

Entrainment Waves in Diesel Jets

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ABSTRACT

Recent measurements in transient diesel jets have shown that fuel in the wake of the injection pulse mixes with ambient gases more rapidly than in a steady jet. This rapid mixing after the end of injection (EOI) can create fuel-lean regions near the fuel injector. These lean regions may not burn to completion for conditions where autoignition occurs after EOI, as is typical of low-temperature combustion (LTC) diesel engines. In this study, transient diesel jets are analyzed using a simple one-dimensional jet model. The model predicts that after EOI, a region of increased entrainment, termed the “entrainment wave,” travels downstream at twice the initial jet propagation rate. The entrainment wave increases mixing by up to a factor of three. This entrainment wave is not specific to LTC jets, but rather it is important for both conventional diesel combustion and LTC conditions. It is shown to be responsible for (i) observed over-mixed regions and (ii) rapid stagnation of mixtures near the injector, (iii) spatial shifts in the location of the onset of soot formation, (iv) increased soot oxidation after the end of injection, (v) decreased penetration of short injections, and (vi) detachment, retreat, and splitting of the liquid part of the vaporizing fuel spray. Finally, the model predicts that a faster ramp-down of injection rate at EOI creates a stronger entrainment wave forming leaner mixtures near the injector more rapidly.

INTRODUCTION

The fluid mechanics of transient fuel-jets are important for subsequent combustion, engine performance, and pollutant emissions of diesel engines. The transient start-up and rate of penetration of liquid fuel and the vapor-phase diesel jet head has been widely studied (for example, [1-3]). From the start of combustion to the end of injection (EOI) is the “quasi-steady” period, which is important for performance and emissions because of significant soot and NO_x formation, has also been well studied (for example, [4-8]). Much less attention has been devoted to the transient ramp-down at EOI, so its importance for conventional diesel combustion is not well established. For low-temperature-combustion (LTC) diesel engines, however, recent measurements show that transient ramp-down processes can be important.

LTC diesel engines [9-18] use dilution to reduce flame temperatures, and thereby to reduce nitrogen oxides (NO_x) formation. The dilution may be achieved either by exhaust-gas recirculation (EGR), or by using locally fuel-lean mixtures. In either case, the fuel is also typically premixed with the in-cylinder gases, especially at lower engine loads, to avoid the fuel-rich regions that form soot. As a result, LTC engines can realize lower emissions of both NO_x and soot relative to conventional diesel engines [9,10,19]. To achieve the necessary premixing before combustion, the fuel is typically injected at a time in the cycle when the in-cylinder gases are cooler and less dense. With lower temperatures,

autoignition of the fuel can be delayed until after EOI, so that more time is available for premixing prior to combustion. With combustion after EOI, the quasi-steady period that is so important for conventional diesel combustion ceases to exist. Therefore, fluid mechanical processes occurring during and after the transient ramp-down become more important for mixture preparation and subsequent pollutant formation in LTC diesel engines.

Recent measurements have shown that combustion and pollutant formation processes in LTC diesel jets are significantly different than in conventional diesel jets. Shown in Fig. 1 are two in-cylinder images of OH planar laser-induced fluorescence (OH-PLIF, green) with simultaneous images of either soot laser-induced incandescence (LII, top, red) or soot luminosity (bottom, red). The top image is for a typical conventional diesel engine operating condition with no EGR and injection near top-dead-center (TDC) of the piston's stroke, taken from [20]. The bottom image in Fig. 1 is for an early-

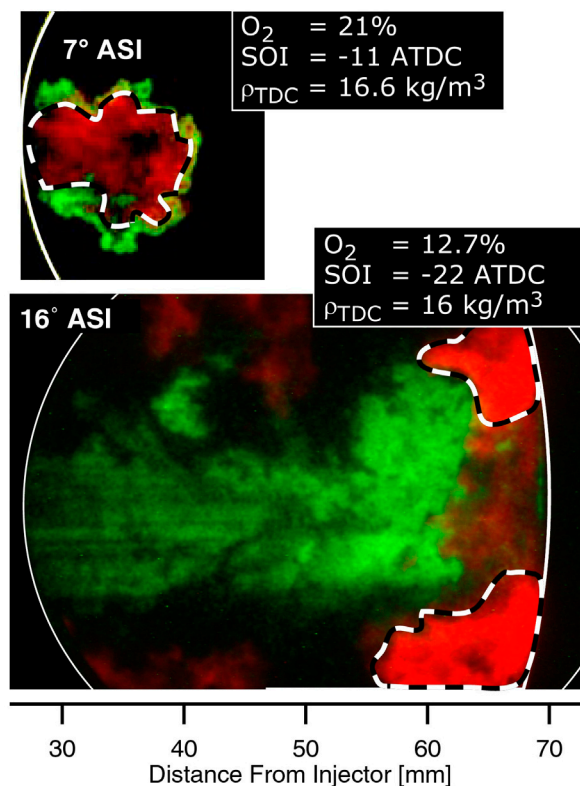


Figure 1. Simultaneous OH-PLIF (green) and soot (red, inside region bounded by dashed line) detected by LII (top) or by natural luminosity (bottom) in a diesel jet penetrating from left to right. Top image: conventional diesel combustion [20]. Bottom image: early-injection LTC [21]. The number in the upper left corner indicates the crank angle degrees after the start of injection when the image was acquired.

injection LTC condition, with the intake stream diluted by nitrogen to an oxygen concentration of 12.7% (equivalent to 50-60% EGR) [21]. For both images, the view is through a window in the cylinder head in place of an exhaust valve. One of the eight diesel jets is visible within the field of view, penetrating from left to right. As indicated by the scale at the bottom of the figure, the injector is outside the field of view, to the left.

For the conventional diesel condition in the top image of Fig. 1, the start of injection (SOI) is at a crank angle of -11.5° after top dead center (ATDC) and ignition occurs about 4° later. By 7° after the start of injection (ASI), the front of the diesel jet is visible through the window, and soot (red) is present throughout the jet cross section, filling the region marked by the dashed line. On the perimeter of the soot-filled jet is a thin layer of OH (green), indicative of a diffusion flame. Soot typically forms when the equivalence ratio ϕ in the jet is greater than about $\phi = 2$ [8], so that the presence of soot indicates that mixtures in the jet cross-section are likely fuel-rich. Indeed, direct vapor fuel measurements by planar laser Rayleigh scattering show that the equivalence ratios are in the range $\phi = 2-4$ where soot forms [4]. OH is most abundant in mixtures of intermediate stoichiometry, centered near $\phi = 1$ [21,22], indicating a thin interface at the diffusion flame between the fuel-rich core of the jet and the surrounding air. Again, direct vapor-fuel measurements confirm that the gradients are steep on the periphery of the jet with a thin region near $\phi = 1$ [4].

For the LTC jet in the lower image, the soot and OH distributions are drastically different. For this condition, fuel injection occurs much earlier, at -22° ATDC, and the main ignition occurs near 12° ASI, which is 3° after the end of injection (AEI). By 16° ASI, when significant soot forms, the jet has penetrated much farther than the conventional diesel jet in the upper image, reaching the cylinder wall. Both the lower in-cylinder density and the much longer time from the start of injection contribute to the greater penetration. More importantly, rather than forming throughout the jet cross-section, soot only forms in the far downstream region of the jet, off to the "sides" of the jet axis (red color bounded by dashed lines in lower image). The OH distribution is also much different than for conventional diesel combustion, with OH filling almost the entire jet cross-section.

The OH and soot distributions strongly suggest that much of the jet cross-section in the LTC jet is of intermediate stoichiometry, somewhere near $\phi = 1$, while only the far downstream region near the cylinder wall is

rich enough for soot formation. This suggestion is surprising because with the intake oxygen concentration reduced from 21% to 12.7 % to achieve low-temperature combustion, mixtures should be more fuel rich overall. For the LTC condition, however, combustion does not occur until after EOI, so there is also more time for premixing. But even with more time for premixing, the width of the upstream LTC jet is nearly the same as for the conventional jet. Therefore, the lack of soot and widespread distribution of OH throughout the LTC jet at just 3° AEI suggests that the jet must very rapidly mix to much leaner conditions without any drastic increase in the local jet dimensions.

To confirm these indications of rapid mixing after EOI, vapor-fuel concentrations were measured under similar thermodynamic conditions, but without combustion (0% intake oxygen). Shown in Fig. 2 are contours of vapor-fuel equivalence ratio measured using toluene-trace fluorescence under non-combusting conditions in a heavy-duty diesel engine [23]. Portions of two of the eight fuel jets are within the field of view, with the injector at the left of each image. From the perspective of the camera, one jet penetrates to the right and upward, while the other penetrates to the right and downward from the injector. The three images in Fig. 2 were acquired at EOI (top), at 2 AEI (middle), and at 5 AEI (bottom). The equivalence ratios are computed for an intake oxygen concentration of 12.7%, which is equivalent to the LTC condition in Fig. 1.

At EOI, the local equivalence ratios are highest near the injector and decrease downstream, which is typical of the roughly inverse proportionality of equivalence ratio with downstream distance in steady diesel jets [1]. Very shortly after EOI, however, as illustrated in the middle and bottom images, the equivalence ratio distribution reverses – regions near the injector become leaner than the downstream regions.

Furthermore, the speed at which the upstream mixtures become leaner far exceeds that expected for steady-jet mixing rates. In the top image of Fig. 2, the circled numbers indicate the approximate time for fuel leaving the injector to reach each position. Each number represents the number of crank angle degrees required for the leading edge of the jet to reach that downstream position. From a Lagrangian perspective (i.e., moving with the flow), the penetration positions give a rough indication of the rate at which fuel mixes with the ambient gases prior to EOI. For instance, after two crank angle degrees, fluid that emanated from the injector will have traveled approximately 25 mm from the

injector to the position of the “2” circle, where centerline equivalence ratios are between 5 and 7. After five crank angle degrees, the fluid will have moved about 45 mm to the “5” circle, where the centerline equivalence ratio is between 3 and 5.

The two images acquired at 2° AEI and 5° AEI show that mixing in the jet after EOI is much faster than would have been expected from the Lagrangian perspective described above. At 2° AEI, centerline equivalence ratios in the jet at the “2” circle position (25 mm from the

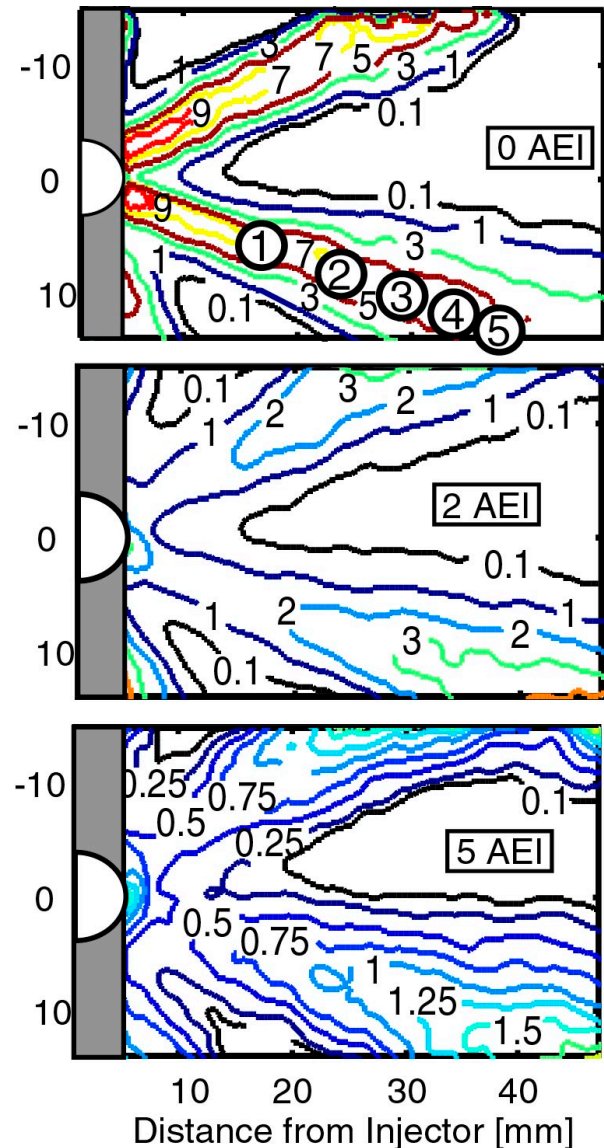


Figure 2. Vapor fuel equivalence ratio measurements by toluene tracer fluorescence under non-combusting conditions [23]. The injector is on the middle-left of the image, and two jets within the field of view. The contours show ensemble-averaged equivalence ratio for 12.7% intake oxygen. The circled numbers show the approximate time in crank-angle degrees for the jet to penetrate to the position of each circle.

injector) are between 2 and 3, compared to 5 to 7 for the steady jet. At 5° AEI, centerline mixtures at the “5” circle position (45 mm from the injector) are near $\phi = 1.5$, compared to 3 to 5 for the steady jet. Mixtures closer to the injector have even lower equivalence ratios, which indicate even greater mixing near the injector after EOI. Clearly, mixing after EOI occurs faster than in the steady jet.

The above observations of drastically increased mixing rates after EOI are consistent with the earlier discussion of the differences in soot and OH distributions between conventional and LTC diesel conditions. As a result of the rapidly increased mixing after EOI, positions in the jet that were rich enough to form soot in the conventional diesel jet (Fig. 1, top) become lean enough that soot does not form (Fig. 1, bottom). In addition, the mixtures in much of the jet are of intermediate stoichiometries by several crank angle degrees after EOI (Fig. 2, bottom). As described earlier, OH is most prominent in mixtures of intermediate stoichiometries, which is where OH is observed in the LTC jet (Fig. 1, bottom). The rapid mixing after EOI shown in Fig. 2 is therefore consistent with the OH and soot distributions for the LTC condition in Fig. 1.

While the rapid mixing after EOI can reduce soot formation in the jet, it can also cause incomplete combustion in the very lean mixtures near the injector. Indeed, late-cycle fuel and formaldehyde, which are indicators of incomplete combustion, can be detected by laser-induced fluorescence in the region near the injector for conditions with ignition occurring after EOI [24-27]. Because of incomplete combustion, these same lean regions near the injector can contribute to unburned hydrocarbon (UHC) and carbon monoxide (CO) emissions [26,27]. Thus, the rapidly increased mixing after EOI can be beneficial for reducing soot formation, but it can also be detrimental to UHC and CO emissions if mixtures become too lean for complete combustion.

Finally, the above observations in diesel jets are similar to those of fundamental single-phase transient gas- and water-jet studies, which show that after a deceleration phase, entrainment can be increased by a factor of two or more compared to steady jets ([28,29]). The changes in entrainment transient jets may be due to alterations in the source fluid for the growth of large structures in the jet flow ([28,30]). For accelerating jets, Kato et al. [30] proposed that more of the vortex engulfment appetite is satisfied by the accelerating jet flow, so that less ambient fluid entrainment is necessary for vortex growth. For decelerating jets, Johari and Paduano [28] proposed

the converse argument, that lesser jet flow is available for the growth of large structures, so that entrainment must increase to compensate.

The goal of the current study is to reveal the underlying fluid-mechanical processes responsible for the increased mixing after EOI. To do so, a simplified one-dimensional numerical model for transient diesel jets is developed and applied to a fuel injection event. In the Results section, the model predictions show that after EOI, entrainment increases by up to a factor of three. To help validate the model, its predictions are compared with experimental data from several previously published transient gas-jet and diesel-jet studies. The implications for mixing, soot formation, and penetration in diesel jets are discussed.

ONE-DIMENSIONAL DIESEL JET MODEL

The simplified jet model proposed here is based on the control volume analysis for steady injections of Naber and Siebers [1]. In their development, they assumed a constant jet spreading angle, no velocity slip between injected fuel and entrained gases, and a constant injection velocity with an instantaneous start. Their analysis was also simplified to one dimension by treating the axial velocity profile as uniform over the jet cross-section. To account for the effects of this simplification on the jet penetration rate, the spreading angle of the jet was reduced to about 2/3 of that measured from Schlieren imaging. By balancing the fluxes of fuel mass and total momentum passing through a single control volume bounding the jet, they derived an expression for the cross-sectionally averaged axial velocity along the jet axis. They also derived an analytical expression for the penetration time as a function of penetration distance as the time-integral of a non-dimensional form of the uniform axial velocity equation. The penetration function agreed well with experimental penetration data over a wide range of conditions [1].

The success of the Naber and Siebers penetration function indicates that the control volume analysis likely captures the important physics affecting starting diesel jets. However, their solution necessarily treats the jet velocity at any axial location as invariant with time, as in a steady jet. Therefore, the dynamics of a jet with an unsteady injection rate cannot be predicted by their solution. One way to relieve the steady-state constraint is to discretize the domain in the axial direction and add transient terms for the jet momentum and fuel mass transport equations in each discretized control volume [31].

Shown in Fig. 3 is an illustration of a one-dimensional array of control volumes encompassing a diesel jet, discretized along the jet axis. The boundaries of the conical jet, as indicated by the dashed lines, span a constant full spreading angle θ . Here, the jet boundary is defined as the radial position where the axial velocity in the model is zero (see equation 3a, introduced below). This effectively corresponds to a position on a Gaussian error function fit to the axial velocity profile at 3% of the centerline value.

Expanding on the steady-jet development of Naber and Siebers [1], transient transport equations for fuel mass and total jet momentum may be solved for the jet geometry in Fig. 3 if seven simplifying assumptions are applied. Some of the assumptions are based on experimental observations from either diesel jets or gas jets. Observations from gas jets are relevant because many aspects of the mixing and transient development of diesel jets are analogous to single-phase transient gas jets [32]. Indeed, many transient gas-jet studies have been motivated by a need to improve understanding of liquid-fuel sprays by studying jet mixing processes in the more tractable environment of single-phase jets [33-35].

1. The jet is non-vaporizing. The effects of evaporative cooling and phase change of the fuel in a vaporizing spray are too complicated to be considered in the simplified model employed here. Therefore, in the model to follow, the fuel is universally treated as liquid. Even with this simplification, Naber and Siebers [1] found that their development for non-vaporizing sprays predicted penetration of vaporizing sprays quite well. Furthermore, Siebers [36] developed a scaling law that predicted the liquid length of vaporizing sprays based on a mixing-limited vaporization assumption, even with a

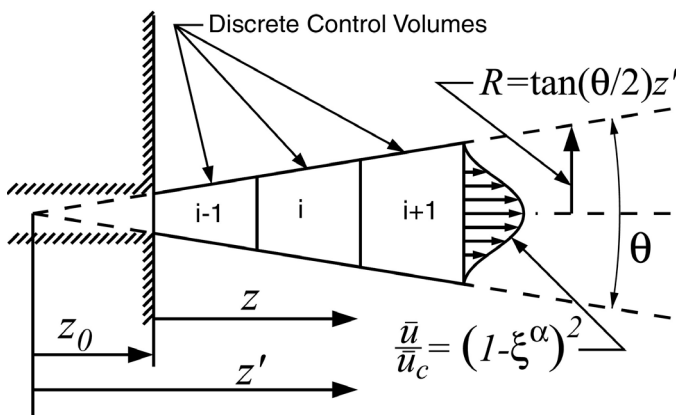


Figure 3: One-dimensional discrete control volume transient diesel jet model.

non-vaporizing model. Here as well, the equivalence ratio solution of the non-vaporizing model described below can also predict mixing-limited liquid-fuel vaporization behavior.

2. The flow is incompressible. For a typical diesel jet, compressibility may be important for the high-velocity flow very near the nozzle, but for most of the downstream jet, the flow is slow enough that compressibility effects are minor.
3. Turbulent (and molecular) viscous forces acting on each control volume are neglected. The normal viscous forces are typically small compared to pressure forces, and viscous shear forces at the outer radius of the jet control volumes can be neglected also, since velocities and their gradients at the boundary are small. That is, the viscous shear forces may be significant in high shear regions of the jet, but these regions are within the control volumes, and therefore do not contribute to any external force.
4. Axial mixing of momentum due to molecular and turbulent diffusion is neglected. As a result, only axial convection is considered in the transient momentum transport equation. The implications of this simplification are addressed at the end of the Discussion section.
5. The net force due to any axial pressure gradient is assumed negligible. This assumption is reasonable for steady jets [37], and the appropriateness of this assumption during and after EOI also is addressed at the end of the Discussion section.
6. The jet spreading angle is treated as constant during and after the EOI transient. During the EOI transient, it is not obvious that the constant spreading-angle observed for steady jets, as illustrated in Fig. 3, would be maintained. For example, it is conceivable that the jet might “neck down” as ambient fluid rushes into the wake of the injection pulse, so that the effective spreading angle would decrease. Although no theoretical justification for a constant spreading angle during the EOI transient is offered here, experimental observations indicate that the spreading angle indeed remains approximately constant, or even increases slightly, during the EOI transient. Observations from six separate experiments support this assumption.

First, in an air jet with a sudden decrease to one half in its initial ejection velocity, hot-wire anemometry

measurements show that during the transient, the jet full width at half maximum (FWHM) of the normalized mean axial velocity profile increased by at most 15% [38,39]. This relative increase in FWHM is magnified somewhat by a small depression of the centerline velocity late in the transition. Nevertheless, the data shows that the jet velocity FWHM increases only slightly as the injection rate decreases, such as during the EOI transient. Similar behavior has also been observed in diesel jets, with the velocity FWHM increasing only slightly during and after EOI [32].

Similar conclusions may be drawn from scalar measurements. At EOI in diesel jets, the scalar jet boundaries remain relatively constant during and after the EOI transient, though the width gradually increases after EOI [23,40]. Shadowgraph movies of non-combusting diesel jets also show that the perimeter of the jet, as indicated by the contrast in the fine-scale density gradients, does not markedly change during the EOI transient, though it does gradually increase thereafter [41]. Finally, as a contrasting argument, the spreading angle has been observed to decrease slightly in an accelerating jet [42], so the opposite trend of a slightly increasing spreading angle during deceleration might be expected. Based on the above consistent experimental observations, the spreading angle during the EOI transient is treated as constant at the same value as that of the steady jet. The implications of this approximation will be examined at the end of the Discussion section.

7. The (normalized) radial profile of mean axial velocity remains unchanged during the EOI transient. Experimental data from at least three separate experiments show that the profile remains approximately unchanged during an EOI transient. First, Atassi et al. [38] observed roughly similar velocity profiles (with a maximum 15% increase in characteristic width) throughout an injection velocity decrease in a single-phase jet. Second, Doudou [32] observed velocity profiles in the tail of a two-phase diesel spray after the EOI transient that were essentially identical to those of the quasi-steady jet. Additionally, the normalized velocity profiles in a variety of modulated jets, which have periodic variations in injection velocity, also remain comparable to those of a steady jet, even with large axial velocity fluctuations [43]. These experimental

data show that exceptional changes in the velocity profile are not observed during the EOI transient.¹

Using the above assumptions, the transient transport equations for fuel mass and total jet momentum in each control volume are developed as follows. The transient transport equations for fuel mass m_f and axial momentum M in each control volume can be written as

$$\frac{\partial m_f}{\partial t} = \dot{m}_{f,in} - \dot{m}_{f,out} \quad (1a)$$

and

$$\frac{\partial M}{\partial t} = \dot{M}_{in} - \dot{M}_{out} \quad (1b)$$

In (1) and (2), the over-dots signify integral fluxes crossing the upstream and downstream faces of each control volume, as indicated by the subscripts *in* and *out*, respectively. Note that according to assumptions three and five above, pressure and viscous forces have been neglected in the momentum equation of (1b).

The integral fuel mass flux in (1a) and the integral momentum flux in (1b) are

$$\dot{m}_f = \rho_f \int \bar{X}_f \bar{u} dA \quad (2a)$$

$$\dot{M} = \int \bar{\rho} (\bar{u})^2 dA \quad (2b)$$

In (2a) and (2b), ρ_f is the density of liquid fuel, and $\bar{u}$, $\bar{\rho}$, and $\bar{X}_f$ are the turbulent mean velocity, density, and fuel volume fraction, respectively. As shown in Fig. 3, the jet cross sectional area $A = \pi(\tan(\theta/2)z')^2$ is a function of the distance z' from the virtual jet origin, as described in [1]. In both (2a) and (2b), only convection terms are included – turbulent or molecular diffusion in the axial direction is neglected, according to assumption four above.

¹ To maintain a constant velocity profile, radial mixing must be relatively rapid, which is somewhat at odds with the fourth assumption above that axial mixing is negligible. However, in the fully developed jet, the radial mean velocity gradient exceeds the axial mean velocity gradient over most of the jet cross section. Therefore, the driving gradient for radial mixing is greater than for axial mixing in much of the jet, though axial mixing may still be significant.

In real jets, both $\bar{X}_f$ and $\bar{u}$ are non-uniform over the jet cross-section. The integral in (2a) may be evaluated using assumed profiles for $\bar{X}_f$ and $\bar{u}$ proposed by Abramovich for both gas jets and for two-phase jets [44]:

$$\frac{\bar{X}_f}{\bar{X}_{f,c}} = \left(1 - \xi^\alpha\right)^2 \quad (3a)$$

$$\frac{\bar{u}}{\bar{u}_c} = \left(1 - \xi^\alpha\right)^2 \quad (3b)$$

In (3a) and (3b), the subscript c denotes the values on the jet centerline, and $\xi = r/R$ is the ratio of the radial coordinate r to the jet width $R = \tan(\theta/2)z'$, as illustrated in Fig. 3. The exponent α in (3a) and (3b) is an adjustable parameter that allows the profiles to evolve from a uniform “top-hat” shape ($\alpha = \infty$) at the nozzle to a fully developed shape ($\alpha = 1.5$) at some distance downstream. The fully developed profiles are similar to a Gaussian error function, which is also similar to the velocity profile shape in fully developed diesel jets [32]. As described in Appendix B, the value of α in the transition region (between the nozzle and the fully developed jet) is selected so that the centerline values in (3a) and (3b) do not exceed the values at the nozzle exit for a steady jet.

Using $dA = 2\pi R^2 \xi \cdot d\xi$, the centerline quantities can be related to the cross-sectionally averaged fuel volume fraction $\bar{\bar{X}}_f$ and velocity $\bar{\bar{u}}$ by

$$\begin{aligned} \bar{\bar{X}}_f &= \frac{\int \bar{X}_f dA}{\int dA} = 2X_{f,c} \int_0^1 \left(\xi - 2\xi^{\alpha+1} + \xi^{2\alpha+1}\right) d\xi \\ &= \frac{\alpha^2}{(\alpha+1)(\alpha+2)} \bar{X}_{f,c} \end{aligned} \quad (4a)$$

$$\bar{\bar{u}} = \frac{\int \bar{u} dA}{\int dA} = \frac{\alpha^2}{(\alpha+1)(\alpha+2)} \bar{u}_c \quad (4b)$$

In (4a) and (4b), the double overbars signify averages over the jet cross-section, as apposed to single overbars, which signify the turbulent mean quantities. With (4a) and (4b), integration of (2a) gives

$$\dot{m}_f = \rho_f \beta \bar{\bar{X}}_f \bar{\bar{u}} A \quad (5a)$$

where

$$\beta = \frac{6(\alpha+1)(\alpha+2)}{(3\alpha+2)(2\alpha+1)} \quad (5c)$$

The factor β accounts for the shape of the fuel volume fraction and velocity profiles, ranging from $\beta = 1$ for a uniform profile (with $\alpha = \infty$) to approximately $\beta = 2$ for a fully developed jet (with $\alpha = 1.5$).

For the momentum relation of (2b), the turbulent mean density $\bar{\rho}$ can vary over the jet cross-section – $\bar{X}_f$ is greater at the jet centerline than at the jet boundaries, so that $\bar{\rho}$ is also higher at the centerline. The density variation over the jet cross-section is significant near the nozzle, but farther downstream where $\bar{X}_f \ll 1$, the effects of density variation on the momentum flux are small. Furthermore, ignoring the density variation across the jet greatly simplifies the mathematics. Here, the density in (2b) is treated as constant at its cross-sectionally averaged value $\bar{\bar{\rho}}$, so that it may be removed from the integral and (2b) can be evaluated as

$$\dot{M} = \bar{\bar{\rho}} \beta (\bar{\bar{u}})^2 A \quad (5b)$$

In (5b), the cross-sectionally averaged density $\bar{\bar{\rho}}$ is a function of the liquid fuel density ρ_f and the ambient gas density ρ_a :

$$\bar{\bar{\rho}} = \rho_f \bar{\bar{X}}_f + \rho_a (1 - \bar{\bar{X}}_f) \quad (6)$$

In the numerical solution, two quantities are tracked in each control volume: the mass of fuel m_f in (1a) and the total momentum M in (1b). The values of $\bar{\bar{X}}_f$ and $\bar{\bar{u}}$ can be determined from m_f and M in each control volume as follows. $\bar{\bar{X}}_f$ is the ratio of the volume of fuel to the volume $A\Delta z$ of each discretized control volume, where Δz is the axial width of the control volume

$$\bar{\bar{X}}_f = \frac{m_f / \rho_f}{A\Delta z} \quad (7a)$$

The cross-sectionally averaged axial velocity $\bar{\bar{u}}$ is the ratio of momentum to mass

$$\bar{\bar{u}} = \frac{M}{\bar{\bar{\rho}} A \Delta z} \quad (7b)$$

With the above development, (1a) and (1b) may be discretized. Using an upwind differencing scheme, the discretized equations for fuel mass and momentum are

$$m_{f,i}^{t+1} = m_{f,i}^t + \rho_f \left[\left(\bar{\bar{X}}_f \bar{\bar{u}} A \right)_{i-1}^t - \left(\bar{\bar{X}}_f \bar{\bar{u}} A \right)_i^t \right] \Delta t \quad (8a)$$

$$M_i^{t+1} = M_i^t + \left[\left(\bar{\rho} \beta (\bar{u})^2 A \right)_{i-1}^t - \left(\bar{\rho} \beta (\bar{u})^2 A \right)_i^t \right] \Delta t \quad (8b)$$

In (8a) and (8b), the superscripts indicate the timestep count (t) and the subscripts indicate the control volume index (i), which increases downstream in the jet as shown in Fig. 3. The values for β and A are evaluated at the axial midpoint of each control volume, and Δt is the time-step size.

The above approach is similar to that developed by Pastor et al. [31]. In their development, they used a Gaussian error function to describe the velocity profile, but the development of the 1-D model is otherwise very similar. The emphasis of Pastor et al. [31] was on the initial jet penetration and quasi-steady behavior, while here, the emphasis is on the jet behavior after the end of injection.

RESULTS

A square-wave injection pulse with duration of 0.5 ms is simulated using the 1-d transient model. The simulation conditions are listed in the inset in the top left corner of Fig. 4. The ambient gas conditions are similar to the LTC condition in Fig. 2, with EGR-diluted ambient gases at 900 K with a density of 19.3 kg/m³ and 12.7% O₂. The orifice discharge and area-contraction coefficients (C_d and C_a , respectively) and spreading angle (θ) were not measured, but are assumed using typical values from [1]. The spatial resolution in the axial direction is 0.1 mm, and the domain is 100 mm long. The predictions of several variables in the model are described in the following sections.

VELOCITY AND EQUIVALENCE RATIO

The cross-sectionally averaged velocity and equivalence ratio predictions from the transient jet model are shown in Fig. 4. The top-left plot (Fig. 4a) shows the cross-sectionally averaged axial velocity $\bar{u}$ in the jet at various times in the simulation up to EOI (at 0.5 ms), as indicated by the annotations on the plot. The bottom-left plot (Fig. 4c) shows the corresponding predictions of the cross-sectionally averaged equivalence ratio $\bar{\phi}$, defined as

$$\bar{\phi} = \frac{\bar{X}_f \rho_f}{(1 - \bar{X}_f) \rho_a} (m_a/m_f)_{Stoich} \quad (9)$$

In (9), $(m_a/m_f)_{Stoich}$ is the stoichiometric ambient gas-to-fuel ratio, by mass. Note that according to (4a) and

(4b), $\bar{u}$ and $\bar{\phi}$ are typically much smaller than the centerline values. For example, in the fully developed region of the jet where $\alpha = 1.5$, (4b) gives $\bar{u} = \bar{u}_c / 3.9$. Also, observe that the ordinate scales for both plots are nonlinear so that the predictions at small values are more clear.

At each instant of the simulation shown in Fig. 4, both $\bar{u}$ and $\bar{\phi}$ vary approximately inversely with axial distance z , as is typical of steady jets. For reference, the steady jet solutions, as derived in Appendix A, are included in Fig. 4 in gray curves. The sharp rise in $\bar{u}$ and $\bar{\phi}$ at each instant in the simulation (vertical line in each curve of Fig. 4a and Fig. 4c) indicates the head of the jet, whose penetration rate in this numerical solution agrees well with the analytical solution (gray curves) of Naber and Siebers [1]. Therefore, up to EOI, the numerical solution agrees with the analytical solution for steady jets, as it should.

The predictions of $\bar{u}$ and $\bar{\phi}$ after EOI are shown in the top-right (Fig. 4b) and bottom-right (Fig. 4d) plots, respectively. Again, the predictions at various instants in the simulation are indicated by the annotations on the plots, and the gray line represents the steady-jet behavior. Three aspects of the predictions after EOI are noteworthy.

First, the model predictions show that after EOI, both $\bar{u}$ and $\bar{\phi}$ decrease very rapidly to zero near the injector. As the simulation proceeds in time, the values of $\bar{u}$ and $\bar{\phi}$ remain anchored at zero near the injector. Farther downstream, the $\bar{u}$ and $\bar{\phi}$ increase nearly linearly to the downstream steady-jet levels. (The linear dependence is not immediately evident in the plots because of the nonlinear ordinate scale).

Second, the departure from steady-jet behavior moves downstream at roughly twice the rate of the initial jet penetration. In the early part of the simulation after EOI, the $\bar{u}$ and $\bar{\phi}$ curves join the steady-jet behavior at some distance downstream. The downstream distance where $\bar{u}$ and $\bar{\phi}$ join the steady jet behavior moves at twice the initial jet penetration rate in the top-left and bottom-left plots. For instance, at EOI (0.5 ms), the head of the jet in Fig. 4a is approximately 45 mm downstream. In Fig. 4b, the departure from steady jet behavior has reached approximately the same location by 0.75 ms in the simulation, which is only 0.25 ms after EOI. Therefore, the EOI disturbance (departure of $\bar{u}$ and $\bar{\phi}$ from steady-jet behavior) reaches a given downstream position in roughly half the time required for the initial penetration of the head of the jet. As a result,