

An Investigation of Contact Angles in Angular Contact Ball Bearings Accounting for High Speed Effects and Preload

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ABSTRACT

Angular contact ball bearings are the most commonly used types of bearings. Although simple in appearance, these bearings have a complicated and nonlinear behavior, and have significant effect on the dynamics of a rotating system. Accounting for high speed effects adds to the complexity of their behavior. The governing equations of the bearing amount to the solution of a set of nonlinear equations for each rolling element at each iteration, which represents a tedious and expensive solution. In this work, a simplified model for angular contact ball bearings under the action of arbitrary loading including high speed effects and preload is presented. The model considers five degrees of freedom for relative displacement of rings and a mean contact angle for inner and outer raceway contacts are introduced which renders the solution for all rolling elements to that for a single one. The use of the simplified model at high speeds is justified by presenting the analysis of a complete spindle-bearing system using the simplified model and comparing the results with those of the full model.

INTRODUCTION

Angular contact ball bearings are the most commonly used types of bearings in high speed rotary applications such as high speed spindles, aerospace mechanisms,

gas turbines, etc. To predict the vibration in rotating mechanical equipment, the role of bearings as the dynamic coupler between the shaft and casing must be completely understood. In spite of their simplicity in appearance and functioning principles, these types of bearings represent complicated nonlinear behavior especially in high speed applications. This nonlinearity originates from load-deflection relationships, geometry and high speed effects.

An early attempt for modeling of the behavior of angular contact ball bearings was proposed by Palmgren (1959) [1]. He presented a two degree of freedom model of the ball bearing and considered relative radial and axial displacements of the bearing rings, but neglected nonlinearity in contact and effects of high rotational speed. Jones (1960) [2] presented a five DOF model of the bearing, three translational and two rotational, and included the effects of rotational speed in the form of centrifugal and gyroscopic loads. This model was formulated by Harris (1984) [3]. Chen (1992) [4] used Harris's model and linearized the forces and displacements around their equilibrium conditions. Houpert (1997) [5] used the nonlinear Hertzian contact theory to represent the contact between rolling elements and raceways within a five DOF model. This model was later modified and presented in matrix form by Hernot et al. (2000) [6]. Cao and Altintas (2001) [7] developed a five DOF bearing element in the finite element context

for modeling of rotor bearing systems; This model is in fact the matrix form of Harris's method; the five DOFs are composed of three translations, one in axial and two in radial directions, and two rotations about radial axes; As a result a five by five stiffness matrix is obtained for each inner and outer rings and the behavior of the whole bearing is described by a combined ten by ten stiffness matrix including the effects of high rotational speed. A coupled thermo-mechanical model for high speed spindles was prepared by Li and Shin (2004) [8] in which the bearing is treated as a nonlinear element of five DOFs with rigid outer ring; Thermal expansions of parts are also accounted for.

Although incorporation of rotational effects in the analysis of bearing behavior increases the accuracy and reliability of the solution, it significantly changes the kinematical attributions of the bearing system, and complicates the governing equations of the system, making their solution costly and time consuming. As a result of speed-related loading, the contact angles between inner raceway and outer raceway contacts will differ. In addition, if the loading is not symmetric, contact angles between individual rolling elements will differ as well. The resulting nonlinear governing equations of the bearing system should be solved using iterative techniques. As the stiffness of the bearings depends on contact angle and the applied preload [9], it is necessary to solve a set of nonlinear equations for all rolling elements in every iteration, making it a rigorous task.

The purpose of this work is to present a simplified model for the behavior of angular contact ball bearings under the action of arbitrary loading including high speed effects and axial preload. The model considers five degrees of freedom for relative displacement of rings, three translations and two rotations, but simplifies the model by introducing a mean contact angle for inner and outer raceway contacts which renders the solution for all rolling elements to that for a single one. A complete spindle-bearing system is considered and analyzed using both detailed approach and mean contact angle approach. The use of this simplified model in a certain speed range is justified based on comparison between the results of the presented model and those of the full model.

MODELLING PROCEDURE

GEOMETRY - The model considers five degrees of freedom for the relative displacement of inner and outer races of an angular contact ball bearing. Since the degrees of freedom express the relative motion between the rings, it is convenient to assume that the outer ring is fixed. Figure 1 shows an angular contact ball bearing, its geometrical parameters and its degrees of freedom: three translations ($\delta_x, \delta_y, \delta_z$) which are displacements in axial (x) and radial (y and z) directions, and two rotations (γ_y, γ_z) about axes (y and z).

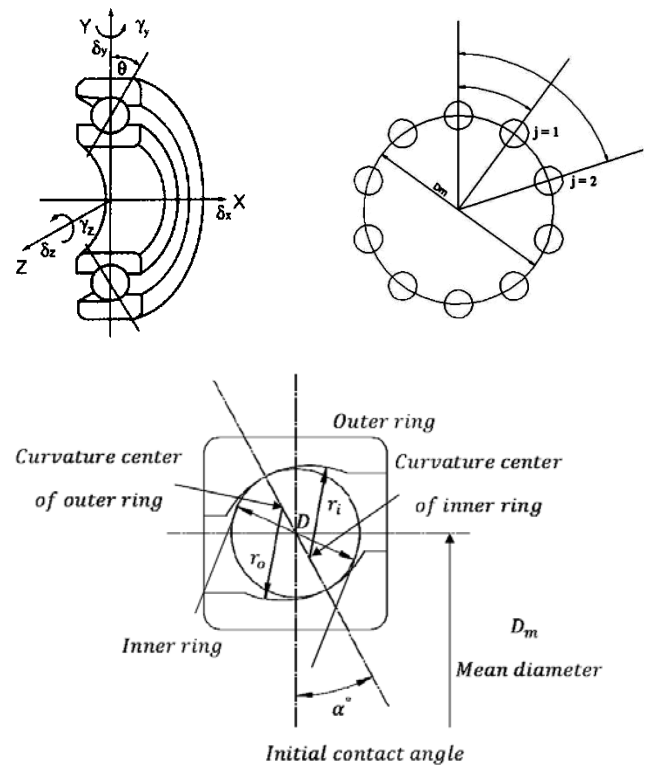


Figure 1: Bearing geometry and degrees of freedom

Figure 2 represents forces acting on the bearing. F_i and F_o are the forces acting on the inner and outer raceway contacts, respectively. F_c is the centrifugal force which is responsible for difference between the contact angles. In addition, in rotation about the bearing axis, the balls will undergo a rolling motion as illustrated in Figure 1. These non-collinear rotations result in a gyroscopic moment [3] that causes the balls to rotate around an axis perpendicular to the plane of figure 2. The gyroscopic moment and the resulting rotation are resisted by friction. The detailed formulation of these high speed loadings, i.e. centrifugal Force and gyroscopic moment, is given in [3] and the results are used here to establish force equilibrium.

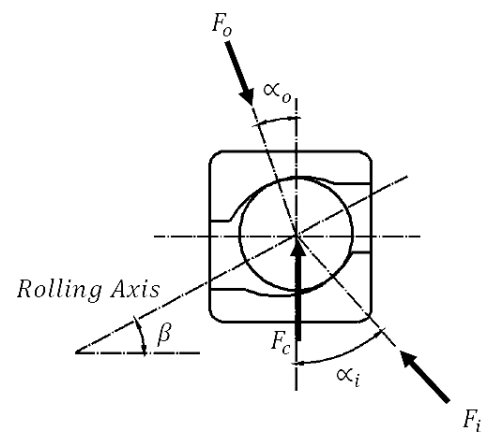


Figure 2: Forces on the bearing

The internal configuration of the bearing and its components i.e. inner ring, outer ring and the bearing ball, are illustrated in figure 3. In order to analyze the system this configuration should be completely defined.

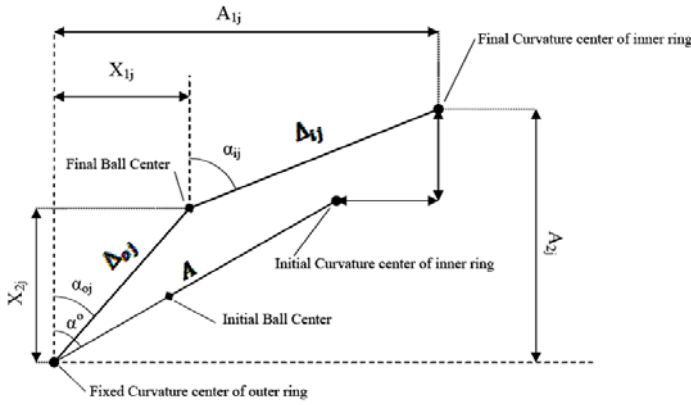


Figure 3: Internal configuration of bearing under application of loads

The initial distance between raceway curvature centers is:

$$A = (r_i + r_o - D) \quad (1)$$

The relative approach of contacting bodies in inner and outer ring contacts are δ_i and δ_o , respectively. Therefore, the distance between bearing ball center and the curvature centers of inner and outer rings are:

$$\Delta_{ij} = r_i - \frac{D}{2} + \delta_{ij} = \left(\frac{r_i}{D} - 0.5\right)D + \delta_{ij} \quad (2)$$

$$\Delta_{oj} = r_o - \frac{D}{2} + \delta_{oj} = \left(\frac{r_o}{D} - 0.5\right)D + \delta_{oj} \quad (3)$$

Where subscript "j" refers to the j_{th} rolling element measured from the most heavily loaded bearing ball.

Using figure 2, the contact angles can be expressed as:

$$\sin \alpha_{oj} = \frac{X_{1j}}{\left(\frac{r_o}{D} - 0.5\right)D + \delta_{oj}} \quad (4)$$

$$\cos \alpha_{oj} = \frac{X_{2j}}{\left(\frac{r_o}{D} - 0.5\right)D + \delta_{oj}} \quad (5)$$

$$\sin \alpha_{ij} = \frac{A_{1j} - X_{1j}}{\left(\frac{r_i}{D} - 0.5\right)D + \delta_{ij}} \quad (6)$$

$$\cos \alpha_{ij} = \frac{A_{2j} - X_{2j}}{\left(\frac{r_i}{D} - 0.5\right)D + \delta_{ij}} \quad (7)$$

in which:

$$A_{1j} = BD \sin \alpha^\circ + \delta_x - \gamma_z R_i \cos \varphi_j + \gamma_y R_i \sin \varphi_j \quad (8)$$

$$A_{2j} = BD \cos \alpha^\circ + \delta_y \cos \varphi_j + \delta_z \sin \varphi_j \quad (9)$$

$$\text{Where } R_i = \frac{1}{2} D_m + \left(\frac{r_i}{D} - 0.5\right)D \cos \alpha^\circ$$

Therefore, the following two geometrical relationships can be established:

$$(A_{1j} - X_{1j})^2 + (A_{2j} - X_{2j})^2 - \Delta_{ij}^2 = 0 \quad (10)$$

$$X_{1j}^2 + X_{2j}^2 - \Delta_{oj}^2 = 0 \quad (11)$$

Forces and equilibrium - Forces acting on a rolling element of the bearing are illustrated in Figure 4.

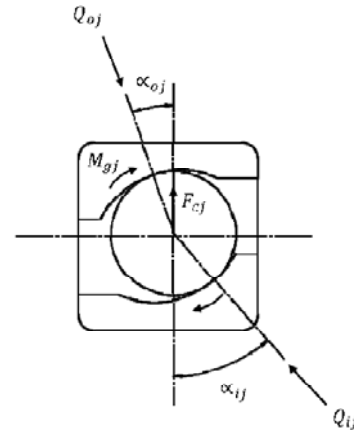


Figure 4- Forces acting on the j_{th} rolling elements

In this figure, Q_i and Q_o are contact forces associated with inner and outer raceways respectively. F_c is the centrifugal force and M_g is the gyroscopic moment. Hertzian contact theory is used to represent the nonlinear load-deflection behaviour of contact zones.

$$Q_{ij} = \chi_{ij} K_{ij} \delta_{ij}^n \quad (12)$$

$$Q_{oj} = \chi_{oj} K_{oj} \delta_{oj}^n \quad (13)$$

In the above equations, coefficient K is a load-deflection factor which depends on material properties and the geometry of contact, δ is the deflection due to applied load, the exponent n equals $3/2$ for ball bearings and χ is zero if $\delta < 0$.

The centrifugal force, F_{cj} , and the gyroscopic moment, M_{gj} , are expressed as follows [3]:

$$F_{cj} = \frac{1}{2} m_b D_m \Omega^2 \left(\frac{\Omega_E}{\Omega}\right)_j^2 \quad (14)$$

$$M_{gj} = J_b \Omega^2 \left(\frac{\Omega_B}{\Omega}\right)_j \left(\frac{\Omega_E}{\Omega}\right)_j \sin \beta_j \quad (15)$$

where m_b and J_b are bearing ball mass and mass moment of inertia, respectively, Ω is the rotational speed of the bearing, Ω_B is the rolling element's orbital speed, Ω_E is the spin speed and β is the angle of the rolling axis



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[3]. Depending on the rotational speed and applied loads, two conditions can exist for the ring that controls the motion of the rolling elements: *inner ring control* or *outer ring control* as discussed in [3] and [7]. The values of Ω_E and β depend on the type of ring control. Writing force equilibrium for rolling elements in axial and radial direction, we have:

$$Q_{oj} \cos \alpha_{oj} - \frac{\lambda_o M_{gj}}{D} \sin \alpha_{oj} - Q_{ij} \cos \alpha_{ij} + \frac{\lambda_i M_{gj}}{D} \sin \alpha_{ij} = F_{cj} \quad (16)$$

$$Q_{oj} \sin \alpha_{oj} + \frac{\lambda_o M_{gj}}{D} \cos \alpha_{oj} - Q_{ij} \sin \alpha_{ij} + \frac{\lambda_i M_{gj}}{D} \cos \alpha_{ij} = 0 \quad (17)$$

λ_i and λ_o determine the portion of contact divided between the inner and outer rings respectively and are determined by specific ring control of the bearing balls [3,7]. It can be assumed for outer ring control that the gyroscopic moment is entirely resisted entirely by frictional force at the ball-outer raceway contact. For other cases it is safe to consider equal resistance between ball raceway contacts [3]; Therefore $\lambda_{ij} = 0$ and $\lambda_{oj} = 2$ for outer raceway control; otherwise $\lambda_{ij} = \lambda_{oj} = 1$ [3]. In this work contact is assumed to be divided between rings by the ratio of contact deformations.

These two equations (16,17) in addition to the two geometrical equations derived in the previous part (10,11) will construct a set of four nonlinear equations for four unknowns: $\{X_{1j}, X_{2j}, \delta_{ij}, \delta_{oj}\}$. Newton-Raphson technique may be used for solving these equations through iterative stages. After solving these equations for all of rolling elements, equilibrium conditions for the whole bearing should be satisfied. This means that:

$$F_{xi} = \sum_{j=1}^Z (Q_{ij} \sin \alpha_{ij} + \frac{\lambda_{ij} M_{gj}}{D} \cos \alpha_{ij}) \quad (18)$$

$$F_{yi} = \sum_{j=1}^Z (Q_{ij} \cos \alpha_{ij} - \frac{\lambda_{ij} M_{gj}}{D} \sin \alpha_{ij}) \cos \varphi_j \quad (19)$$

$$F_{zi} = \sum_{j=1}^Z (Q_{ij} \cos \alpha_{ij} - \frac{\lambda_{ij} M_{gj}}{D} \sin \alpha_{ij}) \sin \varphi_j \quad (20)$$

$$M_{yi} = \sum_{j=1}^Z (R_i (Q_{ij} \sin \alpha_{ij} + \frac{\lambda_{ij} M_{gj}}{D} \cos \alpha_{ij}) - \frac{r_i}{D} \lambda_{ij} M_{gj}) \sin \varphi_j \quad (21)$$

$$M_{zi} = - \sum_{j=1}^Z (R_i (Q_{ij} \sin \alpha_{ij} + \frac{\lambda_{ij} M_{gj}}{D} \cos \alpha_{ij}) - \frac{r_i}{D} \lambda_{ij} M_{gj}) \cos \varphi_j \quad (22)$$

Z is the number of rolling elements in the bearing. The same equations can be established for the outer ring contact.

The forces and moments on the left side of these equations are external forces applied to the bearing through connection with shaft/housing or due to preload. This equations can be arranged in this form:

$$\{F\} = [K]\{\delta\} \quad (23)$$

$\{F\}$ and $\{\delta\}$ are force and displacement vectors respectively. The stiffness matrix of the bearing, $[K]$, is obtained by derivation of force vector components (equations 18-22) with respect to displacement vector components $(\delta_x, \delta_y, \delta_z, \gamma_y, \gamma_z)$. As a result a 5×5 stiffness matrix will be formed which is derived in detail in [7].

This is a set of five nonlinear equations and Newton-Raphson technique can be used again to solve for the unknowns. Having solved the first set, the unknowns here are the five degrees of freedom of the bearing. Thus we have two interconnected sets of nonlinear equations. The complications discussed before arise from the fact that in every iteration of the solution of the second set, the first set should be solved completely for every rolling element. As a result, for a given loading condition including preload, the global displacements of the bearing and its internal configuration and parameters i.e. rolling element contact angles are obtained. In addition, the stiffness matrix of the bearing whose terms depend on the displacement components is calculated.

MEAN CONTACT ANGLE - In order to reduce the computational cost of calculating the full model as detailed above, in this section a mean value for contact angle is introduced that is to be used for all rolling elements simultaneously regardless of the non-symmetrical loading conditions. This assumption is based on the fact that under certain loading and speed conditions, the difference between the contact angles of individual rolling elements may not be large enough to justify the computational cost of calculating them precisely. The mean contact angles is obtained by removing all loading conditions which cause the unsymmetry of the bearing configuration, radial forces and moments and keeping only axial forces. By solving the first set of equations just once (because the situation is the same for all bearing balls), two contact angles will be obtained which can be considered as mean contact angles for inner and outer races, for the general problem in which arbitrary loading is applied. Solving for $\{X_{1j}, X_{2j}, \delta_{ij}, \delta_{oj}\}$, the mean contact angles can be obtained using equations (4 through 7) as:

$$\alpha_{om} = \sin^{-1} \left(\frac{X_{1j}}{\Delta_{oj}} \right) \quad (24)$$

$$\alpha_{im} = \sin^{-1} \left(\frac{A_{1j} - X_{1j}}{\Delta_{ij}} \right) \quad (25)$$

α_{om} is the mean contact angle for outer-raceway contact of all rolling elements and α_{im} is that for inner-raceway contact. Although only one calculated these angles are applied for all rolling elements. These angles are calculated only once, and are applied for all rolling elements in the rest of the analysis.

SPINDLE-BEARING SYSTEM - In order to model an entire spindle bearing system, it is necessary to develop the governing equations of the shaft and housing in addition to the bearings. For this purpose, shaft and housing are modeled as Timoshenko beam elements with gyroscopic and centrifugal effects. The relevant finite element matrices for a 2 node, 10 DOF beam elements may be found, e.g. in [7] and [25]. The global FE equation of motion of the system can be formed by assembling the bearing, shaft and housing elements into the general equation $[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\}$ where $[M]$ is the mass matrix, $[C]$ is the damping matrix, $[K]$ is the stiffness matrix, $\{x\}$ is the displacement vector, $\{\dot{x}\}$ is the velocity vector, $\{\ddot{x}\}$ is the acceleration vector, and $\{F\}$ is the force vector.