of the tool diameter, number of Cutter Location points and toolpath length. Balsamo et al. [1] suggested the use of techniques based on ball plate to determine parametric errors in CMM machines Hwang [4] used a kinematic model to derive equations in closed form for the real toolpath curve for a 4-axis milling machine. For cases when the cutter workpiece interface is not a cutter contact point but a curve, Bohez et al. [2] discussed the errors introduced during 5-axis machining which are higher than the intrinsic repeatability of the machine tool. Author has shown the possibility to identify such systematic errors and compensate for them achieving higher performance. *A group of systematic errors can be compensated for directly in the inverse kinematics equations.* Other systematic errors can be combined and compensated for through the total differentials of the inverse kinematics relations. Bohez et al. [3] discussed a CAM algorithm for geometric error correction for ruled surface 5-axis milling operation. Suh and Lee [6] in their paper measured the machine errors using a ball-bar in a contouring test and then incorporated them into the CAM to optimize the toolpath accuracy. Ramesh et al. [10] reviewed the sources of errors and the methods to eliminate errors through error compensation for machine tools. The review like the past methods focuses on the error measurements and compensation but not on modeling. Sartori and Zhang [11] classify the methods for geometric measurement as direct and self-calibration based on applied standards. One important reference in this regard is the combination of the individual errors in the total error in Shen and Duffie [12]. Algorithms developed by Murty et al. [8] considered tool motion in vertical plane YZ only and Murty et al. in [9] considers no translation error in X-direction and no skew error in XZ-plane in part-loading errors. Please refer to these 2 papers for additional literature in error analysis.

In this paper the authors are focused on compound angle machining on parts loaded with translational and rotational part-loading errors. The objective is to develop an algorithm to compensate for these part-loading errors compounded by the kinematic motions of the part and tool.

2 FORMULATION OF COMPENSATION ALGORITHM

2.1 PROBLEM STATEMENT AND ASSUMPTIONS

As illustrated in Figure 1 a sample block representative of a part is laid on an index table, confined within a fixture and subject to some translation and rotation errors. The scope of errors is depicted in Figure 2. Top center of the movable index table is considered as the reference origin of the (XWY) coordinate system used in CNC machining convention with reference to which all the position coordinates are designated. B -axis is the vertical axis attached to the index table which coincides with the spindle Y -axis in the compound hole machining position.

We first look at the mathematical statement of the problem posed. In an actual compound-angle machining operation with the part accurately loaded into the fixture, the index table is rotated by a specified angle I about the index table B -axis and then the hole is drilled using the

drill at the desired inclination T in the XW -plane and S in the WY -plane. In this situation, the compound-hole axis PQ would be perfect or true. However, as depicted in a blown up picture of the part in Figure 1 the presence of part-loading errors compounded by the kinematic motions of part and tool shifts the compound-hole axis from its true position P_tQ_t to a faulty position P_fQ_f . Note that in both the cases the compound-hole axis remains straight and there is no curvature. Thus for the purposes of an error compensation algorithm the problem reduces to giving the required transformations to the faulty axis in space to make it collinear with the true axis in space. This is because linear movement of the drill tool along the spindle axis can account for collinear separation between the true and faulty axes along the tool movement.

2.2 PART LOADING ERRORS AND COMPENSATION SCHEME

There are three issues to consider. First, the translation and rotational errors at the part loading position are unavoidable in any machining operation. Second, although it is possible to superimpose the effects of these errors on the accuracy of the compound angle axis, what complicates the problem is that these errors are invariably measured with reference to a different datum. So all the errors and the compensation scheme are required to be transformed with reference to a single (XWY) coordinate axes system. Five degrees of freedom namely rotation of the index table, rotation of the spindle axis and three linear motions of the location of the spindle axis are available for correction measures and these are considered sufficient. Figure 2 shows a schematic of the possible part-loading errors, which are translation errors Δ_x , Δ_w , and Δ_y respectively in X , W , and Y direction, and rotation error or skew angle (θ) about X -direction and (β) about W -direction. Figure 3 shows the location of the idealized hole from index table B -axis origin is marked (P_{0x}, P_{0w}, P_{0y}). The skew (rotational) error θ occurs about an axis passing through the locating hole B_6 and parallel to X -direction. and the skew (rotational) error (β) occurs about an axis passing through the locating hole B_6 and parallel to W -direction. Transitional errors Δ_x , Δ_w , and Δ_y , are measured by probing locating holes at C and D and skew errors θ and (β) are measured about locating hole D . Location of locating holes C and D is shown in Figure 1.

To develop an error compensation mechanism it is necessary to determine the direction cosines of the true axis and the faulty axis in space.

2.2.1 Direction Cosines of true axis

To determine the direction cosines of the true axis in space, first it is necessary to determine the position coordinates of the points where the axis intersects the top and bottom centers of the hole or the part. We begin with

0th position where the top center and bottom center of an imaginary *vertical hole* are defined as P_0 and Q_0 . To drill an idealized hole, the part is rotated by an angle I about the index table B -axis and then the part is translated by $-S_z$ along negative X -direction. These two movements would not alter the position coordinates of the top center and bottom center of the hole on the part, which are now defined as P_1 and Q_1 . This is designated as the 1st position. In the next operation, the spindle axis is rotated at an angle T in the ZX -plane and by S in the YZ -plane and the drill bit is placed at the top center of the hole P_2 and the drilling operation is completed. If there were no part loading errors, then the drill would exit from the true point Q_2 located on the part so that the true hole-axis P_2Q_2 is formed in its 2nd position. From the known position coordinates of top center of the hole and the spindle rotation angle T and S , using forward transformation matrices, the position coordinates of the point Q_2 on the part at which the drill exits is determined. From this, the direction cosines of the true axis P_2Q_2 are determined. Note here that the expressions for the direction cosines of the true axis part loading errors do not appear.

2.2.2 Direction Cosines of faulty axis

To determine the direction cosines of the faulty axis in space, first it is necessary to determine the position coordinates of the points where the faulty axis intersects the top and bottom centers of the hole on the part. This is a tricky situation and it is not possible to follow the procedure followed for the true axis straight away. First it is necessary to determine the true position coordinates of the exit point of the true axis at the part loading position where the errors occur. For this, the true axis P_2Q_2 is, rotated by an angle $-I$ about the index table B -axis. Inverse transformation matrices are used to determine the new position coordinates of the point Q_3 to which the point Q_2 on the true axis would move consequent to this reverse motion. However, these motions would not alter the position coordinates of the top center of the hole and so the coordinates of point P_3 would remain the same as P_2 . At this point with the new points on the axis of the hole at the part loading position would be,

$$P_3 = (P_{2x} + \Delta x_{P_2}, P_{2w} + \Delta w_{P_2}, P_{2y}) \quad (1)$$

$$Q_3 = (Q_{2x} + \Delta x_{Q_2}, Q_{2w} + \Delta w_{Q_2}, Q_{2y}) \quad (2)$$

Here Δx_{P_2} , Δw_{P_2} , Δx_{Q_2} , Δw_{Q_2} show incremental changes to the coordinates of point P_2 and Q_2 due to part rotation by an angle $-I$ about B -axis. This would mean that at the part loading position, the true axis is represented by P_3Q_3 . Now we progressively consider the effect of part loading errors on the coordinates of top and bottom centers of the compound hole to determine the position coordinates of the faulty axis. At this point with the consideration of translation errors, the new points on the axis of the hole at the part loading position would be,

$$P_4 = (P_{3x} + \Delta x, P_{3w} + \Delta w, P_{3y} + \Delta y) \quad (3)$$

$$Q_4 = (Q_{3x} + \Delta x, Q_{3w} + \Delta w, Q_{3y} + \Delta y) \quad (4)$$

Now, forward transformation matrices are used to consider effect of the skew error θ about the probe B_6 and an axis parallel to X -direction to give:

$$P_5 = (P_{4x}, P_{4w} + \Delta w_{P_4}, P_{4y} + \Delta y_{P_4}) \quad (5)$$

$$Q_5 = (Q_{4x}, Q_{4w} + \Delta w_{Q_4}, Q_{4y} + \Delta y_{Q_4}) \quad (6)$$

These are the new position coordinates of the top and bottom centers of the hole where Δw_{P_4} , Δy_{P_4} , Δw_{Q_4} , Δy_{Q_4} , are incremental changes to the coordinates of point P_4 and Q_4 due to skew error θ .

Now, again forward transformation matrices are used to consider effect of the skew error β about the probe B_6 and an axis parallel to W -direction to give:

$$P_5 = (P_{5x} + \Delta x_{P_5}, P_{5w}, P_{5y} + \Delta y_{P_5}) \quad (5)$$

$$Q_5 = (Q_{5x} + \Delta x_{Q_5}, Q_{5w}, Q_{5y} + \Delta y_{Q_5}) \quad (6)$$

These are the new position coordinates of the top and bottom centers of the hole where Δx_{P_5} , Δw_{P_5} , Δx_{Q_5} , and Δw_{Q_5} are incremental changes to the coordinates of point P_5 and Q_5 due to skew error β .

The part in this position is rotated by an angle I about the index table B -axis. Next forward transformation matrices are used to determine the new position coordinates of the faulty hole axis as,

$$P_6 = (P_{5x} + \Delta x_{P_5}, P_{5w} + \Delta w_{P_5}, P_{5y}) \quad (7)$$

$$Q_6 = (Q_{5x} + \Delta x_{Q_5}, Q_{5w} + \Delta w_{Q_5}, Q_{5y}) \quad (8)$$

Here Δx_{P_5} , Δw_{P_5} , Δx_{Q_5} , Δw_{Q_5} show incremental changes to the coordinates of point P_5 and Q_5 due to part rotation by an angle I about B -axis.

Mathematically stated these two points represent the position coordinates of the faulty axis. If this faulty axis were to be collinear with the true axis then their direction cosines must be the same in all the planes XW , WY , and YX . Further, if two lines in space are parallel in two planes, they shall also be parallel in the third plane because the sum of squares of the direction cosines is always equal to unity. Thereafter only linear axes movements will be required to make the true and faulty axes coincident.

2.2.3 Determine Δ_I in XW -Plane

We first attempt to align the two axes in XW plane by equating the inclination of lines P_2Q_2 and P_6Q_6 . The inclinations of the true and faulty axes in XW -plane are determined as follows. This would require the faulty axis to be rotated by an incremental angle about B -axis and that after trigonometric transformations would yield the following result. Simplified expression for Δ_I is given at the end.

$$P_2Q_{2xw} = a \tan \left(\frac{P_{2x} - Q_{2x}}{P_{2w} - Q_{2w}} \right) \quad (9)$$



$$P_6 Q_{6xw} = a \tan \left(\frac{P_{6x} - Q_{6x}}{P_{6w} - Q_{6w}} \right) \quad (10)$$

$$\Delta_I = a \tan \left(\frac{(P_{6w} - Q_{6w}) \times \left(\frac{P_{2x} - Q_{2x}}{P_{2w} - Q_{2w}} \right) - (P_{6x} - Q_{6x})}{(P_{6x} - Q_{6x}) \times \left(\frac{P_{2x} - Q_{2x}}{P_{2w} - Q_{2w}} \right) - (P_{6w} - Q_{6w})} \right) \quad (11)$$

Consequent upon rotating the faulty axis by this incremental angle, the true and faulty axes are made collinear in the XW plane but this operation would alter X and W coordinates of the top and bottom centers of the faulty axis. The new coordinates of the top and bottom centers of the faulty axis are determined by using forward transformation matrices and are found to be:

$$P_7 = (P_{6x} + \Delta x_{P6}, P_{6w} + \Delta w_{P6}, P_{6y}) \quad (12)$$

$$Q_7 = (Q_{6x} + \Delta x_{Q6}, Q_{6w} + \Delta w_{Q6}, Q_{6y}) \quad (13)$$

Here Δx_{P6} , Δw_{P6} , Δx_{Q6} , Δw_{Q6} show incremental changes to the coordinates of point P_6 and Q_6 due to part rotation by an incremental angle Δ_I about B -axis to make the true and faulty axes collinear in the XW plane

2.2.4 Determine Δ_S WY -Plane

We then attempt to align the two axes in XY -plane by equating the inclination of lines P_2Q_2 and P_7Q_7 . The inclinations of the true and faulty axes in XY -plane are determined as follows. This would require the faulty axis to be rotated by an incremental angle about S -axis and that is given by the following expression. Simplified expression for Δ_S is given at the end such that true tool inclination is $S' = S + \Delta_S$.

$$P_2 Q_{2wy} = a \tan \left(\frac{P_{2y} - Q_{2y}}{P_{2w} - Q_{2w}} \right) \quad (14)$$

$$P_7 Q_{7wy} = a \tan \left(\frac{P_{7y} - Q_{7y}}{P_{7w} - Q_{7w}} \right) \quad (15)$$

$$\Delta_S = a \tan \left(\frac{(P_{7w} - Q_{7w}) \times \left(\frac{P_{2y} - Q_{2y}}{P_{2w} - Q_{2w}} \right) - (P_{7y} - Q_{7y})}{(P_{7y} - Q_{7y}) \times \left(\frac{P_{2y} - Q_{2y}}{P_{2w} - Q_{2w}} \right) - (P_{7w} - Q_{7w})} \right) \quad (16)$$

2.2.5 Determine (S'_x, S'_w, S'_y) to position the tool on the part to drill the hole.

In the previous step we determined the true inclination of the spindle axis. We shall now determine the position coordinates of the top center of the hole at which the tool must be positioned to drill the correct hole. For this purpose we start from the position coordinates of the top center of the hole with part loading errors $P_{51} = (P_{51x}, P_{51w}, P_{51y})$. Since the part is now being rotated by I' , we determine the new displaced top center of the

hole. $P_8 = (P_{8x} = P_{51x}, P_{8w} = P_{51w}, P_{8y} = P_{51y})$ and then $P_9 = (P_{9x}, P_{9w}, P_{9y})$ resulting from using forward transformation matrices with an angle of $I' = I + \Delta_I$. This will give us the true coordinates of the top center of the hole at which the tool needs to be positioned. This is to be considered in both XW , and WY planes separately.

2.2.6 Mathematical check

Check the correctness of the correct coordinates for the spindle position S'_w and S'_y thus computed by checking the angle formed by the line P_9Q_7 in WY -plane which must be the same as S' and in XW -plane which must be the same as angle T which is proved by expressions on MathCAD file. Note that the variables used in Mathcad are: (X, W, Y) as against our coordinate system of (X, Y, Z) on AutoCAD.

$$\text{atan} \left(\frac{\text{NewP9y} - \text{NewQ7y}}{\text{NewP9w} - \text{NewQ7w}} \right) = 19.738296 \text{ deg}$$

$$\text{atan} \left(\frac{\text{NewP9y} - \text{NewQ7y}}{\text{NewP9w} - \text{NewQ7w}} \right) = 0.344498 \text{ rad}$$

$$\text{atan} \left(\frac{\text{NewP9x} - \text{NewQ7x}}{\text{NewP9w} - \text{NewQ7w}} \right) = 40 \text{ deg}$$

$$\text{atan} \left(\frac{\text{NewP9x} - \text{NewQ7x}}{\text{NewP9w} - \text{NewQ7w}} \right) = 0.698132 \text{ rad}$$

$$\text{atan} \left(\frac{\text{NewP9x} - \text{NewQ7x}}{\text{NewP9y} - \text{NewQ7y}} \right) = 66.848038 \text{ deg}$$

$$\text{atan} \left(\frac{\text{NewP9x} - \text{NewQ7x}}{\text{NewP9y} - \text{NewQ7y}} \right) = 1.166718 \text{ rad}$$

2.2.7 Determine (S''_x, S''_w, S''_y) with the lengths of spindle L_S and tool L_T

And last, the effect of the gage length and tool length is schematically described in Figure 4. Obviously as expected these parameters only affect the correction in W and Y -directions. Now we consider the position coordinates of the spindle axis and the lengths of the spindle and tool to derive final expressions for the new spindle axis location (S''_x, S''_w, S''_y) as follows.

$$S''_x = S'_x \quad (19)$$

$$S''_w = S'_w + (L_S + L_T) \times \cos(S') \quad (20)$$

$$S''_y = S'_y + (L_S + L_T) \times \sin(S') \quad (21)$$



3 EXPRESSIONS FOR A GENERAL CASE WITH 3-TRANSLATION AND 2-SCREW ERRORS WHERE TOOL MOTION IS UNCONSTRAINED IN SPACE

3.1.1 Incremental equations

The following incremental equations are derived for various parameters.
(22)

$$\text{NewIncremental} := \text{atan} \left(\frac{(\text{NewP6x} - \text{NewQ6x}) - (\text{NewP6w} - \text{NewQ6w}) \cdot \tan \text{New}\theta 1}{(\text{NewP6w} - \text{NewQ6w}) + (\text{NewP6x} - \text{NewQ6x}) \cdot \tan \text{New}\theta 1} \right)$$

$$\text{NewSIncremental} := \text{atan} \left(\frac{(\text{NewP7y} - \text{NewQ7y}) - (\text{NewP7w} - \text{NewQ7w}) \cdot \tan \text{New}\theta 5}{(\text{NewP7w} - \text{NewQ7w}) + (\text{NewP7y} - \text{NewQ7y}) \cdot \tan \text{New}\theta 5} \right)$$

(23)

Complete expressions are available on MathCAD file.

$$\Delta S_x = (S_x'' - S_x) \quad (24)$$

$$\Delta S_w = (S_w'' - S_w) \quad (25)$$

$$\Delta S_y = (S_y'' - S_y) \quad (26)$$

3.1.2 Absolute equations

Absolute equations as given below are determined by algebraically adding the incremental corrections to the correct values without part loading errors. Eventually, the machine controller will impart these movements to the machine axes to arrive at the true axis for the compound hole. After simplification, the absolute equations in terms of the known parameters are as follows.

$$I' = I + \Delta_I \quad (27)$$

$$S' = S + \Delta_S \quad (28)$$

$$S_x'' = (S_x + \Delta S_x) \quad (29)$$

$$S_w'' = (S_w + \Delta S_w) \quad (30)$$

$$S_y'' = (S_y + \Delta S_y) \quad (31)$$

3.1.3 Sign convention for angular corrections

The skew angle errors θ and β may be in either direction. Further, the expressions for Δ_I and Δ_S , given in Eq. (22) and Eq. (23) may result in positive or negative numbers. If the skew error is counter clockwise, then θ or β is considered negative. If expression for Δ_I is positive then index table is incremented in the counter clockwise direction by this amount. If Δ_S is found positive then the spindle axis rotation is incremented by this amount in the counter clockwise direction.

4 COMPENSATION ALGORITHM WHERE TOOL MOTION IS UNCONSTRAINED IN SPACE

Transformation equations are used in conjunction with input variables that become input values upon use. The

following lists the steps required in computing the final solution. It is essential to follow the following steps in the proper order with proper regard to the sign convention while feeding the algorithm to the machine controller and if not done so, the results will go wrong.

- Compute Δ_I using the input data and *incremental offset equation* for Δ_I .
- Compute Δ_S using the input data, Δ_I computed in (1) above and *incremental offset equation* for Δ_S .
- Compute s' using the input data and Δ_I computed in (1) and *absolute offset equation* for S' .
- Compute S_x'' , S_w'' , and S_y'' , using the input data, Δ_I , and s' computed in (1), and (3) above, and *absolute offset equations* for S_x'' , S_w'' , and S_y'' .
- Compute ΔS_x , ΔS_w , and ΔS_y , using the input data, S_x'' , S_w'' , and S_y'' computed in (4) above, and *incremental offset equations* for ΔS_x , ΔS_w , and ΔS_y .

Note: The steps are required to be followed in the proper order because expression for Δ_S contains Δ_I which must be determined first. This has occurred because true and faulty axes were made parallel in XW - plane first. It is also possible to first equate the angles in WY -plane and in that case expression for Δ_I will contain the term of Δ_S , which will have to be determined first with a different set of equations.

5 VERIFICATION OF ALGORITHM

In the absence of a specific part, the algorithm is verified using AutoCAD tool.

5.1 EXAMPLE 1

5.1.1 Input parameters for the compound hole

The position coordinates of the top center of the true hole are $P_{0z} = 254 \text{ mm}$, $P_{0x} = 355.6 \text{ mm}$, and $P_{0y} = 304.8 \text{ mm}$. Part loading errors: Translational errors are $\Delta_z = -0.24097 \text{ mm}$, $\Delta_x = 0.230607 \text{ mm}$, $\Delta_y = -0.226619 \text{ mm}$, and Rotational errors are $\theta = -0.160168 \text{ deg}$, $\beta = 1.047895 \text{ deg}$.

5.1.2 Assumed known parameters

The position coordinates of the D locating hole are $D_z = 25.4 \text{ mm}$, $D_x = 159.5501 \text{ mm}$, and $D_y = 127 \text{ mm}$. The spindle rotation angle is $S = 20 \text{ deg}$, $T = 40 \text{ deg}$. The index table rotation is $I = -(180 \text{ deg} + 20 \text{ deg})$. Position coordinates of the spindle axis are $S_z = -254 \text{ mm}$, $S_x = -304.8 \text{ mm}$, $S_y = 203.2 \text{ mm}$.

5.1.3 Output from Mathcad Numerical Tool (Figure 5)

The results obtained using are obtained in Mathcad program are reproduced below. Coordinate System: (XWY) . Δ_I , Δ_S , ΔS_x , ΔS_y , and ΔS_w are represented on

MathCAD using variables NewIncremental, NewSincremental, NewXincremental, NewWincremental, and NewYincremental respectively.

$$\begin{aligned}\Delta_I &= \text{NewIncremental} = 0.289094 \text{ deg}, \\ \Delta_S &= \text{NewSincremental} = -0.261704 \text{ deg} \\ \Delta S_x &= \text{NewXincremental} = -101.986393 \text{ mm} \\ \Delta S_w &= \text{NewWincremental} = 53.833858 \text{ mm}, \text{ and} \\ \Delta S_y &= \text{NewYincremental} = 104.970633 \text{ mm}.\end{aligned}$$

5.1.4 Output from AutoCAD

Coordinate system: (XYZ)

$$\begin{aligned}\Delta_I &= \text{NewIncremental} = 0.289094 \text{ deg}, \\ \Delta_S &= \text{NewSincremental} = -0.261705 \text{ deg} \\ \Delta S_x &= \text{NewXincremental} = -101.986393 \text{ mm} \\ \Delta S_w &= \text{NewWincremental} = 53.833858 \text{ mm}, \text{ and} \\ \Delta S_y &= \text{NewYincremental} = 104.970633 \text{ mm}.\end{aligned}$$

See Figure 7 for the AutoCAD output.

$$\begin{aligned}P_{7x} &= -355.986393 \text{ mm} \\ P_{7w} &= -250.966142 \text{ mm} \\ P_{7y} &= 308.170663 \text{ mm}\end{aligned}$$

Note that the variables on MathCAD file have are prefixed with "New". They are different from the AutoCAD coordinate system.

6 RESULTS

6.1 STUDY ON THE EFFECT OF TRANSLATION ERROR Δ_x

Figure 8 shows Table of values showing the effect of Δ_x error on Δ_I , Δ_S , corrections in spindle position Δ_{Sx} or $X_{Incremental}$ or $(S_x'' - S_x)$ -, and $W_{Incremental}$ or Δ_{Sw} or $(S_w'' - S_w)$ and Δ_{Sy} or $Y_{Incremental}$ or $(S_y'' - S_y)$.

Figure 9 (a) shows that for small values of Δ_x error the plot Δ_x vs. Δ_{Sx} is linear and the equation of the linear regression line is $X_{Incremental} = -0.9398\beta - 4.1924$. Figure 9(b) shows that for small values of Δ_x error the plot Δ_x error vs. Δ_{Sw} is linear and the equation of the linear regression line is $W_{Incremental} = 0.3148\beta + 2.2338$. Figure 9(c) shows that for small values of Δ_x error the plot of Δ_x error vs. $I_{Incremental}$ (Δ_I) is linear and the equation of the linear regression line is $I_{Incremental} = 0.0154\beta$, and Δ_x error vs. $S_{Incremental}$ (Δ_S) is also linear with the trend line $S_{Incremental} = 0.1869\beta$. These corrections in angular corrections are observed to be independent of Δ_x error.

6.2 STUDY ON THE EFFECT OF SKEW ERROR β

Figure 10 shows Table of values showing the effect of β error on Δ_I , Δ_S , corrections in spindle position Δ_{Sx} or $X_{Incremental}$ or $(S_x'' - S_x)$ -, and $W_{Incremental}$ or Δ_{Sw} or $(S_w'' - S_w)$ and Δ_{Sy} or $Y_{Incremental}$ or $(S_y'' - S_y)$.

Figure 10(a) shows that plot of small values of β error vs. $I_{Incremental}$ (Δ_I) is linear and equation of the regression line is $I_{Incremental} = 0.2652\beta + 0.0138$. Likewise same plot shows that for small values the plot of β error vs. $S_{Incremental}$ (Δ_S) is linear and equation of the regression line is $S_{Incremental} = -0.4242\beta + 0.1854$.

Figure 10(b) shows that plot of small values of β error, the plot β vs. $X_{Incremental}$ is linear and equation of the regression line is $X_{Incremental} = 0.1592\beta - 4.1829$, the plot of β error vs. $W_{Incremental}$ is linear and equation of the regression line is $W_{Incremental} = -0.1073\beta + 2.2311$ and the plot of β vs. $Y_{Incremental}$ is linear and equation of the regression line is $Y_{Incremental} = 0.1571\beta + 3.9689$. The Figure 11(a) and Figure 11(b) show that when the values of β error are very high, the relationship between β and other corrections in plots in Figure 10 is not observed. However since in most of the practical applications the rotational errors are of the same magnitude and small, the linear relationships given by equations in Figure 10 can be used.

7 CONCLUSION

In this paper the authors studied the effects of part loading errors on the accuracy of a compound hole axis. Three translation errors Δ_x , Δ_w , and Δ_y and two rotational or skew errors θ and β are considered. These errors are compensated through corrective motions Δ_I , Δ_S , ΔS_x , ΔS_w , and ΔS_y . It is observed that Δ_x error has no effect on Δ_I , and Δ_S but does change ΔS_x , ΔS_w , and ΔS_y . For small values of rotational (skew) errors θ and β if only β is varied, the corrections Δ_I , Δ_S , ΔS_x , ΔS_w , and ΔS_y show linear relationship. Attached figures show the trend line for all the corrections. Therefore, within the possible range of acceptable skew errors the trend lines can be used to predict corrections. However if the algorithm is used, it will give accurate corrections irrespective of the magnitude of the errors both translational and rotational.

It is shown that mathematical approach to the solution provided the required compensation algorithm with no human intervention irrespective of the magnitude and quality of the translation and rotation errors. Both absolute and incremental solutions are derived. In the incremental solution, the machine is positioned to the point where machining should occur without errors, and then incremental adjustments are made from this idealized point. Two rotary axes, S (Spindle body) and I (Rotary Index Table), are adjusted as well as three linear axes: X, w, and Y. The top center of the table is taken as the