

High Frequency Vibro-Acoustic Analysis Using Energy Finite Element Method

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ABSTRACT

As an alternative method to Statistical Energy Analysis (SEA), Energy Finite Element Method (EFEM) offers several unique advantages for vibro-acoustic analysis of structural-acoustic systems. In this paper, the theory of the energy finite element method is overviewed. The main developments of a recently available EFEM code are presented. This is followed by the investigation of several example problems using EFEM; (a) the acoustic pressure computation in an acoustical duct, (b) the sound transmission loss of an automotive dash, and (c) the vibro-acoustic analysis of a truck cab. The EFEM predictions are compared to the analytical solutions, SEA predictions or test data and good correlations are observed. Further, the advantages of EFEM in the solution of high and middle frequency vibro-acoustic problems are discussed.

INTRODUCTION

Numerical methods are used extensively for structural vibration and acoustic analysis in automotive and aerospace structures. At high frequency, when the dimension of the structure is large with respect to the wavelength, conventional Finite Element Method (FEM) and Boundary Element Method (BEM) become computationally expensive or even impracticable. In addition, structural-acoustic systems are highly sensitive to changes in environmental or boundary conditions at high frequency. In order to account for these variations, the averaged response with respect to frequency, space or across an ensemble is utilized. Statistical Energy

Analysis (SEA) is a relatively mature and established approach for predicting the average response of structural-acoustic systems at high frequency¹⁻³. In SEA, a vibro-acoustic system is divided into subsystems of similar modes. The lumped averaged energy within each subsystem comprises the primary SEA variable and the power transferred between subsystems is expressed in terms of coupling loss factors. SEA models result in few equations and are easy to solve. However, no information is available on the spatial variation of the energy within a subsystem. In addition, SEA models cannot be developed directly from CAD data, local damping or excitation cannot be accounted for accurately, and the model development requires specialized knowledge.

As an alternative method to SEA, Energy Finite Element Method (EFEM) is a recently developed tool for vibro-acoustic analysis⁴⁻⁶. EFEM is based on deriving governing energy flow differential equations in terms of energy density variables and employing a finite element approach for solving them numerically. In EFEM, structural and acoustic systems are modeled by finite elements. At the interface of different elements or same elements connected at locations of physical and/or material discontinuity, a special joint element based on power flow continuity across the junction is utilized to couple the discontinuous energy densities. EFEM formulations have been applied to structure-borne vibration and noise prediction in automotive⁷⁻⁸, naval structures⁹⁻¹⁰, and noise control treatment analysis¹¹⁻¹². A numerical code, Comet EnFlow, has been developed recently based on EFEM formulations¹³. Unlike SEA, FEM mesh can be directly used in EFEM, and as a

result, EFEM requires much less modeling effort. Since the EFEM results are not adversely affected by the use of finer mesh, both low frequency and high frequency analyses may be performed using a single FEM model¹².

In this paper, the EFEM formulations are overviewed first. Subsequently, the validation of EFEM is conducted by investigating several example problems. In the first example, the acoustic pressure distribution in an acoustical duct is computed. The duct is subjected to pressure excitation at one end and has two types of termination conditions at the other end. Next, the sound transmission loss of an automotive dash is computed. Finally, the acoustic and structural response of truck cab subjected to acoustical excitation is predicated. The EFEM predictions are performed using Comet EnFlow™ code and the results are compared to analytical solutions, SEA solutions or test data. Further, the advantages of EFEM in the solution of high and mid frequency vibro-acoustic problems are discussed in the conclusions.

OVERVIEW OF EFEM FORMULATION

Two approaches can be utilized to derive the relationship between the time and space averaged energy density and the intensity; one is considering a lossy plane wave solution to governing differential equations of an acoustic space or a thin plate⁷, and the other is assuming the response of a plate or an acoustic space as a linear superposition of incoherent orthogonal plane waves^{10,15}. The relationship between energy density and intensity is derived as:

$$\langle \bar{I} \rangle = -\frac{c_g^2}{\eta\omega} \nabla \langle \underline{D} \rangle \quad (1)$$

where $\langle \underline{D} \rangle$ is the time and space averaged energy density, $\langle \bar{I} \rangle$ is the time and space averaged energy intensity, c_g is the group velocity, η is the damping loss factor, and ω is the frequency of the harmonic excitation. At steady state, the conservation of power over a differential control volume can be expressed as:

$$\langle \underline{\Pi}_{in} \rangle = \langle \underline{\Pi}_{diss} \rangle + \nabla \langle \bar{I} \rangle \quad (2)$$

where $\langle \underline{\Pi}_{in} \rangle$ is the input power and $\langle \underline{\Pi}_{diss} \rangle$ is the time and space averaged dissipated power. The dissipated power is related to the energy density as:

$$\langle \underline{\Pi}_{diss} \rangle = \eta\omega \langle \underline{D} \rangle \quad (3)$$

The governing differential equation is derived by substituting equations (1) and (3) into equation (2) as:

$$-\frac{c_g^2}{\eta\omega} \nabla^2 \langle \underline{D} \rangle + \eta\omega \langle \underline{D} \rangle = \langle \underline{\Pi}_{in} \rangle \quad (4)$$

Finite element method is utilized for numerically solving the above governing equation. A linear matrix equation for the representative element is developed as:

$$[S^e] \{D^e\} + \{P^e\} = \{F^e\} \quad (5)$$

where superscript “e” indicates element-based quantities, $[S^e]$ is the element system matrix, $\{D^e\}$ is the nodal vector of energy density, $\{F^e\}$ is the nodal input power vector, and $\{P^e\}$ is the vector of power flow across the element boundary. The vector, $\{P^e\}$ provides a mechanism for connecting elements across discontinuities and to account for the intensity boundary conditions. Global matrix assembly is accomplished using standard finite element matrix assembly procedures except at discontinuities where the energy density is discontinuous. A specialized joint element equation is developed to formulate the connection between discontinuous primary variables at the joint. At junctions, the energy density is discontinuous while the power flow is continuous. The joint formulation is developed in detail in reference 11. The final equation is:

$$\left(\begin{bmatrix} S_s & \\ & S_a^{nct} \end{bmatrix} + [JC]^{nct} \right) \begin{Bmatrix} D_s \\ D_a \end{Bmatrix} = \begin{Bmatrix} F_s \\ F_a \end{Bmatrix} \quad (6)$$

where $[S_s]$ and $[S_a]$ are the system matrices for structural elements and acoustic elements, $\{D_s\}$ and $\{D_a\}$ are related to the nodal vector of structural and acoustic energy density, $\{F_s\}$ and $\{F_a\}$ are structural and acoustic nodal input power vector, and $[JC]$ are joint matrices the connection between discontinuous primary variables. The joint matrices are functions of power transmission coefficients. In the above equation, superscript “nct” represents the effects of noise control treatments.

The global equation of EFEM can be obtained by assembling equation (6) as,

$$[S] + \sum [JC] \{D\} = \{F\} \quad (7)$$

where $[S]$ is the global system matrix, $\sum [JC]$ is the summation of all joint matrices, $\{D\}$ is the vector of nodal energy density, and $\{F\}$ is the vector of input power.

DEVELOPMENT OF A EFEM COMPUTER CODE

A numerical code, Comet EnFlow, and its graphical user interface Comet EnVision is developed recently¹³⁻¹⁴. The main developments include:

- Direct loading finite element mesh from third party commercial FEM codes such as Nastran, Ansys and Abaqus.
- Automatic identification of point, line and area junctions at geometric and material discontinuities.
- Incorporation of direct and indirect transmission paths for interior noise prediction.
- Incorporation of extensive library of structural (beam, isotropic plate, orthotropic plate, honeycomb structure, and shell) and noise control (elastic porous, rigid porous, limp porous, fibrous, resistive screen, impervious screen, and perforated panel) elements.
- Incorporation of power transfer mechanism of extensive joint elements (beam/beam, plate/plate, beam/plate, plate/acoustics, and plate with passive noise control treatment and leakage).
- Incorporation of energy boundary element method for the modeling of acoustic space.
- Incorporation of a hybrid method for the accurate modeling of local excitation.
- Incorporation of efficient solution methods including node numbering, bandwidth optimization and iterative solvers for large scale systems.
- Incorporation of the non-conservative joints and the user defined joints.

VALIDATION EXAMPLES

A. ACOUSTICAL DUCT

The acoustic pressure distribution in a duct (Figure 1) is computed here. The dimension of duct is 0.5m*0.5m*2.0m and a unit pressure excitation is applied at one end. Two types of the termination conditions are considered at other end of the duct; rigid wall and noise control treatment (NCT) layer backing by the rigid wall. In this example, a 10 mm melamine foam layer is utilized as the noise control treatment. The relevant properties of the melamine foam are listed in Table 1. The acoustic damping is assumed as 1%. The EnFlow model is shown in Figure 2. The input power is applied at the excitation end and absorption boundary condition due to the noise control treatment is applied at the other end. For comparison, the analytical method is utilized to compute the acoustic pressure for both configurations. The noise control treatment is simulated as normal impedance boundary condition in the analytical method which is regarded as exact solutions.

The comparison of results is performed at three excitation frequencies (1000 Hz, 2000 Hz and 3000 Hz). The acoustic pressure distribution along the longitudinal axis of the duct at the three frequencies is plotted in Figure 3 to Figure 5, respectively. To compare properly with the pressure retrieved from EnFlow results, the averaged pressure over time and space by the analytical method is also plotted in the figures. The EnFlow results are almost identical with the analytical results for both rigid and NCT termination conditions. The excellent correlation is anticipated since a plane lossy wave solution is assumed in the analytical method which exactly matches the assumption that is used in the derivation of the relationship between energy density and energy intensity in Equation (1) of the EFEM formulations. The results also indicate that the decay of the acoustic pressure is more obvious as the excitation frequency increases. EnFlow predictions capture correctly the decay of acoustic pressure in the continuous acoustic space of the duct.

B. SOUND TRANSMISSION LOSS OF AN AUTOMOTIVE DASH

The sound transmission loss of an automotive dash is calculated by EFEM. The dash is divided into five regions as shown in Figure 6. The panel properties and noise control treatment arrangement of different regions are listed in Table 2. Except for the cutout region, noise control treatments are applied at all other regions. The layer configuration of the noise control treatments are described in Table 3.