

Synthesis of a Dynamically Loaded Structure with Topology Optimization

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ABSTRACT

In this paper, a Hybrid Cellular Automaton (HCA) algorithm has been utilized to develop an efficient methodology for synthesizing structures under a dynamic loading event. This method utilizes the cellular automata computing paradigm with the nonlinear transient finite element analysis. Previous work in topology optimization for structural design has concentrated on modeling assuming static loading conditions due to the complexities associated with dynamic loading. An HCA method has been developed to address more complicated cases that involve dynamic events, such as impacts and collisions. To illustrate this technology, a generic design domain that represents an automotive crush structure is presented to demonstrate the efficiency of the proposed methodology in synthesizing crashworthy, constant cross section structures. Influence of mass fraction on final topology and application of this methodology for multiple load cases is presented.

INTRODUCTION

Previous research in structural optimization has concentrated on solving issues using static loading conditions. However, many industrial applications require non-linear dynamic modeling of loads. In a dynamic case, such as one involving an impact or collision between two bodies, the loading is a function of time. Thus, a transient finite element analysis is useful to

obtain the numerical dynamic response of the system. The complexities and expensive computational time associated with this analysis have hindered research in the optimization of structures designed for dynamic loading conditions.

In the proposed methodology, topology optimization is used to generate design concepts for structures subjected to dynamic loading. Topology optimization, a method for generating structural concepts, is an iterative process that determines the best arrangement of material so as to obtain an optimal mechanical performance of the concept design. These design issues generally involve a large number of variables and thus can lead to large computational efforts for gradient-based optimizers. Therefore a hybrid cellular automaton (HCA) algorithm is developed to synthesize structures that are subjected to nonlinear transient loading.

METHODOLOGY

DYNAMIC LOADING - The traditional class of problems that have been addressed using topology optimization are independent of time. These types of analyses are referred to as being static. Therefore static structural analysis can be used to evaluate the response of a structure at each optimization iteration. However, many practical events are time-dependent and dynamic in nature. In static loading, the structural analysis is performed for a steady-state response. In a dynamic event, such as one involving an impact or collision between two bodies, the loads are applied over a short

period of time where transient effects must be considered. For example, with application to the automotive industry, a vehicle crash is appropriately described as a dynamic event [1, 2].

To obtain the numerical dynamic response of the system, a dynamic FEA (finite element analysis) is required. FEA is a numerical technique that is typically utilized in topology optimization for structural analysis of the collision-impact events to solve for the displacements and ultimately obtain structural performances for the topology at each iteration. In this method, a structure is discretized into N finite elements that are made up of nodes; then displacements are interpolated between the nodes. The response of a dynamic system can be expressed by the state equation,

$$\mathbf{M}\ddot{\mathbf{d}}(t) + \mathbf{C}\dot{\mathbf{d}}(t) + \mathbf{K}\mathbf{d}(t) = \mathbf{R}(\mathbf{d}, t) \quad (1)$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the mass, damping, and stiffness matrices respectively. Where only the elastic forces ($\mathbf{F}_E = \mathbf{K}\mathbf{d}$) are accounted for in static problems, the transient analysis accounts for inertial effects due to acceleration-dependent inertia forces ($\mathbf{F}_I = \mathbf{M}\ddot{\mathbf{d}}$) and velocity-dependent damping forces ($\mathbf{F}_D = \mathbf{C}\dot{\mathbf{d}}$) [3]. The parameters $\mathbf{d}(t)$, $\dot{\mathbf{d}}(t)$, and $\ddot{\mathbf{d}}(t)$ are the displacement vector, velocity vector, and acceleration vector respectively as functions of time. The dynamic analysis can be solved using an *implicit* or *explicit* integration to solve the equation of motion in Eq. (1). Using an implicit time scheme to find the displacements at each time step is generally more accurate as compared to the explicit integration method, although it requires more computational time. For the explicit time scheme, the displacements are solved by direct numerical integration of the equation, meaning that prior to numerical integration, no transformation of the equations is required. In this paper, a nonlinear dynamic finite element analysis using explicit integration is utilized for topology synthesis.

TOPOLOGY OPTIMIZATION - The field of topology optimization has been developed extensively over the past two decades. In some respects, topology optimization can be viewed as a method for developing an initial, or concept design. The optimization process systematically eliminates and re-distributes material throughout the domain to minimize or maximize a specified objective. This computational technique for the optimal distribution of material in continuum structures was first introduced by Bendsøe and Kikuchi [4]. Early work in topology optimization generally dealt with simple cases that had the assumptions of linearly elastic material properties and static loading conditions. The objective and constraints considered are a global structural response such as mean compliance, von Mises stresses, eigenfrequencies, or geometrical parameters such as volume (or mass) or perimeter. Topology optimization has been solved using several

approaches. In this paper, a HCA method is utilized for the topology synthesis.

HCA FORMULATION AND METHODOLOGY – The HCA based topology optimization approach is inspired by the biological process of bone remodeling as presented by Tovar [5-7]. In this approach material is appropriated to uniformly distribute the effects of a specified mechanical stimulus, or field variable, throughout the design domain based on local CA rules. While the HCA method is not explicitly an optimization technique, the local rules produce this behavior. A cellular automaton is a discrete model studied in computability theory and mathematics [8]. It consists of an infinite, regular grid of cells, or lattice, where each is characterized by a finite number of states. The state of each cell at a given time, or generation, is a function of the states of a finite number of neighboring cells, called the neighborhood, as illustrated in figure-1. Every cell has the same set of rules, which are applied based on information in its neighborhood. These rules are applied to the entire CA.

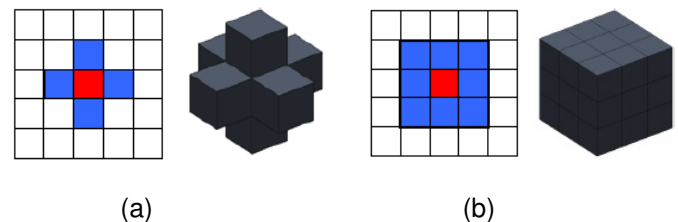


Figure 1: Typical CA Neighborhoods: (a) vonNeumann (2D: $\hat{N}=4$, 3D: $\hat{N}=6$) and (b) Moore (2D: $\hat{N}=8$, 3D: $\hat{N}=26$).

The HCA method utilizes the CA computing paradigm along with FEM to act as an optimization tool for the synthesis of optimal topologies. As done in traditional topology optimization methods, the structural design domains are discretized into material elements. To use the FEM for structural analysis, the design domain is represented using a finite element model that is discretized using continuum finite elements (FE). The states of the material elements in the design domain are represented using a lattice of CAs, where a one-to-one correspondence between CAs and FEs generally exist, although this is not a requirement. However, uniformity in the CA discretization is required. A set of local rules is used to determine material distribution. These rules are applied to the local information collected in the neighborhood of each CA. At a discrete position i and time/iteration k , a CA is defined by a set of states that are operated on by a set of rules belonging to a given neighborhood of the CA.

Objective – One of the objectives in the design of dynamic loaded structures is to absorb maximum energy. For a collision-impact event involving dynamic loading, the total energy absorbed is given by the total internal energy (IE). This is the total elastic and plastic work done during the whole loading cycle. In this paper, the goal is to synthesize structure so that all elements in

the structure should contribute to the energy absorption through plastic deformation. This is an extension of the idea of a fully stressed design (FSD) which states that “for the optimum design each member of the structure that is not at its minimum gage is fully stressed under at least one of the design load conditions” [9]. This philosophy is used in the HCA based topology optimization to find a density distribution within the design domain so that the resulting design will sustain the applied dynamic load as efficiently as possible. Most often the density function is discretized according to the finite element mesh. The effective internal energy density (IED) ($\bar{S}_i$) for an element is found by taking the average of its own IED and its neighborhood,

$$\bar{S}_i = \frac{1}{\hat{N}} \sum_{j=0}^{\hat{N}} S_j \quad (2)$$

where $\hat{N}$ is the number of elements in the neighborhood.

This quantity is utilized as a measure of the effective contribution of each finite element to the internal energy state of the current design. The higher an element's effective internal energy the more that element contributes to the overall energy absorption of the structure, and thus are of greater importance to the performance of the topology. To drive the IED of each element, material distribution rules and the CA paradigm are used.

Material Distribution Rules – The material distribution rules drive the topology synthesis to achieve the specified objective. For continuum structures, the homogenization and density approaches are the two most popular material parameterizations. The homogenization method can be viewed as an interpolation model for void and full material. The density approach, which is used in this paper deals with the more direct approach of associating just one design variable for each individual material element. The material model is defined to allow the material to assume intermediate property values by defining an interpolation function. The relative density is used to interpolate the material property (elastic modulus, yield strength, hardening modulus etc) of each element. This interpolation is most commonly done using the solid isotropic material with penalization (SIMP) model. The penalization in SIMP is used as a technique to drive away intermediate densities to generate a concept topology with a 0-1 design (0 being no density and 1 being full density) [2,10,11].

To ensure the even distribution of the internal energy density, a target internal energy density, also termed as setpoint, is evaluated after each iteration. Then all the elemental IEDs are compared with this target value. If the element is found inefficient i.e. having less IED than the target value, the elements density is reduced. On the contrary, if the element is found to be efficient, i.e. having

more IED than the target value, then the density of the element is increased. At the k^{th} iteration, this material update rule can be expressed as,

$$\Delta x_i^{(k)} = K_p (\bar{S}_i^{(k)} - S_i^{*(k)}) \quad (3)$$

where $\Delta x_i^{(k)}$, $\bar{S}_i^{(k)}$ and $S_i^{*(k)}$ are the change in density, the effective IED and specified IED setpoint respectively for the i^{th} element, and K_p is a scaling parameter. The maximum allowable change in relative density of any element per iteration is 0.1 to provide stability [2,10,11].

Material distribution can oscillate between iterations, especially in synthesis of dynamic events. To reduce these oscillations in the material distribution, information from previous iterations is used. In the proposed method, the CA field state (S_i) of each element is represented by an exponential moving average of the IED state at the current iteration and n previous iterations. This information gives each element "memory" of its previous states. This can be expressed as,

$$S_i^{(k)} = \frac{1}{\sum_{k=0}^n w_i^k} \sum_{j=0}^n w_i^j U_i^{(k-j)} \quad \text{where } w_i = (x_i - x_{\min})^2 \quad (4)$$

The amount of memory possessed by an element differs for each element and is a function of its relative density (x). Elements at the lower bound retain no memory of the IED state from the previous n iterations. In this investigation weighted information from the previous three iterations ($n = 4$) is used to calculate the field state of each element at the current iteration [2]

Mass Control – In practice, a constraint on the mass of the synthesized structure is often required to generate concept designs. To accommodate a mass constraint the global setpoint value (applied to all material elements) that produces the required mass must be determined at each iteration. This is done by iterating on the HCA update rule and updating the setpoint until the correct mass results after applying the design rule expressed in Eq. 4. The setpoint for the $(k+1)^{th}$ HCA iteration is found by iterating on the update,

$$S^{*(j+1)} = S^{*(j)} \left(\frac{M_f^{(k+1)}}{M_f^*} \right) \quad (5)$$

where j is an iterator for the sub-loop on the material distribution rule and M_f^* is the mass fraction target. The mass fraction of a design domain is defined as,

$$M_f = \frac{\sum_{i=1}^N x_i}{N} \quad (6)$$