

Vibration Analysis of Powertrain Mounting System with a Combination of Active and Passive Isolators with Spectrally-varying Properties

Jae-Yeol Park and Rajendra Singh
The Ohio State University

Copyright © 2009 SAE International

ABSTRACT

Most of the prior work on active mounting systems has been conducted in the context of a single degree-of-freedom even though the vehicle powertrain is a six degree-of-freedom isolation system. We seek to overcome this deficiency by proposing a new six degree-of-freedom analytical model of the powertrain system with a combination of active and passive mounts. All stiffness and damping elements contain spectrally-varying properties and we examine powertrain motions when excited by an oscillating torque. Two methods are developed that describe the mount elements via a transfer function (in Laplace domain). New analytical formulations are verified by comparing the frequency responses with numerical results obtained by the direct inversion method (based on Voigt type mount model). Eigensolutions of a spectrally varying mounting system are also predicted by new models. Complex eigenvalue problem formulation with spectrally-varying properties provides a closer match with experimental results than the real eigenvalue formulation with frequency-independent mounts. Given the spectral variance in the mount properties, a simple roll mode decoupling scheme is suggested for the powertrain isolation system. Then, the role of active path is clarified by comparison with no actuator operation. Multi-dimensional motions (especially coupling) are predicted and in particular the effects of active mount parameters such as the orientation angle, location and actuator input are investigated from the motion coupling perspective.

INTRODUCTION

Prior analyses of rigid body isolation systems, including vibration transmissibility, natural frequency placement and motion decoupling studies [1 – 7], assume that the stiffness (k) and viscous damping (c) properties are constant. However, real-life mount elements inherently (and some even by design) exhibit considerable frequency- and amplitude-dependency, express $k = k(\omega, x)$ and $c = c(\omega, x)$ where ω is the angular frequency (rad/s) and x is the amplitude of excitation [8 – 11]. The rigid body mounting systems are typically modeled in terms of a six degree of freedom (6-DOF) inertial body that is supported by tri-axial isolation elements at 3 or 4 locations along with a rigid foundation [1 – 7]. Yu, Naganathan, and Dukkupati [12], He and Singh [13], and Jeong and Singh [14] have suggested that it is necessary to incorporate the spectrally-varying properties in the engine isolation system models.

Several active or smart engine mounting devices [15 – 19] have been designed to reduce noise and vibration, especially under high dynamic torque conditions. Typical design criteria [1 – 7] include decoupling of powertrain motions and motion control. Motion control is achieved through reduction in resonant peaks, natural frequency placement, reduced vibration transmissibility and increased acoustic comfort. Even though the powertrain (rigid body) is a 6 degree-of-freedom (DOF) isolation system, most of analytical work has been conducted in the context of single-degree-of-freedom systems [15 – 19]. In several cases, high damping solutions have been implemented to control resonance(s) at lower frequencies, and then more compliant mounts have been sought at higher frequencies to minimize the force transmitted in the

base; this implies an introduction of frequency-dependent isolators through passive, adaptive or active means. The above-mentioned design may not satisfy the powertrain motion decoupling considerations, and thus the selection of an active mount in the context of a 6-DOF system remains an empirical science. We seek to overcome this deficiency by proposing a new analytical model that will examine a combination of active and passive mounts.

The literature on multi-degree-of-freedom active isolation systems is sparse. For example, Gardonio, Elliott, and Pinnington [20], Kim and Lee [21], and Royston and Singh [22] have limited their analyses to 3-DOF systems (e.g. transverse-axial-pitch motions or pitch-roll-bounce motions). Also, prior researchers have attempted to minimize the forces transmitted into the rigid or compliant foundations without considering the multi-dimensional motion control and coupling issues. In particular, Gardonio, Elliott, and Pinnington [20] and Kim and Lee [21] have suggested that passive and active mount parameters should be properly selected prior to the isolation control problem, especially at the lower frequencies (say up to 50 Hz). This article extends the prior multi-degree-of-freedom active isolators work [20 – 22] by focusing on the eigensolutions, coupling dynamics and motion control issues.

PROBLEM FORMULATION

Figure 1 illustrates a typical 6-DOF rigid body isolation system composed of an inertial body (engine and transmission), a rigid base (chassis), and three or four linear time invariant active and passive mounts such that each isolation element could be arbitrarily placed at any exterior point and oriented in any direction. The dynamic characteristics of isolators are generally represented in terms of the complex-valued cross point dynamic stiffness, $K(j\omega)$, from non-resonant dynamic tests [8] where j is the imaginary unit. Figure 2 shows sample measurements of $k(\omega)$ and $c(\omega)$ for two example cases (rubber and hydraulic engine mounts). We could embed these or similar $k_i(\omega)$ and $c_i(\omega)$ properties in one or more mount elements of Fig. 1. The governing equations of the mounting system in frequency domain (ω) are as follows, where $\mathbf{q}(\omega)$ is the dynamic displacement vector, and $\mathbf{f}(\omega)$ is the external excitation (force/torque) vector

$$[-\omega^2 \mathbf{M} + j\omega \mathbf{C}(\omega) + \mathbf{K}(\omega)]\mathbf{q}(\omega) = \mathbf{f}(\omega) \quad (1)$$

Here, $\mathbf{M}$ is the inertial (mass) matrix, and $\mathbf{K}(\omega)$ and $\mathbf{C}(\omega)$ are the stiffness and viscous damping matrices. For an internal combustion powertrain system, the main excitation, $\mathbf{f}(t)$, comes from the pulsating torque that is generated by multi-cylinder engines and it could be either periodic (under steady state) or transient (under start-up or switching conditions). Frequency responses could be numerically calculated by the direct inversion method (designated in this article as Method I), where

we could simply use different k and c values at each frequency (essentially a look-up table scheme). However, Method I in Eq. (1) cannot directly lead to the analytical modal expansion for response predictions and motion coupling analyses. In order to overcome this limitation, Jeong and Singh [14] had formulated several spectral eigenvalue problems and then suggested modal superposition procedures. However, their method is valid for a special class of problems (where the dynamic stiffness is given in a specific form), and their approximation cannot be extended to a more general spectrally-varying vibrating system.

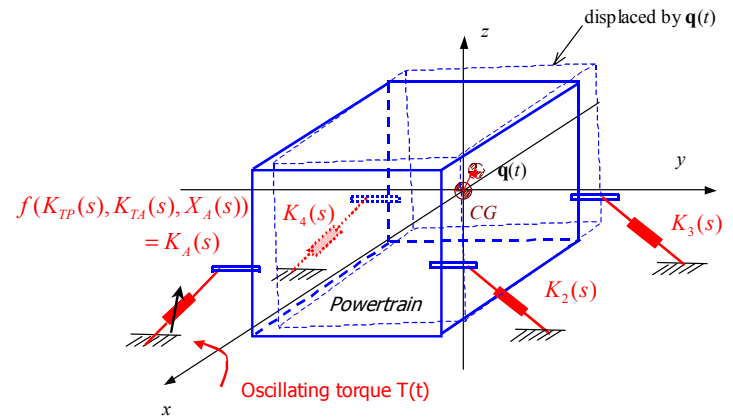


Fig. 1 Multi-degree-of-freedom powertrain isolation system with one active mount and three passive mounts. Each mount is described by tri-axial elements with frequency dependent stiffness $k(\omega)$ and damping $c(\omega)$ properties. Here, $K_i(s)$, $i = 2, 3$, and 4 ($= n$: number of mounts) and $K_A(s)$ are the dynamic stiffness terms of passive and active elements (in a specific direction) respectively.

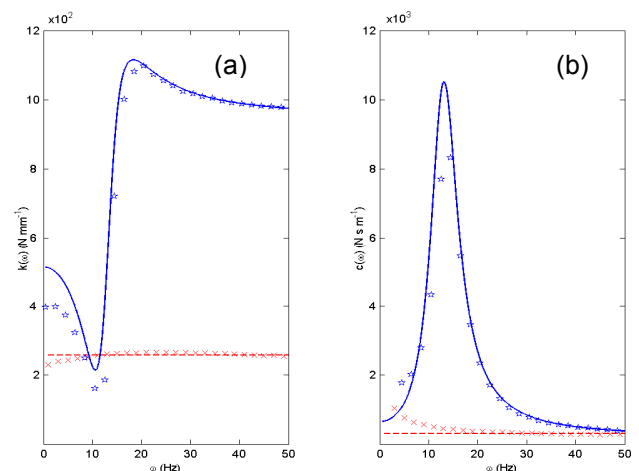


Fig. 2 Mount examples with spectrally-varying stiffness and viscous damping properties. Measured data for rubber and hydraulic mounts are compared, respectively, with transfer function (TF) models given by Eq. (2). (a) Stiffness spectra, $k(\omega) = \text{Re}[K(j\omega)]$; (b) Viscous damping spectra, $c(\omega) = \text{Im}[K(j\omega)]/\omega$. Key:

☆, measured data for a hydraulic mount with an excitation amplitude of $x = 1.5 \text{ mm}$; ✕, measured data for a rubber mount; —, second order TF for the hydraulic mount; —, zeroth order TF for the rubber mount ($k = 280 \text{ N mm}^{-1}$, $c = 300 \text{ N s m}^{-1}$).

The cross-point dynamic stiffness (in Laplace domain s) of a typical engine mount at a certain excitation amplitude, under a specific mean load, could be given by the following expression [8]:

$$K_i(s) = \frac{\alpha_b s^b + \dots + \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0}{\beta_a s^a + \dots + \beta_2 s^2 + \beta_1 s + \beta_0} \quad (2)$$

Here, a and b are the order of denominator and numerator, respectively, and the coefficients, α_k and β_k , are determined by experimental or numerical (analytical) method reflecting the internal fluid structure and rubber material properties. Observe that Eq. (2) matches well with the experimental results of Fig. 2 for hydraulic (with $a = 2$, $b = 3$) and rubber (with $a = 0$, $b = 1$) mounts. Employment of Eq. (2) to the powertrain mounts in Fig. 1 is designated here as Method II. After verification by comparison with experimental and numerical results, this formulation will be used here for vibration and decoupling analysis of powertrain mounting system with combination of active and passive mounts with spectrally-varying properties.

Active and passive mount models that are depicted in Fig. 3. Effective control of powertrain motions is essential over the lower frequency range up to 50 Hz since the rigid body modes significantly dominate and could couple with other vehicle system modes [20, 21]. Actuator displacement in active mounts is adjusted sinusoidally [23, 24] by a displacement actuator, $x_A(t) = X_A e^{j(\omega t + \phi_A)}$ where X_A is active displacement amplitude, ω angular active displacement input frequency, and ϕ_A phase angle with respect to the external torque excitation. Consideration of constant X_A and ϕ_A (at a time) would allow us to make the active mount as analytically tractable in the context of a multi-degree-of-freedom isolation system. Our model will analytically examine the parameters of active and passive mounts (such as stiffness, damping and active input), their locations and orientation angles. Analysis is limited to only the engine torque excitation.

Consider a 6-DOF isolation system consisting of a rigid body (with powertrain mass m , and inertia I_{ij} , i and $j = x, y, z$) under an oscillating torque ($T(t)$) and 4 tri-axial mounts which are assumed to be attached to a rigid base. Out of these, one is an active mount (given by subscript A) with dynamic stiffness $K_A(s) = f[K_{TP}(s), K_{TA}(s), X_A(s), X(s)]$ in a specific direction where s is the Laplace variable, $X(s)$ is the powertrain displacement and $X_A(s)$ is the actuator displacement. The other three are passive devices with dynamic

stiffness $K_i(s)$, $i = 2, 3, 4$ in a specific direction. Each mount element (in any direction) is assumed to have frequency-dependent, stiffness $k(\omega)$ and viscous damping $c(\omega)$ properties.

Governing equations in matrix form are as follows, where (s) implies the Laplace domain, $\mathbf{q}(s) = [x, y, z, \theta_x, \theta_y, \theta_z]^T(s)$ is the displacement vector, and $\mathbf{f}(s)$ is the external force vector (primarily the torque $T(s)$ excitation),

$$[s^2 \mathbf{M} + \mathbf{K}(s)] \mathbf{q}(s) = \mathbf{f}(s), \quad (3)$$

where $\mathbf{M}$ is the mass matrix (powertrain mass and inertia), and $\mathbf{K}(s)$ is the stiffness matrix that includes the transfer function (dynamic stiffness) models of active and passive mounts. Passive mounts are modeled as shown in Fig. 3(a) in terms of the cross point stiffness [8, 13] $K_T(s) = F_T(s) / X(s)$. Here, $K_T(s)$ could be either analytically available from a fluid mount model [8] or experimentally measured by a non-resonance mount test [13, 25]. This type of transfer function model is valid in the lower frequency range (say up to 50 Hz).

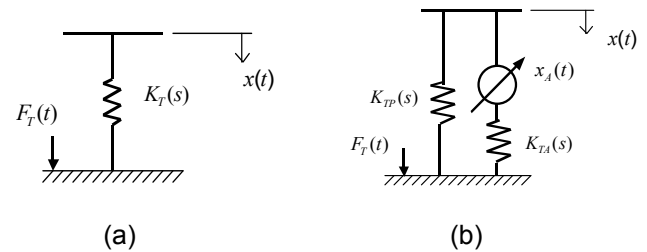


Fig. 3 Passive and active engine mounts given displacement input $x(t)$. (a) Passive mount; (b) Active mount with piston displacement type input ($x_A(t)$). In both cases, force transmitted to the rigid base is $F_T(t)$. Stiffness terms are given in the Laplace domain.

The chief objective of this article is to investigate the complex eigensolutions and frequency responses of the powertrain system of Fig. 1, when excited by an oscillating torque, with one active mount and 3 passive mounts. The active mount model of Fig. 3(b) is first formulated for fluid piston type active device such as a hydraulic engine mount [1, 3 - 5]. Our method will be validated by comparing analytical predictions in frequency domain with the direct inversion (numerical) method where we could simply use different k and c values at each frequency. The second objective is to examine multi-directional motion control and vibration isolation issues in the context of a multi-degree-of-freedom mounting system.

Genesseeux [26] examined for active isolation schemes and concluded that an actuator structure in

$$F_T(s) = F_{TP}(s) + F_{TA}(s). \quad (4)$$

The individual transfer functions, $K_{TP}(s)$ and $K_{TA}(s)$, of the passive (primary) and active (secondary) paths are defined by the following:

$$K_{TP}(s) = \frac{F_{TP}(s)}{X(s)} \quad \text{and} \quad K_{TA}(s) = \frac{F_{TA}(s)}{X_A(s)}. \quad (5a, b)$$

Here, $X(s)$ is the powertrain displacement's principal direction component of the active mount, and $X_A(s)$ is the actuator displacement in Laplace domain. For a single-degree-of-freedom isolation system of Fig. 4, the governing equation is:

$$[ms^2 + K_{TP}(s)]X(s) = F(s) - K_{TA}(s)X_A(s). \quad (6)$$

Here, the active force, $F_{TA}(s) = K_{TA}(s)X_A(s)$, can be viewed as an “additional external” excitation since it acts independent of $X(s)$. Therefore, the eigensolutions and passive dynamics are governed by the passive path ($K_{TP}(s)$). While an adaptive mount is usually designed to have at least two passive transfer functions depending on designated operating conditions [25], only one passive transfer function is assigned to an active mount. Therefore, detailed dynamic analysis of the passive element in an active mounting system is crucial.

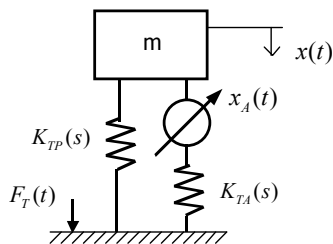


Fig. 4 Single-degree-of-freedom system with an active mount.

Two active mount concepts, where the actuator element is in parallel with the passive rubber element, will be examined. First, consider an active hydraulic device as shown in Fig. 5. The active mount consists of three control volumes (upper and lower chambers are designated by #1 and #2, respectively, and the inertia

track is represented by $\#i$). The momentum equation for rubber mass, m_r , is:

$$m_r \ddot{x}(t) = F(t) - k_r x(t) - c_r \dot{x}(t) - A_r p_1(t). \quad (7)$$

The continuity equations for the lower and upper chambers are:

$$A_r \dot{x}(t) - q_i(t) = C_1 \dot{p}_1(t) + A_A \dot{x}_A(t), \quad (8)$$

$$q_i(t) = C_2 \dot{p}_2(t). \quad (9)$$

The momentum equation for the inertia track is:

$$p_1(t) - p_2(t) = I_i \dot{q}_i(t) + R_i q_i(t). \quad (10)$$

Since $F(t) = F_T(t) (= F_{TP}(t) + F_{TA}(t))$ in the lower frequency range [8], the transfer functions for this active mount are derived from Eqs. (7) - (10) as follows:

$$\begin{cases} K_{TP}(s) = \frac{F_{TP}(s)}{X(s)} = m_r s^2 + c_r s + k_r + \frac{\alpha_2 s^2 + \alpha_1 s + \alpha_0}{\beta_2 s^2 + \beta_1 R_i s + \beta_0} A_r \\ K_{TA}(s) = \frac{F_{TA}(s)}{X_A(s)} = -\frac{\alpha_2 s^2 + \alpha_1 s + \alpha_0}{\beta_2 s^2 + \beta_1 R_i s + \beta_0} A_A \end{cases}, \quad (11a, b)$$

$$\alpha_2 = C_2 A_r I_i, \quad \alpha_1 = C_2 A_r R_i, \quad \alpha_0 = A_r^2, \quad (11\text{c-e})$$

$$\beta_2 = I_i C_1 C_2, \quad \beta_1 = C_1 C_2 R_i, \quad \beta_0 = C_1 + C_2. \quad (11\text{f-h})$$

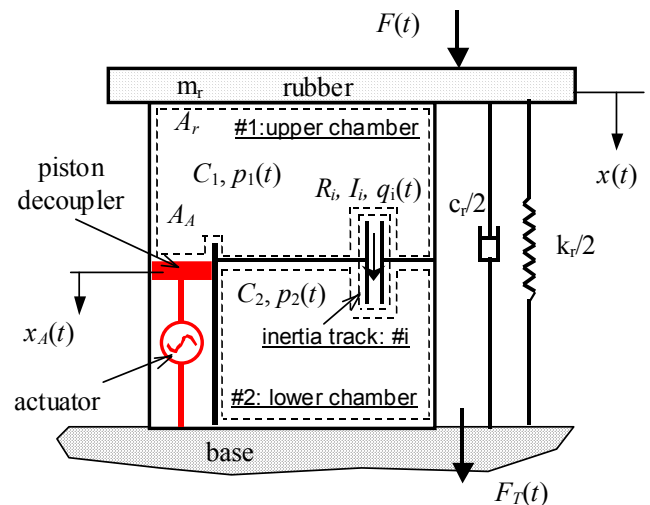


Fig. 5 Schematic of an active engine mount based on hydraulic mount. Here, C_1 and C_2 are the compliances of upper and lower chambers, $p_1(t)$ and $p_2(t)$ are the chamber pressures, R_i , I_i , and $q_i(t)$ are the fluid resistance, inertance, and flow rate in the inertia track, A_r is the equivalent piston area of rubber, A_A is the actuator piston area, and k_r and c_r are the rubber stiffness and viscous damping terms (Voigt model assumed).