

Robust Yaw Moment Control for Vehicle Handling and Stability

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ABSTRACT

This paper presents a robust controller design method for improving vehicle lateral stability and handling performance. In particular, the practical load variation will be taken into account in the controller synthesis process such that the controller can keep the vehicle lateral stability and handling performance regardless of the load variation. Based on a two-degree-of-freedom (2-DOF) lateral dynamics model, a model-based Takagi-Sugeno fuzzy control strategy is applied to design such a controller and the sufficient conditions for designing such a controller are given in terms of linear matrix inequalities (LMIs) which can be solved efficiently using currently available numerical software. Numerical simulations are used to validate the effectiveness of the proposed control approach.

INTRODUCTION

A considerable research and subsequent developments have been done to improve vehicle handling and stability. Among them, the yaw moment control has proved its effectiveness to improve vehicle handling and stability during severe driving manoeuvres. In general, the motivation for the development of yaw moment control can be understood by examining the driver's disabilities to control vehicle directional dynamics under critical conditions. For example, in a turning manoeuvre with high lateral acceleration, where tyre forces are approaching or at the limit of road adhesion, the vehicle sideslip angle grows and the effectiveness of the vehicle steering angle in generating a yaw moment becomes significantly reduced because of the saturation of tyre force. The decrease in restoring yaw moment generated

by the tyre lateral force when the sideslip angle increases may cause the unstable motion of the vehicle called the spin motion. Providing the required external yaw moment will therefore recover the stability of the vehicle.

A practical approach to generate a required external yaw moment can be the transverse distribution of the vehicle braking forces between the left and right wheels. This strategy known as differential braking can be achieved using the main parts of the common antilock braking system (ABS). Nowadays, how to design the required controller based on the measured or estimated variables to achieve the desired performance is attracting much more attention. Several authors have proposed different control methods. For example, optimal control [1,2], fuzzy logic control [3], H_∞ yaw-moment control [4], internal model control [5], multi-objective control [6,7], LQR and sliding mode control [8], etc. As the vehicle is operating under a wide range of conditions of speed, load, friction etc., the active yaw moment control should guarantee the stability performance robustly in the presence of uncertainty arising from different operating situations. In addition, the physical constraints of control input resulting from the actuation system and the tyre-road conditions, etc., may cause the saturation of the control input with possible deteriorations of the control performance. Therefore, the controller design for yaw moment control should consider both robust stability and control saturation aspects. These issues, however, are ignored or partly dealt with in those studies.

This paper concerns with the robust controller design for a two-degree-of-freedom (2DOF) vehicle lateral dynamics model considering vehicle load uncertainty

and control input constraint. The focus of the paper is to design a state feedback control law such that the closed-loop system is asymptotically stable with the optimal disturbance attenuation subject to a variation of vehicle mass and control input constraint. To deal with the vehicle load uncertainty, the vehicle lateral dynamics model is first represented by a Takagi-Sugeno (T-S) fuzzy model using "Sector Nonlinearity" approach. Then, a robust T-S fuzzy controller is designed based on the parallel distributed compensation (PDC) scheme. Sufficient conditions for designing such a controller are given in terms of linear matrix inequalities (LMIs), which can be solved very efficiently using available software like Matlab LMI Toolbox. Simulation results are used to validate the control performance.

This paper is organised as follows. In Section 2, the linear vehicle lateral dynamics model is briefly presented. In Section 3, the control problem is formulated considering the vehicle load uncertainty and control input constraint. In Section 4, the fuzzy controller development is presented. Then, in section 5, the results of the simulation for different steering manoeuvres are shown and discussed. Finally, conclusions are presented in section 6.

LINEAR VEHICLE MODEL

A vehicle's handling dynamics in the horizontal plane are represented by a linear two-degree of freedom (2DOF) bicycle model with states of sideslip angle β at the centre of gravity (CG) and yaw rate r as shown in Figure 1.

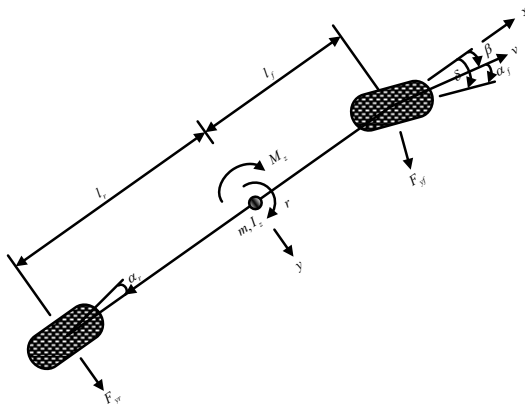


Figure 1. Bicycle model

The equations governing the lateral and yaw motions for the linear bicycle model follows from the force and moment balance can be expressed as [3]

$$\begin{aligned} mv(\dot{\beta} + r) &= C_f \left(\beta + l_f \frac{r}{v} - \delta \right) + C_r \left(\beta - l_r \frac{r}{v} \right), \\ I_z \dot{r} &= l_f C_f \left(\beta + l_f \frac{r}{v} - \delta \right) - l_r C_r \left(\beta - l_r \frac{r}{v} \right) + M_z. \end{aligned} \quad (1)$$

where I_z is the moment of inertia of the vehicle about its yaw axis, m is the vehicle mass, l_f and l_r are distance of the front and rear axles from the centre of gravity (CG). C_f and C_r are cornering stiffnesses of the front and rear tyres. δ is the steering angle. M_z is yaw moment and v is vehicle velocity.

Considering the vehicle velocity to be time invariant, the following state space equation can be formulated as:

$$\dot{x} = Ax + B_1 w + B_2 u, \quad (2)$$

where x is the state variable, u is the control input (yaw moment M_z), which is generated from the difference of the brake forces between the left and right drive wheels. w is the disturbance (steering angle δ). The variables and matrices are given as

$$\begin{aligned} x &= \begin{bmatrix} \beta \\ r \end{bmatrix}, \quad A = \begin{bmatrix} -\frac{C_f + C_r}{mv} & -1 - \frac{l_f C_f - l_r C_r}{mv^2} \\ -\frac{l_f C_f - l_r C_r}{I_z} & -\frac{l_f^2 C_f + l_r^2 C_r}{I_z v} \end{bmatrix}, \\ B_1 &= \begin{bmatrix} \frac{C_f}{mv} \\ \frac{l_f C_f}{I_z} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ \frac{1}{I_z} \end{bmatrix}. \quad w = \delta, u = M_z. \end{aligned}$$

PROBLEM FORMULATION

LOAD VARIATION - In practice, the vehicle mass can be varied in terms of the loading conditions such as the number of riding persons and payload, etc. Since the uncertain vehicle mass m is bounded by its minimum value $m_{\min}$ and its maximum value $m_{\max}$ in practice, using the idea of "sector nonlinearity" [9], it can thus be represented by

$$\frac{1}{m} = h_1(\xi) m_{s\max} + h_2(\xi) m_{s\min}, \quad (3)$$

where $\xi = \frac{1}{m}$ is defined as a premise variable (i.e., the variable x used in the if-part of a single fuzzy if-then rule

like “if x is A then y is B ”), $m_{s\max} = \frac{1}{m_{\min}}$, $m_{s\min} = \frac{1}{m_{\max}}$,

$h_1(\xi)$ and $h_2(\xi)$ are membership functions which are defined as

$$h_1(\xi) = \frac{\frac{1}{m} - m_{s\min}}{m_{s\max} - m_{s\min}}, h_2(\xi) = \frac{m_{s\max} - \frac{1}{m}}{m_{s\max} - m_{s\min}}. \quad (4)$$

We name the above mentioned two membership functions as “Light” and “Heavy”, respectively, the uncertain vehicle handling model (2) can then be represented by the following fuzzy models.

Model Rule 1:

IF ξ is light,

THEN $\dot{x} = A_1x + B_{11}w + B_2u$.

Model Rule 2:

IF ξ is heavy,

THEN $\dot{x} = A_2x + B_{12}w + B_2u$.

And then, the Takagi-Sugeno (T-S) fuzzy model which exactly represents the uncertain vehicle handling model (2) under the assumption on bounds of vehicle mass $m \in [m_{\min}, m_{\max}]$ is obtained as:

$$\dot{x} = \sum_{i=1}^2 h_i(\xi) [A_i x + B_{1i} w] + B_2 u, \quad (5)$$

where $h_i(\xi) \geq 0, i = 1, 2$, and $\sum_{i=1}^2 h_i(\xi) = 1$.

In practice, the vehicle mass m can be measured, thus, the T-S fuzzy model (5) can be realised.

INPUT CONSTRAINT - On the other hand, the control law designed should be found in a way that some physical constraints of control input resulting from both the actuation system and the tyre-road conditions are satisfied. That is the calculated yaw moment must remain below the maximum admissible value determined by the maximum difference in the longitudinal forces that can be generated by the left and right wheels during every cornering manoeuvre. This indicates that the control saturation may exist in some emergency conditions and should be accounted for in the controller design. Then, equation (5) is expressed as

$$\dot{x} = \sum_{i=1}^2 h_i(\xi) [A_i x + B_{1i} w] + B_2 \bar{u}, \quad (6)$$

where $\bar{u}$ is the bounded input expressed as $\bar{u} = \text{sat}(u)$, where $\text{sat}(u)$ is a saturation function of control input u and is defined as

$$\text{sat}(u) = \begin{cases} -u_{\lim} & , \quad \text{if } u < -u_{\lim}, \\ u & , \quad \text{if } -u_{\lim} \leq u \leq u_{\lim}, \\ u_{\lim} & , \quad \text{if } u > u_{\lim}, \end{cases} \quad (7)$$

where $u_{\lim}$ is a control input limit.

In order to avoid the large number of inequalities problem involved in the controller design, the norm bounded approach is used to handle the saturation nonlinearity defined in (7). Hence, the equation (6) will be written as:

$$\begin{aligned} \dot{x} &= \sum_{i=1}^2 h_i(\xi) [A_i x + B_{1i} w] + B_2 \bar{u} \\ &= \sum_{i=1}^2 h_i(\xi) [A_i x + B_{1i} w] + B_2 \frac{1+\varepsilon}{2} u + B_2 \left(\bar{u} - \frac{1+\varepsilon}{2} u \right) \\ &= A_h x + B_{1h} w + B_2 \frac{1+\varepsilon}{2} u + B_2 \bar{v}, \end{aligned} \quad (8)$$

where

$$A_h = \sum_{i=1}^2 h_i(\xi) A_i, B_{1h} = \sum_{i=1}^2 h_i(\xi) B_{1i}, \bar{v} = \bar{u} - \frac{1+\varepsilon}{2} u.$$

And, we have the following lemma.

Lemma 1 For the saturation constraint defined by (7), as long as $|u| \leq \frac{u_{\lim}}{\varepsilon}$, we have

$$\left\| \bar{u} - \frac{1+\varepsilon}{2} u \right\| \leq \frac{1-\varepsilon}{2} \|u\|,$$

and hence

$$\left[\bar{u} - \frac{1+\varepsilon}{2} u \right]^T \left[\bar{u} - \frac{1+\varepsilon}{2} u \right] \leq \left(\frac{1-\varepsilon}{2} \right)^2 u^T u,$$

where $0 < \varepsilon < 1$.

CONTROLLER DESIGN

The robust fuzzy controller design for the uncertain handling model (8) is carried out based on the so-called