

Acoustic Analysis of Exhaust Muffler of a 4-Stroke Engine

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ABSTRACT

A one-dimensional analysis was performed to analyze a three-pass muffler with perforated tubes for Transmission Loss, using numerical decoupling approach. Effect of mean flow on transmission loss inside the muffler was studied. To account for the three-dimensional nature of acoustic waves at higher frequencies, a three dimensional finite element analysis was done using SYSNOISE. The Transmission loss results of the three-dimensional analysis were compared with those of one-dimensional analysis for no flow case and shown to agree reasonably for lower frequency range.

INTRODUCTION

Exhaust noise of internal combustion engines is considered as one of the biggest pollutant of the present day urban environment. Fortunately, this noise can be reduced sufficiently by means of a well-designed muffler. For a 4-stroke engine, exhaust pulses appear with frequency equal to one-half the number of revolutions of the crankshaft, also called as the firing frequency. The exhaust noise appears at this fundamental frequency and its integral multiples.

One-dimensional acoustic wave propagation in the perforated pipes and cavities of straight through and cross flow silencer elements is in the form of coupled differential equation. Thus to obtain solution, the equations are to be decoupled. Jayaraman and Yam [1] developed a decoupling method to obtain a closed form analytical solution based on the assumption of equal mean flow in the two ducts. Munjal and Rao [2] developed a decoupling method by extracting the roots

of the characteristic polynomial of the system numerically. Peat [3] utilized eigen values and eigenvectors to decouple the coupled equations thereby eliminated the numerical instability caused by other methods. As the majority of mufflers used in the automobiles are three pass in nature, such configurations have been of more recent interest. Ross [4] attempted the analysis of a three-pass muffler using the finite element method. Selamet and Easwaran [5] presented a quasi-one-dimensional approach to analyze a three-pass muffler with perforated elements using numerical decoupling approach.

In this study, a quasi one-dimensional theoretical model, based on transfer matrix and numerical decoupling, is described for the analysis of a three pass perforated duct muffler. The results of transmission loss from one-dimensional analysis are compared with the results from three-dimensional finite element analysis, calculated using SYSNOISE.

The present study involves the analysis of muffler of a typical four cylinder four stroke gasoline engine.

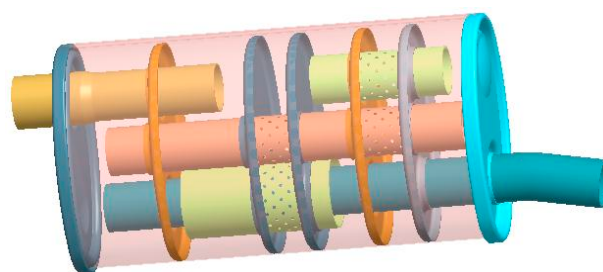


Figure 1: Three dimensional view of Muffler

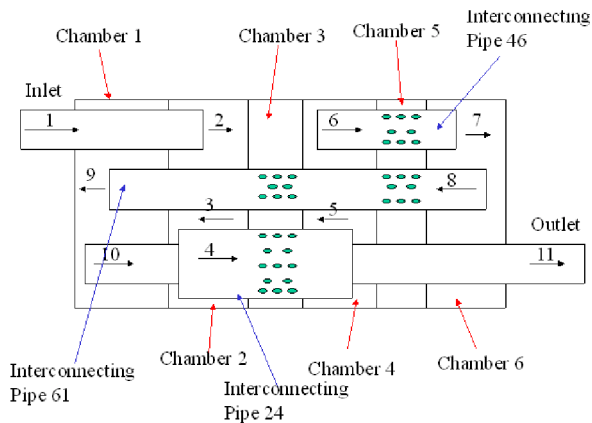


Figure 2: Cross sectional view of Muffler

Figure 1 shows the three-dimensional model of the muffler. A sectional view of the muffler is shown in Figure 2. It is a three-pass muffler consisting of five pipes (one inlet, one outlet and three interconnecting pipes). Each interconnecting pipe is perforated and connects two chambers. Chamber 3 and chamber 5 form two “three ducts reverse flow expansion chambers”.

Arrowheads with numbers show the path follows by the exhaust gas inside the muffler. Flow starts from inlet and expands in chamber 2. It then enters into chamber 4 through interconnecting pipe 24. Through interconnecting pipe 46, it enters into chamber 6. Flow then reverses in direction and enters into chamber 1 through interconnecting pipe 61. It again reverses and leads to exit through outlet pipe. The cross section of the muffler is elliptical. Interconnecting pipe 24 is concentric with the outlet pipe. This provides annular passage for flow from chamber 2 to chamber 4.

The muffler can be seen as a combination of various small elements, which when put together gives the same transmission loss as the muffler as a whole. The geometry can be seen as a combination of the following three basic elements:

- Three duct reverse flow expansion chamber
- Sudden area change
- Uniform tube

MATHEMATICAL MODEL

THREE-DUCT REVERSE FLOW EXPANSION CHAMBER

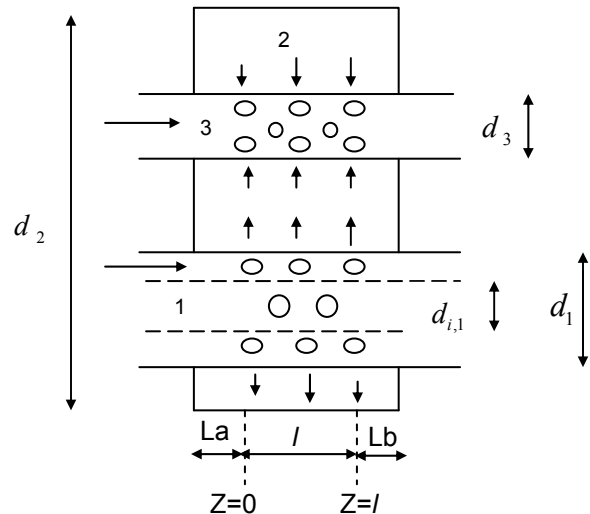


Figure 3: Three duct reverse flow expansion chamber element

Figure 3 shows the case of three interacting ducts. Duct 1 is an annular passage with d_1 and $d_{i,1}$ as outer and inner diameter respectively. Analysis of this arrangement involves writing down the basic governing equations of mass continuity and momentum balance for all the ducts, and then eliminating the velocity variables to obtain the coupled ordinary differential equations in terms of acoustic pressure variables. For the three ducts 1, 2 and 3 as shown above, continuity and momentum equations are

$$\rho_0 \frac{\partial u_1}{\partial z} + U_1 \frac{\partial \rho_1}{\partial z} + \frac{4}{d} \rho_0 u_{1,2} = - \frac{\partial \rho_1}{\partial t} ; d = \frac{d_1^2 - d_{i,1}^2}{d_1} \quad (1)$$

$$\rho_0 \frac{\partial u_1}{\partial t} + \rho_0 U_1 \frac{\partial u_1}{\partial z} = - \frac{\partial p_1}{\partial z} \quad (2)$$

$$\rho_0 \frac{\partial u_2}{\partial z} + U_2 \frac{\partial \rho_2}{\partial z} - \frac{4d_1 \rho_0 u_{1,2}}{d_2^2 - d_1^2 - d_3^2} + \frac{4d_3 \rho_0 u_{2,3}}{d_2^2 - d_1^2 - d_3^2} = - \frac{\partial \rho_2}{\partial t} \quad (3)$$

$$\rho_0 \frac{\partial u_2}{\partial t} + \rho_0 U_2 \frac{\partial u_2}{\partial z} = - \frac{\partial p_2}{\partial z} \quad (4)$$

$$\rho_0 \frac{\partial u_3}{\partial z} + U_3 \frac{\partial \rho_3}{\partial z} - \frac{4}{d_3} \rho_0 u_{2,3} = - \frac{\partial \rho_3}{\partial t} \quad (5)$$

$$\rho_0 \frac{\partial u_3}{\partial t} + \rho_0 U_3 \frac{\partial u_3}{\partial z} = - \frac{\partial p_3}{\partial z} \quad (6)$$

$$\text{Where, } u_{1,2} = \frac{p_1 - p_2}{\rho_0 a_0 \xi_1} \text{ and } u_{2,3} = \frac{p_2 - p_3}{\rho_0 a_0 \xi_2}$$

For an incompressible isentropic flow, relation between acoustic pressure and density perturbation in the three ducts is given by the equations

$$\rho_1 = \frac{p_1}{a_0^2}, \quad \rho_2 = \frac{p_2}{a_0^2} \text{ and } \rho_3 = \frac{p_3}{a_0^2}$$

Assuming time dependence of all variables $u_1, u_2, u_3, p_1, p_2, p_3, u_{1,2}, u_{2,3}$ is of type $e^{j\omega t}$ and eliminating velocity and density perturbations terms from the above equations, it gives

$$D^2 p_1 + \alpha_1 D p_1 + \alpha_2 p_1 + \alpha_3 D p_2 + \alpha_4 p_2 = 0 \quad (7)$$

$$\alpha_5 D p_1 + \alpha_6 p_1 + D^2 p_2 + \alpha_7 D p_2 + \alpha_8 p_2 + \alpha_9 D p_3 + \alpha_{10} p_3 = 0 \quad (8)$$

$$\alpha_{11} D p_2 + \alpha_{12} p_2 + D^2 p_3 + \alpha_{13} D p_3 + \alpha_{14} p_3 = 0 \quad (9)$$

$$\text{Where: } \alpha_1 = \frac{-M_1 \left(\frac{4}{d \xi_1} + 2k_0 j \right)}{(1 - M_1^2)}; \alpha_2 = \frac{-\left(\frac{4k_0 j}{d \xi_1} - k_0^2 \right)}{(1 - M_1^2)};$$

$$\alpha_3 = \frac{4M_1}{d \xi_1 (1 - M_1^2)}; \alpha_4 = \frac{4k_0 j}{d \xi_1 (1 - M_1^2)}$$

$$\alpha_5 = \frac{4M_2 d_1}{(d_2^2 - d_1^2 - d_3^2) \xi_1 (1 - M_2^2)};$$

$$\alpha_6 = \frac{4k_0 d_1 j}{(d_2^2 - d_1^2 - d_3^2) \xi_1 (1 - M_2^2)}$$

$$\alpha_7 = \frac{-M_2 \left(\frac{4d_1}{\xi_1 (d_2^2 - d_1^2 - d_3^2)} + \frac{4d_3}{\xi_2 (d_2^2 - d_1^2 - d_3^2)} + 2k_0 j \right)}{(1 - M_2^2)}$$

$$\alpha_8 = \frac{-\left(\frac{4d_1 k_0 j}{\xi_1 (d_2^2 - d_1^2 - d_3^2)} + \frac{4d_3 k_0 j}{\xi_2 (d_2^2 - d_1^2 - d_3^2)} - k_0^2 \right)}{(1 - M_2^2)}$$

$$\alpha_9 = \frac{4M_2 d_3}{(d_2^2 - d_1^2 - d_3^2) \xi_2 (1 - M_2^2)};$$

$$\alpha_{10} = \frac{4k_0 d_3 j}{(d_2^2 - d_1^2 - d_3^2) \xi_2 (1 - M_2^2)}; \alpha_{11} = \frac{4M_3}{d_3 \xi_2 (1 - M_3^2)};$$

$$\alpha_{12} = \frac{4k_0 j}{d_3 \xi_2 (1 - M_3^2)}$$

$$\alpha_{13} = \frac{-M_3 \left(\frac{4}{d_3 \xi_2} + 2k_0 j \right)}{(1 - M_3^2)}; \alpha_{14} = \frac{-\left(\frac{4k_0 j}{d_3 \xi_2} - k_0^2 \right)}{(1 - M_3^2)}$$

Putting equations 7, 8 and 9 in the matrix form

$$\begin{bmatrix} D^2 + \alpha_1 D + \alpha_2 & \alpha_3 D + \alpha_4 & 0 \\ \alpha_5 D + \alpha_6 & D^2 + \alpha_7 D + \alpha_8 & \alpha_9 D + \alpha_{10} \\ 0 & \alpha_{11} D + \alpha_{12} & D^2 + \alpha_{13} D + \alpha_{14} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (10)$$

Or,

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} -\alpha_1 & -\alpha_3 & 0 & -\alpha_2 & -\alpha_4 & 0 \\ -\alpha_5 & -\alpha_7 & -\alpha_9 & -\alpha_6 & -\alpha_8 & -\alpha_{10} \\ 0 & -\alpha_{11} & -\alpha_{13} & 0 & -\alpha_{12} & -\alpha_{14} \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix} \quad (11)$$

Representing the above matrix in abbreviated form as

$$\{Y'\} = [R]\{Y\}$$

The above equation is in coupled form and to solve it, the equation has to be decoupled. Applying numerical decoupling technique [3] to decouple the equation, it yields

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \rho_0 a_0 u_1 \\ \rho_0 a_0 u_2 \\ \rho_0 a_0 u_3 \end{bmatrix} = \begin{bmatrix} \psi_{41} e^{\lambda_1 z} & \psi_{42} e^{\lambda_2 z} & \psi_{43} e^{\lambda_3 z} & \psi_{44} e^{\lambda_4 z} & \psi_{45} e^{\lambda_5 z} & \psi_{46} e^{\lambda_6 z} \\ \psi_{51} e^{\lambda_1 z} & \psi_{52} e^{\lambda_2 z} & \psi_{53} e^{\lambda_3 z} & \psi_{54} e^{\lambda_4 z} & \psi_{55} e^{\lambda_5 z} & \psi_{56} e^{\lambda_6 z} \\ \psi_{61} e^{\lambda_1 z} & \psi_{62} e^{\lambda_2 z} & \psi_{63} e^{\lambda_3 z} & \psi_{64} e^{\lambda_4 z} & \psi_{65} e^{\lambda_5 z} & \psi_{66} e^{\lambda_6 z} \\ -\psi_{11} e^{\lambda_1 z} & -\psi_{12} e^{\lambda_2 z} & -\psi_{13} e^{\lambda_3 z} & -\psi_{14} e^{\lambda_4 z} & -\psi_{15} e^{\lambda_5 z} & -\psi_{16} e^{\lambda_6 z} \\ jk_0 + M_1 \lambda_1 & jk_0 + M_1 \lambda_2 & jk_0 + M_1 \lambda_3 & jk_0 + M_1 \lambda_4 & jk_0 + M_1 \lambda_5 & jk_0 + M_1 \lambda_6 \\ -\psi_{21} e^{\lambda_1 z} & -\psi_{22} e^{\lambda_2 z} & -\psi_{23} e^{\lambda_3 z} & -\psi_{24} e^{\lambda_4 z} & -\psi_{25} e^{\lambda_5 z} & -\psi_{26} e^{\lambda_6 z} \\ jk_0 + M_2 \lambda_1 & jk_0 + M_2 \lambda_2 & jk_0 + M_2 \lambda_3 & jk_0 + M_2 \lambda_4 & jk_0 + M_2 \lambda_5 & jk_0 + M_2 \lambda_6 \\ -\psi_{31} e^{\lambda_1 z} & -\psi_{32} e^{\lambda_2 z} & -\psi_{33} e^{\lambda_3 z} & -\psi_{34} e^{\lambda_4 z} & -\psi_{35} e^{\lambda_5 z} & -\psi_{36} e^{\lambda_6 z} \\ jk_0 + M_3 \lambda_1 & jk_0 + M_3 \lambda_2 & jk_0 + M_3 \lambda_3 & jk_0 + M_3 \lambda_4 & jk_0 + M_3 \lambda_5 & jk_0 + M_3 \lambda_6 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{bmatrix} \quad (12)$$

Here, $[\psi]$ is a matrix, whose columns is the eigen vectors and λ is a diagonal matrix, composed of eigen values of matrix $[R]$. $[f]$ is constant. Equation 12 can be represented in abbreviated form as

$$\{p\}_z = [\Omega(z)]\{f\} \quad (13)$$

