

Reconstruction of Time-Resolved Vehicle Emissions Measurements by Deconvolution

James D. Pakko
Ford Motor Company

Copyright © 2009 SAE International

ABSTRACT

A thorough understanding of vehicle exhaust aftertreatment system performance requires time-resolved emissions measurements that accurately follow driving transients, and that are correctly time-aligned with exhaust temperature and flow measurements. The transient response of conventional gas analyzers is characterized by both a time delay and an attenuation of high-frequency signal components. The distortion that this imposes on transient emissions measurements causes significant errors in instantaneous calculations of aftertreatment system efficiency, and thus in modal mass analysis. This creates difficulties in mathematical modeling of emissions system performance and in optimization of powertrain control strategies, leading to suboptimal aftertreatment system designs.

A mathematical method is presented which improves the response time of emissions measurements. This begins with development of a model of gas transport and mixing within the sampling and measurement system. Deconvolution techniques are described which solve the inverse of the model, i.e. the input to the system is estimated when the output is known. In this way, an improved signal is reconstructed by removing the effects of the transient response of the measurement system. Model parameters are estimated by exciting the system with a series of square pulses and fitting the model response to the resulting data. Finally, reconstruction is applied to transient vehicle emissions data, showing significant recovery of detail.

INTRODUCTION

Designers of modern automotive powertrain systems must satisfy an increasing number of regulatory, market and

business requirements. Governments around the world continue to enact regulations reducing allowed exhaust emissions. Customers want increasing levels of power and performance. Customer demand and government regulation both exert pressure to reduce fuel consumption. An increasingly competitive business environment demands improved quality, at reduced cost. Manufacturers have responded with added reliance on computer-aided engineering (CAE) tools to improve design optimization while minimizing requirements for costly prototype testing. They have also developed increasingly sophisticated control systems, offering unprecedented freedom to control almost every aspect of powertrain operation.

After the cold-start period, most vehicle exhaust emissions result from transient events in which the control system is momentarily unable to maintain the optimum value of some important parameter, such as air/fuel ratio. Understanding the conditions that lead to such events, calibration of the control system to prevent their occurrence, and the ability of CAE tools to predict their effects requires detailed time-resolved measurements of mass emissions. Unfortunately, conventional gas analyzers are not designed for fast transient response, and they perform poorly in this role. Fast-response analyzers exist,¹ but they are not currently in widespread routine use for vehicle testing.

The constant volume sampling (CVS) system is the standard for emission regulation compliance certification. As a vehicle is driven over a specified driving speed cycle on a chassis dynamometer, the CVS system dilutes the tailpipe exhaust with ambient air, and a sample of this diluted exhaust is continuously drawn into a bag. At the end of the test, the contents of the bag are analyzed for concentrations of pollutants. This is the task for which conventional gas

analyzers are designed, and while it requires a high degree of accuracy, precision and sensitivity, it does not require fast transient response.

While CVS testing measures the total masses of emissions integrated over the entire drive cycle, modal mass analysis uses a time-resolved record of emissions mass flowrates throughout the test. This can then be integrated over segments of the test to determine the emissions contributions of specific events. Instead of being drawn into a bag, a gas sample is drawn directly into the gas analyzer to produce instantaneous measurements of emissions concentrations. The concentration is multiplied by the exhaust mass flow rate to obtain the emission mass flow rate. However, gas concentration measurements are delayed by the need to transport the gas sample from the vehicle to the analyzer, and the slow transient response typical of gas analyzers distorts the concentration signal. Thus, the concentration measurement is no longer synchronized with either the flow measurement or the relevant powertrain inputs. This creates errors in the emission mass flow rate calculation, and makes it difficult to correlate powertrain events to emissions production. The engine-out emissions mass flow rates are used as inputs to CAE tools that predict tailpipe emissions, given various aftertreatment hardware configurations. These models cannot produce correct outputs if the inputs are erroneous, or are not synchronized with other relevant inputs, such as exhaust temperature, flowrate, or air/fuel ratio. Therefore, a way to improve the transient response of emissions concentration measurements would be valuable. (While this work considers only emissions concentrations, other measurements can also be significantly affected by slow instrument transient response, e.g. exhaust temperature measurements as described by Son and Kolasa.²)

Assuming the emissions sampling and measurement system behaves as a linear time-invariant system as represented in Figure 1, the system responds to an input $x(t)$ with the output $y(t)$ according to the convolution³

$$y(t) = \int_{-\infty}^{\infty} x(u) g(t-u) du$$

or, using more concise notation,

$$y(t) = x(t) * g(t) \quad (1)$$

where “*” denotes the convolution operation. In this example, $x(t)$ is the system input, or the concentration in the exhaust pipe, $y(t)$ is the output, or the concentration measured by the analyzer, and $g(t)$ is the impulse response of the sampling and measurement system. Convolution can be computed using Fourier analysis. According to the

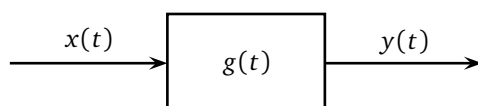


Figure 1: A linear time-invariant system.

convolution theorem:⁴

$$\mathcal{F}(x(t) * g(t)) = (\mathcal{F}x(t)) (\mathcal{F}g(t))$$

where $\mathcal{F}$ is the Fourier transform operator. In other words, the Fourier transform of the convolution of functions is the product of the Fourier transforms of the functions. The utility of this relationship becomes apparent if Equation 1 is rewritten in the frequency domain:

$$\hat{y}(s) = \hat{x}(s) \hat{g}(s) \quad (2)$$

where s is the cyclical frequency (in hertz), and the hats ($\hat{\cdot}$) denote the Fourier transform (or spectrum) of a function, $\hat{x}(s) = \mathcal{F}x(t)$. The spectrum of the impulse response function, $\hat{g}(s)$, is called the frequency response function. From Equation 2, if the output $y(t)$ of the system is known, the input $x(t)$ can be found using the inverse Fourier transform:

$$x(t) = \mathcal{F}^{-1} \left(\frac{\hat{y}(s)}{\hat{g}(s)} \right) \quad (3)$$

Just as multiplication in the frequency domain corresponds to convolution in the time domain, division in the frequency domain corresponds to deconvolution in the time domain. This allows us to remove the effects of the measurement system to find the “true” concentration signal by dividing the spectrum of the measured signal by the system frequency response and taking the inverse Fourier transform of the quotient. First, a model of the gas sampling system will be formulated, in order to determine its frequency response function.

SAMPLING SYSTEM MODEL

INTRODUCTION - When emissions from a vehicle are measured, an exhaust gas sample is transported through a tube from the vehicle to the gas analyzers. This tube is typically about 6mm in diameter, and can be over ten meters long, requiring several seconds for the gas to reach the analyzers. If the gas was transported by perfect plug flow, then the only effect on a measured concentration would be a delay between the time the gas was sampled in the exhaust pipe and when it reached the analyzer. However, while the gas is in transit, it is subject to a diffusion process,^{1,6} resulting in dispersion of the gas. Sharp concentration gradients become less sharp, and narrow peaks are widened. In addition, the system usually includes one or more volumes such as filter housings or condensate traps where turbulence promotes mixing, further distorting the measured signal. Diffusion and mixing processes will first be considered separately, and then combined to form an overall frequency response function for the system.

SAMPLE LINE TRANSPORT - To analyze transport through the sample line, imagine incompressible inviscid flow through a tube of infinite length, $-\infty < z < \infty$. The flowing gas is observed as a function of time at positions $z = 0$ and $z = L$, where L is the length of the sample line. The governing equation is the convection-diffusion equation:

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial z^2} + v \frac{\partial c}{\partial z} \quad (4)$$

where c is the gas concentration, D is the diffusivity, v is the flow velocity, and z and t are the space and time variables, respectively. Equation 4 is usually solved given an initial condition $c(z, 0)$, but since concentration is observed as a function of time at only two values of z ($z = 0$ and L), it would be more convenient to provide an input $c(0, t)$. Some manipulation of this equation gives an alternate form more amenable to such a solution. For $v > 0$, $v = \partial z / \partial t$, so the substitutions $\partial t = (1/v) \partial z$, $\partial z^2 = v^2 \partial t^2$, and $\partial z = v \partial t$ can be made in Equation 4. Then, dividing both sides of the equation by v gives:

$$\frac{\partial c}{\partial z} = \frac{D}{v^3} \frac{\partial^2 c}{\partial t^2} - \frac{1}{v} \frac{\partial c}{\partial t}$$

Taking the Fourier transform⁴ with respect to t of both sides gives:

$$\frac{\partial}{\partial z} \hat{c}(z, s) = \hat{c}(z, s) \left(-4\pi^2 s^2 D / v^3 - i 2\pi s / v \right)$$

where $i = \sqrt{-1}$. This is an ordinary differential equation in z , and the solution is:

$$\hat{c}(z, s) = \hat{c}(0, s) e^{-4\pi^2 s^2 D z / v^3} e^{-i 2\pi s z / v}$$

Since the impulse response is the output of the system when excited by a unit impulse, the initial condition $c(0, t)$ is set as the impulse function, or the Dirac delta function, $\delta(t)$. Then the impulse response $g_L(t)$ is the measured concentration at $z = L$, $g_L(t) = c(L, t)$ and the frequency response is its Fourier transform, $\hat{g}_L(s) = \hat{c}(L, s)$. The Fourier transform of $\delta(t)$ is one,⁵ thus $\hat{c}(0, s) = 1$. Only the concentration at the end of the sample tube is of interest, so the substitution $z = L$ is now made. The sample time delay is defined as $t_d \equiv L/v$, and also $\sigma \equiv \sqrt{2DL/v^3}$. Making these substitutions, the sample line frequency response is:

$$\hat{g}_L(s) = e^{-2\pi^2 s^2 \sigma^2} e^{-i 2\pi s t_d} \quad (5)$$

The impulse response is the inverse Fourier transform of the frequency response, hence:

$$g_L(t) = \frac{1}{\sigma \sqrt{2\pi}} e^{-(t-t_d)^2 / 2\sigma^2} \quad (6)$$

Now the impulse response for the sample tube can be recognized as the Gaussian function with its central ordinate at t_d and standard deviation σ . This accounts for the observations of McClure,⁷ who empirically inferred the contribution of the Gaussian function to the sampling system response.

MIXED VOLUME - In addition to the sample line, the system may include one or more volumes where mixing occurs. The outlet concentration of an instantaneously and perfectly mixed volume is determined by continuity:

$$\frac{d}{dt}(Vc) = q_{in}c_{in}(t) - q_{out}c_{out}(t) \quad (7)$$

where V is the mixing chamber volume, q is the volume flow rate and c is the concentration. The inlet and outlet values

are denoted by the subscripts, while the unsubscripted symbols represent values within the mixing chamber. The outlet concentration is the same as the concentration within the mixing chamber, $c_{out} = c$. If the mixing chamber volume is constant and there is no net storage, $q_{in} = q_{out}$, Equation 7 becomes:

$$\frac{dc}{dt} = \frac{q}{V} (c_{in}(t) - c)$$

Setting the input as the impulse function, $c_{in}(t) = \delta(t)$ and taking the Fourier transform of both sides:

$$i 2\pi s \hat{c}(s) = \frac{q}{V} (1 - \hat{c}(s))$$

Since the input is the impulse function, the frequency response is $\hat{g}_V(s) = \hat{c}(s)$. Defining $\tau \equiv V/q$ and rearranging, the frequency response function is:

$$\hat{g}_V(s) = \left(\frac{1}{1 + i 2\pi s \tau} \right) \quad (8)$$

The impulse response is the inverse Fourier transform of the frequency response:

$$g_V(t) = \Phi(t) \frac{1}{\tau} e^{-t/\tau} \quad (9)$$

where $\Phi(t)$ is the Heaviside step function. The impulse response of the mixed volume is a one-sided exponential with time constant τ .

COMBINED SYSTEM FREQUENCY RESPONSE - The frequency response function for the complete system will be assembled by combining the frequency response functions for the sample line and the mixed volume. A diagram of the complete system model is shown in Figure 2. This includes a long sample tube with a characteristic sample delay and dispersion effect as previously described. The magnitude of the diffusion is characterized by σ , the standard deviation of the Gaussian function which describes the impulse response of the sample tube as shown in Equation 6. The gas then enters a mixing volume which, in contrast to the perfect mixing volume described above, incorporates two characteristic time constants. The "fast" time constant, τ_f , characterizes the bulk of the flow which is mixed and passes quickly through the volume. The "slow" time constant, τ_s , represents that portion of the flow which is trapped for a period of time in recirculation zones, or boundary layers, or is temporarily adsorbed on surfaces. In fact, one could easily introduce any number of time constants into the model to represent each one of these, and other processes.

This imperfect mixing is modeled as parallel perfectly mixed volumes, as shown in Figure 3. A fraction r of the gas passes through a volume with a "slow" characteristic time constant (e.g. a recirculation zone), while the remainder is subjected to the "fast" time constant. Since q is the total volume flowrate, the rate of exchange into and out of the recirculation zone is qr , while the remainder, $q(1-r)$, comprises the bulk flow.

From Figure 3 it can be seen that the output of the imperfect mixing model is an average of the output streams of two

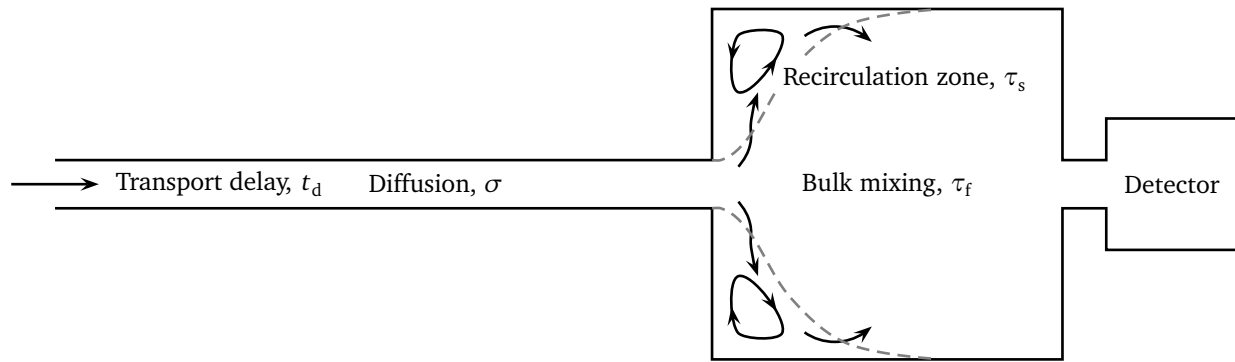


Figure 2: The sampling system model.

perfectly mixed volumes, weighted by r . If the two perfectly mixed volumes are linear time-invariant systems with impulse response functions $g_f(t)$ and $g_s(t)$, and the input to both systems is $x_L(t)$, then:

$$y(t) = r(x_L(t) * g_s(t)) + (1-r)(x_L(t) * g_f(t))$$

The input to the imperfect mixing model is the output of the sample line, $x_L(t) = x(t) * g_L(t)$, so by substitution:

$$y(t) = r((x(t) * g_L(t)) * g_s(t)) + (1-r)((x(t) * g_L(t)) * g_f(t))$$

And, since convolution is both associative and distributive over addition,⁴ this can be simplified to:

$$y(t) = x(t) * (g_L(t) * (r g_s(t) + (1-r) g_f(t)))$$

Using this result, and substituting the results from Equations 6 and 9, the total impulse response of the sampling system model is:

$$g(t) = \left(\frac{1}{\sigma \sqrt{2\pi}} e^{-(t-t_d)^2/2\sigma^2} \right) * \left[\Phi(t) \left(\frac{r}{\tau_s} e^{-t/\tau_s} + \frac{1-r}{\tau_f} e^{-t/\tau_f} \right) \right] \quad (10)$$

The sampling system frequency response function is the Fourier transform of the impulse response. Using the results shown in Equations 5 and 8, the frequency response is:

$$\hat{g}(s) = e^{-2\pi^2 s^2 \sigma^2} e^{-i2\pi s t_d} \left(\frac{r}{1 + i2\pi s \tau_s} + \frac{1-r}{1 + i2\pi s \tau_f} \right) \quad (11)$$

INVERSE MODEL SOLUTION

INSTABILITY AND REGULARIZATION - Referring to Equation 3 to solve the inverse problem of obtaining the input to a system using knowledge of its output, the difficulties to this approach may not be immediately apparent. Indeed,

with perfect knowledge of both $y(t)$ and $g(t)$, $x(t)$ can be determined exactly. However, experimental data invariably includes some degree of error, and the system model is acknowledged to be a simplification of the true physics. Moreover, the model includes a number of parameters which must be estimated.

Relating $y(t)$ and $g(t)$ to their errors:

$$y(t) = y_{\text{true}}(t) + \varepsilon_y(t)$$

$$g(t) = g_{\text{true}}(t) + \varepsilon_g(t)$$

where $y_{\text{true}}(t)$ and $g_{\text{true}}(t)$ are the true, or exact values of $y(t)$ and $g(t)$, and $\varepsilon_y(t)$ and $\varepsilon_g(t)$ are their respective errors. Rewriting Equations 2 and 3 including these error terms:

$$\hat{y}_{\text{true}}(s) = \hat{x}(s) \hat{g}_{\text{true}}(s) \quad (12)$$

and,

$$x(t) = \mathcal{F}^{-1} \left(\frac{\hat{y}_{\text{true}}(s) + \hat{\varepsilon}_y(s)}{\hat{g}_{\text{true}}(s) + \hat{\varepsilon}_g(s)} \right) \quad (13)$$

From Equation 12, it's clear that as values of $\hat{g}_{\text{true}}(s)$ approach zero, the corresponding values of $\hat{y}_{\text{true}}(s)$ must also approach zero. Under this condition, the solution of Equation 13 becomes dominated by the error terms such that the resulting $x(t)$ bears little resemblance to the true

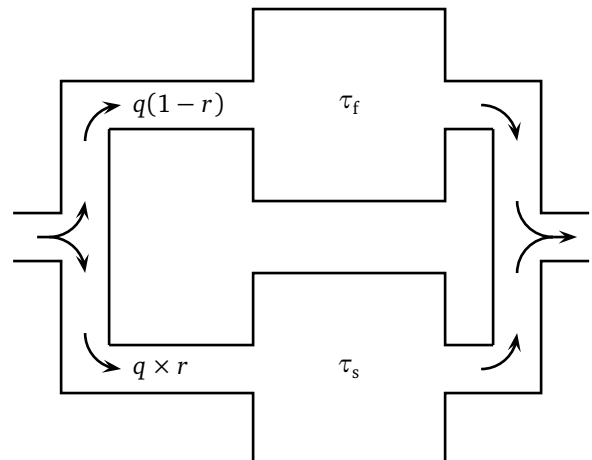


Figure 3: The mixing model, incorporating parallel mixing volumes.