

Modeling a Slider-Crank Mechanism With Joint Wear

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ABSTRACT

The paper presents a study on the prediction of wear for systems in which progressive wear affects the operating conditions responsible for the wear. A simple slider-crank mechanism with wear occurring at one of the joints is used to facilitate the study. For the mentioned mechanism, the joint reaction force responsible for the wear is, itself, affected by the progression of wear. It is postulated that the system dynamics and the wear are coupled and evolved simultaneously.

The study involves integrating a dynamic model of the slider-crank mechanism (with an imperfect joint) into a wear prediction procedure. The prediction procedure builds upon a widely used iterative wear scheme. The accuracy of the predictions is validated using results from an actual slider-crank mechanism.

INTRODUCTION

Clearances at the joints of multibody systems (usually due to manufacturing tolerance) have been noted to affect the performance and service life of mechanical systems. This may be attributed to the increased vibration, excessive wear and dynamic force amplification as discussed by Dubowsky [1]. Due to the significance of the problem, numerous studies have been conducted with the goal of understanding the dynamic response of these systems in the presence of joint clearances [1-16]. These studies have evolved from the analysis of less complex planar multibody systems [1-3, 5-11, 13,14,16] to more complex spatial systems [4, 15] as well as from rigid multibody analysis [1,2,5-10,12-16] to flexible multibody analysis [3,4,11]. The

studies have demonstrated that the presence of clearances alter the response of the system appreciably.

Although these studies will go a long way into allowing designers to take into account joint clearance, the findings may be limited to the idealized case in which wear is assumed to be nonexistent. This is contrary to a realistic scenario in which wear is expected to increase the clearance size and thus further alter the system response. This research seeks to address this issue by allowing the joint clearance to vary as dictated by the wear. As a result, the system dynamics will evolve with the wear and this evolution is captured by an integrated model presented in the article. The effect of the wear on the system dynamics and conversely the effect of the evolving dynamics on the wear can then be studied.

In the first part of the paper a wear prediction procedure is presented. The procedure presented is based on a widely used finite-element-based iterative wear prediction procedure. In the next part, modeling of a perfect and imperfect joint is discussed. Two different kinds of imperfect joints are discussed. The first being a general imperfect joint model in which the two components of the joint are allowed to move relative to each other depending on the dynamic behavior of the system. The second model is a simplified and more specific case to the study (slider-crank mechanism), in which the two components of the joint are in continuous contact. Next, the wear prediction procedure is integrated with the model that describes the imperfect joint. Only the simplified joint case is considered for the integration. In the final part of the report, the experimental validation of the integrated model is discussed.

WEAR PREDICTION

In the case of a revolute joint of a mechanical system, wear would occur when the components of the joint are in contact and in relative motion. The amount of wear at such a joint is affected by the type of material the components are made of, the relative sliding distance and the operating conditions. Here, the operating conditions refers to the amount of reaction force developed at the joint and the condition of the joint which could be dry, lubricated, or contaminated with impurities.

A lot of effort has been placed in developing models to predict wear occurring in components similar to those of the revolute joint. Majority of these models are based on the Archard's wear law, first published by Holm [17] in 1946. One form of the equation is express mathematically as follows:

$$\frac{hA}{s} = kF_N, \quad (1)$$

where s is the sliding distance, k is a wear coefficient, h is the wear depth, A is the contact area and F_N the applied normal force. Equation (1) can further be simplified by noting that the contact pressure may be expressed with the relation $p = F_N/A$ so that the wear model is expressed as

$$\frac{h}{s} = kp. \quad (2)$$

The wear process is generally considered to be a dynamic process (rate of change of the wear depth with respect to the sliding distance) so that the differential form of Eq. (2) can be expressed as

$$\frac{dh}{ds} = kp(s), \quad (3)$$

where the sliding distance is considered as a time in the dynamic process. A numerical solution for the wear depth may be obtained by estimating the differential form in Eq. (3) with a finite divide difference to yield the following updating formula for the wear depth:

$$h_i = h_{i-1} + kp_i \Delta s_i. \quad (4)$$

In Eq.(4), h_i refers to the wear depth at the i^{th} cycle while h_{i-1} represents the wear depth at the previous cycle. The last term in Eq. (4) is the incremental wear depth which is a function of the contact pressure (p_i) and the incremental sliding distance (Δs_i) at the corresponding cycle. Thus if the wear coefficient, the

contact pressure and the incremental sliding distance are available at every cycle, the overall wear can be estimated. To that end, the value of the wear coefficient can be obtained through experiments as discussed in the literature,[22, 24-26] where as the contact pressure can be calculated using Finite element analysis (FEA) or the Winkler Surface model [22]. Since the Finite element method is more superior than the FEA, with regard to accuracy, only the FEA method is adopted in this reserach. The incremental sliding distance can be obtained as a result of the FEA or may be specified explicitly. In this work the commercial finite element program ANSYS has been employed in conjunction with the corresponding design language, Ansys Parametric Design language (APDL).

A number of papers [18-24] which demonstrate the implementation of Eq. (4) in estimating wear have been published. Although the details of the various procedures differ, three main steps are common to all of them. These include the following:

- Computation of the contact pressure resulting from the contact of bodies.
- Calculation of the incremental wear amount based on the wear model.
- Geometry update by moving the contact boundary to reflect the wear and to provide the new geometry for the next cycle (this allows for a more accurate and realistic prediction of the wear process).

The wear prediction procedure employed in this research incorporates the three steps mentioned. It should, however, be mentioned that the wear simulation procedure is a computationally expensive process. This is due to the number of cycles that need to be simulated (usually greater than 10,000 cycles) each of which requiring some type of analysis to determine the contact pressure and sliding distance. In order to mitigate the computational costs, an extrapolation procedure was used. This involves calculating the incremental wear depth for a representative cycle and then extrapolating this wear depth over N fixed cycles. The use of an extrapolation results in a modification of the updating formula (4). The new equation is expressed as:

$$h_i = h_{i-1} + kA_E p_i \Delta s_i, \quad (5)$$

where A_E is the extrapolation factor. The choice of the extrapolation is critical to the efficiency and stability of the simulation. The use of large extrapolations will cause the simulation to be unstable and compromise the accuracy of the simulation. On the other hand using small extrapolation sizes will result in a less than optimum use of resources. A complete study on

extrapolations, its effect on stability of wear prediction and optimizing its selection can be found in our previous work [24].

A flowchart summarizing the simulation procedure is shown in Fig.1. The procedure will later be integrated with the dynamic model for the imperfect slider-crank mechanism, which is discussed in the next section.

MODELING IMPERFECT REVOLUTE JOINTS

Revolute joints in mechanical systems are generally imperfect. This means that the joints have some amount of clearances usually due to constraints and requirements in manufacturing. In addition, throughout the service life of the system the clearances increase in size due to wear. As was mentioned earlier, the clearance affects the dynamics of system. In this section two imperfect joint models that have the capability of accounting for changes in system dynamics due to clearances size changes are discussed.

To facilitate the study, a slider-crank mechanism has been used, due to its simplicity. A diagram of the slider-crank mechanism to be used in the study is shown in Fig. 2. The study is simplified by eliminating friction and wear from all connection points in the mechanisms except for one joint, shown as the joint of interest in Fig. 2. This joint essentially consists of a pin that is attached to the crank (drive-link Fig. 2.) and a bushing attached to the connecting rod (driven-link). The pin is made of hardened steel and is assumed to be hard enough so that no appreciable wear occurs on its surface. The bushing on the other hand is made of poly-tetra-fluoro-ethene (PTFE) which is soft and will experience considerable wear. A spring is attached to the slider which serves as a means to increase the joint reaction force and hence accelerated the wear occurring at the joint.

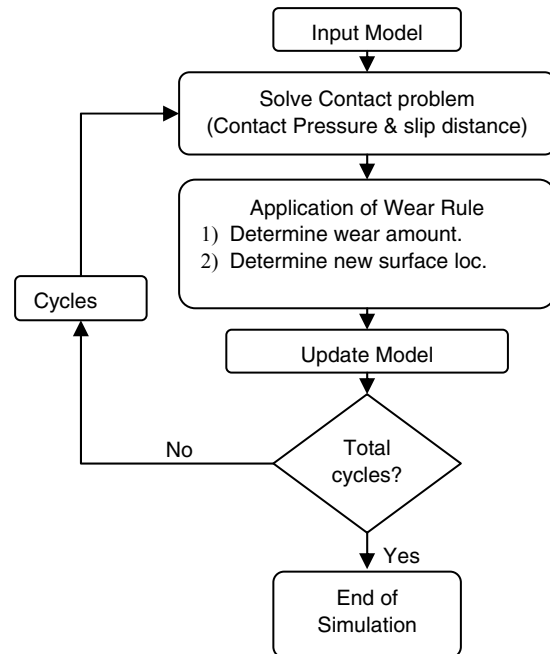


Figure 1: Wear simulation flow chart.

In order to successfully study the dynamic response of the mechanism under joint wear, it is necessary to develop a formulation for the slider-crank system that estimates the changes to the system dynamics when the joint clearance is changed. In what follows, a formulation for the dynamic and kinematic analysis of the slider-crank mechanism with perfect joints is presented. Based on this formulation, the model for the slider-crank mechanism with an imperfect joint will be developed.

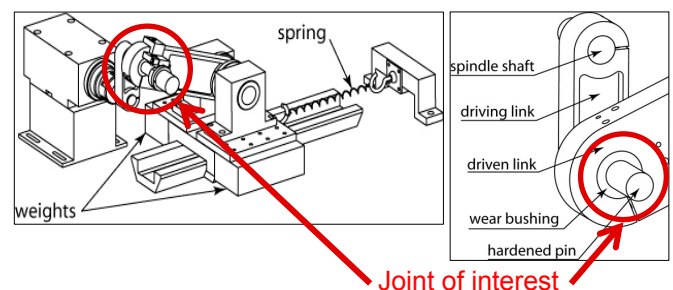


Figure 2: Slider-crank mechanisms to be used in the wear study.

DYNAMICS OF A SLIDER-CRANK MECHANISM WITH PERFECT JOINTS - In a slider-crank mechanism with perfect joint, the pin is assumed to fit perfectly in the bushing. Consequently the pin and bushing centers coincide at all times. The slider-crank system is assumed to consist of three rigid bodies with planar motion as depicted in Fig. 3. The three disassembled components of the mechanism (link-1, link-2 and a slider) are shown in the global axis. Each component can translate and rotate in the plane.

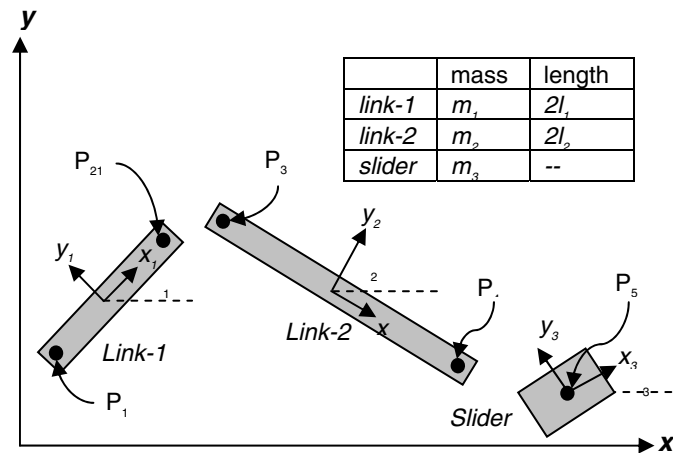


Figure 3: Components of the slide crank mechanisms.

The kinematics of the system is determined by imposing constraints on the motion of the components. The constraints corresponding to the slider-crank mechanism shown in Fig. 3. consist of nine nonlinear simultaneous equations expressed as:

$$\Phi = \begin{bmatrix} x_1 - l_1 \cos \phi_1 \\ y_1 - l_1 \sin \phi_1 \\ x_2 - 2l_1 \cos \phi_1 - l_2 \cos \phi_2 \\ y_2 - 2l_1 \sin \phi_1 - l_2 \sin \phi_2 \\ x_3 - 2l_1 \cos \phi_1 - 2l_2 \cos \phi_2 \\ y_3 - 2l_1 \sin \phi_1 - 2l_2 \sin \phi_2 \\ y_3 \\ \phi_3 \\ \phi_1 - \omega t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (6)$$

The first two constraints in Eq. (6) confine point P1 on link-1 to the origin. The next two constraints ensure that points P2 on link-1 and P3 link-2 coincide at all times. This condition is synonymous to a perfect joint and later will be relaxed when modeling the imperfect joint. The fifth and sixth constraints in Eq. (6) represent the perfect revolute joint between Link-2 and the slider. The next two constraints ensure that the slider remain on the x-axis without rotation. The final constraint, known as the

driving constraint, is an external input such as a servo motor that specifies the motion of one of the links. For the current case a constant angular velocity ω is imposed in link-1.

It can be seen from the set of simultaneous equations above, that the number of equations exactly equals the number of unknowns. The unknowns are the DOFs of the components at the center of masses. This is denoted by vector q , as

$$q = [x_1, y_1, \phi_1, x_2, y_2, \phi_2, x_3, y_3, \phi_3]^T. \quad (7)$$

The set of simultaneous nonlinear equations (Eq. (6)) can be solved simultaneously to determine the slider-crank mechanism component positions at any instant. The velocities and accelerations may also be determined using the following relations:

$$\dot{q} = -\Phi_q^{-1} \Phi_t \quad (8)$$

$$\ddot{q} = \Phi_q^{-1} \left(-(\Phi_q \dot{q})_q \dot{q} - 2\Phi_{qt} \dot{q} - \Phi_{tt} \right) \quad (9)$$

Once the accelerations have been computed, the reaction forces can be obtained through the process of reverse dynamics. However, if we desire to obtain the response of the system due to externally applied forces, such as a spring force (see Fig. 2.) or a torque applied instead of the drive constraint, then a dynamic analysis is required. This involves assembling and solving the differential-algebraic equations of motion (DAE). This equation is expressed as follows;

$$\begin{bmatrix} \mathbf{M} & \Phi_q^T \\ \Phi_q & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_A \\ \Phi_q \ddot{q} \end{bmatrix}, \quad (10)$$

where $\mathbf{M}$ is a diagonal mass matrix, Φ_q is the Jacobian of the constraint vector, $\ddot{q}$ is the acceleration vector, λ is a vector of Lagrange multipliers and $\mathbf{Q}_A$ is the vector of externally applied forces. The solution procedure for this equation, to obtain the dynamics of a system, is well documented in the literature [14, 28,29]. An summary of the solution procedure is shown in Fig. 4.

DYNAMICS OF A SLIDER-CRANK MECHANISM WITH AN IMPERFECT JOINT - Two kinds of the imperfect joints for the slider-crank mechanism, a general imperfect joint and a simplified imperfect joint are discussed. The first imperfect joint can be found in the literature [13,14,16] and is only briefly discussed. It is used as a reference for comparison with the simplified imperfect joint.