

A Modal-Based Derivation of Transient Pressure Distribution Along the Tyre-Road Contact

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ABSTRACT

The two-dimensional, frictional tyre-road contact interaction is investigated. A transient contact algorithm is developed, consisting of an analytical belt model, a non linear sidewall structure and a discretized viscoelastic tread foundation. The relationship between the magnitude/shape of the predicted two-dimensional pressure distribution and the corresponding belt deformation is identified. The effect of vertical load and the role of sidewall non linearity are highlighted. The modal expansion/reduction method is proposed for the increase of the computational efficiency and the effect of the degree of reduction on the simulation accuracy is presented. The qualitative results are physically explained through the participation of certain modes in the equilibrium solution, offering directions for the application of the modal reduction method in shear force oriented tyre models.

INTRODUCTION

Tyres play a crucial role in a vehicle's dynamic behaviour potential, as the forces developed in the tyre-road contact area are responsible for any change in the vehicle's motion. The developed forces simultaneously act as a transient excitation of the tyre structure and a closed loop of interaction is established with the developed response affecting the tyre force potential. The tyre's contact non linearity and frictional nature, though, impose severe difficulty in the simulation of the tyre-road interaction mechanism.

The tyre contact strain and stress fields along the interface area have been the topic of extensive research, as the resulting pressure distribution directly

affects the shear force potential of the tyre. Numerous methods for the experimental derivation of the contact pressure distribution have been proposed, such as pressure sensing contact surfaces [1, 2, 3], light absorption/reflection techniques [4] and ultrasonic based measurements [5]. The accurate simulation, though, of the pressure distribution development still comprises a challenging research aim for reasons associated with the tyre's shape, structural anisotropy and the viscoelastic phenomena related to the interaction.

The rapid development of the finite element methods enabled the representation of tyre contact stress and strain field with great accuracy [6, 7], but the necessary fine discretization imposes a huge computational load. If this shortcoming is overcome or neglected, the discretized modelling approach can be extended towards accurate transient tyre models [8]. If the transient nature of the contact area development is, though, neglected, analytical carcass deformation representations can predict the equilibrium pressure distribution. A membrane shell model, for example, was combined with a contact stiffness foundation in [9, 10].

The well established ring on elastic foundation model, introduced in [11], still represents the state-of-the-art of two-dimensional tyre carcass modelling [12]. Its vibrational behaviour correlation with the actual tyre structure has been experimentally verified in numerous studies [13, 14, 15]. Although an analytical, ring based, contact solution has been presented in [1], in the majority of the ring model applications, the solution is acquired through the modal expansion/reduction technique [16], offering great computational efficiency to the approach.

Ring based models have been often proposed as tyre belt representations under the steady state [17] or the transient approach [18]. For computational reasons, though, the contact pressure field is not usually calculated as part of the contact algorithm but artificially superimposed as an input to the friction calculation sub-model ([18], SWIFT model described in [19]). Analytical parabolic/trapezoidal shaped vertical pressure distribution is the common assumption along the contact length when the friction potential has to be calculated, as in [20]. This approach is believed to moderate the transient validity and potential of the model, in comparison to studies where the contact pressure field is incorporated in the simulation process [21].

Although the necessary modal/frequency range of study for the tyre behaviour simulation in structural vibrations and vibrations' transmission studies has been long ago identified and discussed [22, 23, 24], the effect of the breadth of the modal range in the accuracy of the representation of the contact has not been investigated in depth. In the present study, the interaction between the belt frictional contact deformation and the predicted two-dimensional contact pressure distribution is investigated and the crucial role of the vertical load/deformation is identified. The study of this interaction is carried out with the development of a physical-ring based- tyre model which enables the qualitative study of the modal reduction effect on the distribution simulation accuracy. This is accomplished through the identification of the participation of each mode in the tyre contact response.

A fully transient two-dimensional tyre model has been developed and exploited for this investigation. The nature of the developed algorithm and the qualitative conclusions on the acceptable reduction level entitle the future transformation of the approach into a physical shear force generation simulation tool with acceptable computational demands. The present state of the research may offer guidelines for the application of modal expansion/reduction method in various tyre models, the transient nature of which imposes the real time calculation of the contact interaction stresses [8, 20]. The huge computational load of such discretized approaches can be significantly reduced without major sacrifice of the prediction accuracy.

MODEL DESCRIPTION

The model is composed by three interacting sections:

- belt
- sidewall structure

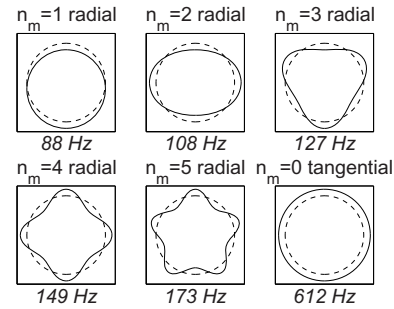


Figure 1 – The mode shapes and the natural frequencies of the $n_m = 1$ to $n_m = 5$ radial and the $n_m = 0$ -breathing- tangential modes.

- tread foundation

BELT REPRESENTATION The core of the proposed approach is the analytical ring on elastic foundation model. Although the solution is acquired through the modal expansion method, the simulation is accomplished in the time domain. The nature of the examined problem and the friction induced non linearity prevent the *a priori* association with a certain frequency range or a certain modal response and necessitates the time domain solution [25]. The common ring inextensibility assumption [26] is not applied here, as the proposed modal range of solution is too broad for this assumption to be valid. Additionally, the simulation of inflation, permitted only with the inclusion of an extensible belt, is vital for the solution in the non linear sidewall case. The equations of motion of the pretensioned, extensible ring may be written as [27]:

$$\frac{D}{R^4}(u_r'''' - u_\theta''') + \frac{K}{R^2}(u_r + u_\theta') + \frac{N}{R}(2u_\theta' - u_r'') + k_r u_r + b d \rho \ddot{u}_r = q_r + \frac{N}{R} \quad (1a)$$

$$\frac{D}{R^4}(u_r''' - u_\theta'') - \frac{K}{R^2}(u_r' - u_\theta') - \frac{N}{R}(2u_r' + u_\theta) + k_\theta u_\theta + b d \rho \ddot{u}_\theta = q_\theta \quad (1b)$$

$u_r(\theta)$	radial deformation
$u_\theta(\theta)$	tangential deformation
b	width of the ring cross section
d	height of the ring cross section
ρ	material density
R	ring radius
$D = E b d^3 / 12$	bending stiffness of the ring
$K = E b d$	membrane stiffness of the ring
P_0	inflation pressure
$N = P_0 b$	inflation tension force
k_r	linear radial sidewall stiffness
k_θ	linear tangential sidewall stiffness
$q_r(\theta)$	radial excitation
$q_\theta(\theta)$	tangential excitation
$(\cdot)$	angle derivative
$(\ddot{\cdot})$	time derivative

Double axis symmetry of the tyre imposes harmonic functions for the deformation shapes of the modes. Each of them participates in the expanded representation of the model twice with a $\frac{\pi}{2n_m}$ angle separating the corresponding mode shapes (n_m being the modal number of the mode). Two modal groups are predicted by the expansion method. The first, low frequency, group corresponds to primarily radial deformation, while the second, high frequency, one is associated with tangential deformation [16]. The mode shapes and the natural frequencies for some of the first modes are presented in fig.1.

For the radial and the tangential deformation of the j^{th} mode it may be respectively written:

$$\begin{aligned} R_j(\theta) &= A_j \cos(n_m(\theta - \varphi)) \\ \Theta_j(\theta) &= B_j \sin(n_m(\theta - \varphi)) \end{aligned} \quad (2)$$

$R_j(\theta), \Theta_j(\theta)$ mode shape functions
 A_j, B_j mode shape amplitudes
 φ excitation defined orientation angle

Each mode participates in the response through its modal participation factor, the time (t) differential equation of which reads [16]:

$$\ddot{\eta}_j(t) + \frac{\lambda_j}{\rho \cdot h} \cdot \dot{\eta}_j(t) + \omega_j^2 \cdot \eta_j(t) = F_j^{modal}(t), \quad 1 \leq j \leq m \quad (3)$$

η_j modal participation factor
 λ_j viscous damping coefficient
 ω_j mode natural frequency
 F_j^{modal} modal force
 m total number of solution participating modes

Although the belt modes are analytically derived [16], a constant space discretization process is applied to the ring. This process coincides with the discretization of the sidewall and tread foundations. In other words, tread and sidewall representing elements correspond to every ring node (n in total). This permits the expression of the excitation function, in the space domain, as a vector of constant dimension, which corresponds to the number of degrees of freedom ($2n$):

$$\mathbf{F}_{2n \times 1}(t) = \mathbf{F}_{inflation}(t) + \mathbf{F}_{contact}(t) \left\{ + \mathbf{F}_{sidewall}(t) \right\} \quad (4)$$

where n is the total number of nodes. Each node is associated with one radial and one tangential degree of freedom (horizontal/vertical in the global system of axis).

Under the linear sidewall assumption, the sidewall effect is incorporated in the ring equations of motion (eqn.1a and 1b) and the sidewall corresponding term is not included in eqn.4. Under the non linear sidewall assumption, though, the expansion method is

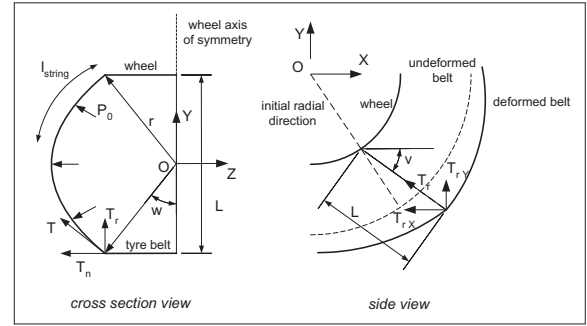


Figure 2 – The radial string geometry and the resulting sidewall forces.

applied on a ring without sidewall foundation and the corresponding effect is incorporated through the force term.

The solution transformation between the modal and the space domain is achieved through the constant eigenvectors matrix $\Phi_{2n \times m}$. The modal mass of the j^{th} mode is defined as:

$$M_j = \rho \cdot d \cdot R \cdot b \cdot \sum_{i=1}^{2n} \Phi_{ij}^2 \quad (5)$$

The modal force of each mode can be calculated as:

$$\begin{aligned} \mathbf{G}^{modal}(t) &= \Phi^T \times \mathbf{F}^{space}(t) \Rightarrow \\ F_j^{modal}(t) &= \frac{1}{M_j} G_j^{modal}(t) \end{aligned} \quad (6)$$

Finally, the deformation and velocity responses in the space domain are calculated from the modal participation vector according to the equations:

$$\mathbf{U}(t) = \Phi \times \begin{bmatrix} \eta_1(t) \\ \vdots \\ \eta_m(t) \end{bmatrix}, \quad \dot{\mathbf{U}}(t) = \Phi \times \begin{bmatrix} \dot{\eta}_1(t) \\ \vdots \\ \dot{\eta}_m(t) \end{bmatrix} \quad (7)$$

SIDEWALL REPRESENTATION Although the ring equations of motion incorporate a two-dimensional linear stiffness foundation (k_r, k_θ), representing the sidewall of the tyre, this approach is known to result in contact reaction force overestimation [11, 24, 28, 29]. The radial buckling-softening- behaviour of the actual sidewall offers a force reduction mechanism. This behaviour is commonly modelled by a string under tension foundation, an approach also followed here, under the inextensible discretized assumption.

The tension force of an inextensible string is given by:

$$T = P_0 \cdot r \quad (8)$$

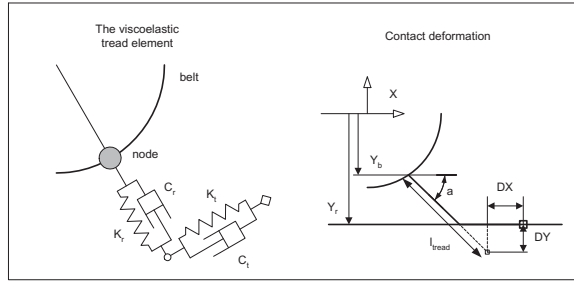


Figure 3 – The tread element and its contact deformation.

while the geometry presented in fig.2 proposes:

$$\left. \begin{aligned} \frac{\pi-2w}{l_{string}} &= \frac{1}{r} \\ \cos w &= \frac{L}{2 \cdot r} \end{aligned} \right\} \Rightarrow L(\pi - 2w) = 2l_{string} \cdot \cos w \quad (9)$$

T	tension force
r	string radius
w	string angle (fig.2)
l_{string}	string length
L	wheel to belt distance

The non linear eqn.9 is solved by an iteration procedure in every calculation step, using as inputs the relative wheel to road displacement and the deformation of the belt. The resulting string forces are presented in fig.2 and their values are calculated as:

$$\begin{aligned} T_{rX} &= -P_0^* \cdot l_{string} \frac{\sin w}{\pi-2w} \cos v \\ T_{rY} &= P_0^* \cdot l_{string} \frac{\sin w}{\pi-2w} \sin v \end{aligned} \quad (10)$$

The string sidewall behaviour is decoupled from the discretization effect by the normalization of the inflation pressure value:

$$P_0^* = P_0 \frac{2 \cdot \pi \cdot R}{n} \quad (11)$$

The tangential sidewall behaviour is assumed to remain linear, so the corresponding stiffness is in any case incorporated in the tangential equation of motion (eqn.1b).

TREAD REPRESENTATION The tread modelling approach is a viscoelastic (Kelvin-Voigt) extension of the winker contact stiffness theory, established in numerous studies, as a stand alone tyre model [19], or in combination with a separate carcass representation [17, 14, 21]. The tread elements are attached to the belt nodes initially with radial orientation. Slip motion potential is included in the model through the incorporation of an additional viscoelastic element and a corresponding degree of freedom. The second tread element is dimensionless under no strain condition and is attached to the free end of the radial element. Its deformation direction is vertical to the first one, while

the road contact is realized at the free end of this element.

The tread viscoelastic properties are represented by the stiffness and damping matrices:

$$\mathbf{K} = \begin{bmatrix} -K_r & 0 \\ 0 & -K_t \end{bmatrix}, \mathbf{C} = \begin{bmatrix} -C_r & 0 \\ 0 & -C_t \end{bmatrix}$$

K_r	radial stiffness
K_t	tangential stiffness
C_r	radial viscous damping
C_t	tangential viscous damping

The contact reaction forces, in global terms, can be calculated according to:

$$\begin{bmatrix} F_{CX} \\ F_{CY} \end{bmatrix} = \mathbf{T}^T(\alpha) \mathbf{K} \mathbf{T}(\alpha) \begin{bmatrix} DX \\ DY \end{bmatrix} + \mathbf{T}^T(\alpha) \mathbf{C} \mathbf{T}(\alpha) \begin{bmatrix} D\dot{X} \\ D\dot{Y} \end{bmatrix} \quad (12)$$

where $\mathbf{T}$ is the angular transformation matrix:

$$\mathbf{T}(\alpha) = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

The tread global orientation angle α is the sum of the belt node rotation around the wheel centre, imposed by the corresponding tangential deformation and the torsional local belt deformation-expressed by the slope of the corresponding radial deformation.

$$\alpha = u_\theta + u'_r \quad (13)$$

The total vertical deformation and deformation velocity of the combined element is kinematically defined by the relative road-wheel distance and the belt deformation (fig.3):

$$\begin{aligned} DY &= Y_r - (Y_b + l_{tread} \sin \alpha) \\ D\dot{Y} &= \dot{Y}_r - \dot{Y}_b \end{aligned} \quad (14)$$

DY	vertical tread deformation
Y_r	road vertical profile
Y_b	belt node vertical position
l_{tread}	initial radial length

Horizontal slip motion is controlled by the differential equation of the horizontal degree of freedom, which, under the massless assumption, transforms into a simple force equilibrium equation:

$$\Sigma F_X = m_t D\ddot{X} \Rightarrow F_{tread} + F_{friction} = 0 \quad (15)$$

$F_{tread} = -F_{CX}$	tread viscoelastic force
$F_{friction}$	friction force
DX	horizontal deformation
$D\dot{X}$	horizontal def. velocity

The integration of the relative slip velocity value entitles the calculation of horizontal deformation (slip):

$$DX = \int D\dot{X} dt \quad (16)$$

A number of friction laws can be incorporated in the model. The classic Coulomb one ([30]) is adopted here for simplicity reasons.