

The Acoustic Impedance of a Wide Side Branch Orifice: Experimental Determination Using Three-Port Methodology

Mikael Karlsson

KTH, The Marcus Wallenberg Laboratory for Sound and Vibration Research

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ABSTRACT

The acoustic impedance of a circular, confined, side branch orifice subjected to grazing flow is studied. Two geometries are tested. In both geometries, the side branch dimension is of the same order as that of the main duct. The system is viewed as an acoustic three-port, whose passive properties are described by a system matrix.

The impedance is studied with the acoustic field incident at different ports, which is shown to influence the results significantly. When excited from the leading edge or from the side branch, an interaction of the hydrodynamic and acoustic fields is triggered, while excitation from the trailing edge does not trigger such an interaction.

For both the resistance and the reactance (here expressed as an end correction) the results vary in the three possible excitation cases. In the quasi-stationary limit the resistance is given by a loss coefficient times the Mach number, and the end correction collapses to a single value. A simple empirical model is presented, based on the results.

INTRODUCTION

Side branches subjected to grazing flow are a common design element in a variety of engineering applications. Common examples are perforated liners in jet engines and silencers, junctions in duct networks, and different types of resonators. The acoustic characteristics of the side branch can change significantly with the introduction of grazing mean flow.

Passive acoustic properties are often characterized in terms of acoustic impedance, which is the ratio of a

potential to flow quantity (often the ratio of acoustic pressure to volume velocity in duct acoustics). This is a complex quantity where the real part can be attributed to the exchange of energy between the flow and the acoustic field, while the imaginary part characterizes the apparent mass at the orifice, often expressed in terms of an end correction.

Analytical models for the acoustic impedance of an orifice with grazing flow have been given, for example, by Howe [1] and Jing et al. [2]. However, the agreement with experimental data is generally not satisfactory (see Peat et al. [3]). Hence, numerous experimental studies have been performed [2, 4-10] and a number of empirical models are available [4, 9, 11]. Generally, each model works satisfactorily for a specific test case but has limited application for other configurations (Lee and Ih [9]). We will explore some of the reasons for that in this paper.

The most common measurement technique used in previous experimental investigations is the two-microphone in situ technique (Dean [12]). Although variations exist, it is based on measuring the fluctuating pressure just outside the tested sample and relating it to another pressure signal measured in a well-defined cavity backing the sample. Another common technique is based on impedance-tube-type measurements, either directly, by simply determining the reflection coefficient (see, e.g., Sun et al. [13]), or with a microphone on the other side of the sample (Dickey and Selamet [5]). More recently, inverse methods have been developed [14, 15], whereby a measurement of a sample is compared to a numerical propagation model, and the impedance parameters in the model are fitted to the measured data via an iterative procedure.

Wide side branches, where the dimension of the side branch is of the same order as that of the main duct, are of primary interest here. This relates to applications such as T-junctions in duct systems or the neck of Helmholtz or quarter-wave resonators. Literature on this subject is scarce, with most previous work focused on perforates—that is, orifices that are small compared to the duct to which they are connected.

An alternative measurement methodology, recently described in a paper by Karlsson and Åbom [16], is used in this work. It is based on active three-port methodology—that is, the test object is seen as a lumped element system with three ports. The passive properties of the test object can be described with a matrix (here the scattering matrix formulation is used). The impedance of the orifice, as seen from any port, is easily derived from the matrix. The methodology and the more generic phenomena are described in Ref. 16, while this work discusses the impedance specifically and gives an empirical formula for a wide side branch subjected to grazing mean flow. This work should prove a useful engineering tool for anyone interested in the design of flow duct networks.

EXPERIMENTAL SETUP AND METHODOLOGY

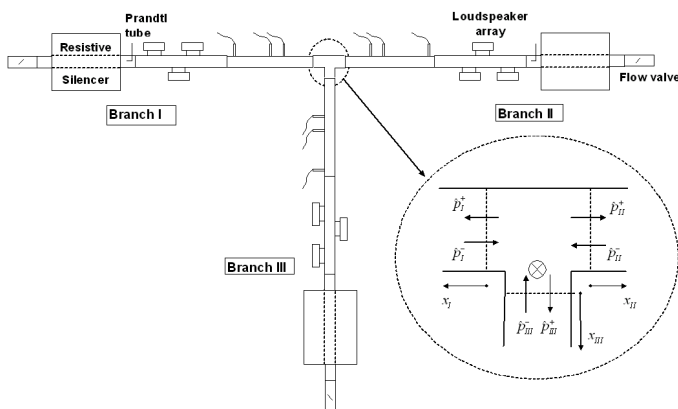


Figure 1. Experimental set up with three-port definition. The inset indicates the positions to which the three-port is reduced.

Any linear time-invariant N-port can be described by relating N input state variables of choice to the corresponding N output variables. The result is an NxN matrix. For the type of problem studied here, a convenient choice for the state variables is the forward and backward travelling acoustic plane wave pressure amplitudes p^+ and p^- . The matrix is then in scattering form, $\mathbf{S}$. This describes the passive part of the system—that is, the reflection and transmission characteristics of the system. With the definition of the coordinate system given in Figure 1, the elements of the scattering matrix $\mathbf{S}$

represent the reflection and transmission coefficients of the system, R and T , respectively, as:

$$\begin{bmatrix} \hat{p}_I^+ \\ \hat{p}_{II}^+ \\ \hat{p}_{III}^+ \end{bmatrix} = \underbrace{\begin{bmatrix} R_I & T_{II,I} & T_{III,I} \\ T_{I,II} & R_{II} & T_{III,II} \\ T_{I,III} & T_{II,III} & R_{III} \end{bmatrix}}_{\mathbf{S}} \begin{bmatrix} \hat{p}_I^- \\ \hat{p}_{II}^- \\ \hat{p}_{III}^- \end{bmatrix} \quad (1)$$

As shown in the next section, the side branch impedance is readily derived from the obtained scattering matrix.

Two different configurations were tested, both with the main duct (Branch I and Branch II) diameter $D = 0.057\text{m}$. In the first configuration, Case I, the side branch had the same diameter as the main duct, while in Case II it had a smaller diameter: $d = 0.0245\text{m}$. As discussed by Kooijman et al. [8] and Knotts and Selamet [17], the geometry of the edge of the orifice has an influence. Here sharp edges that were well-defined and easy to implement were chosen. The mean flow velocity profile approaching the orifice also influences the results (Kooijman et al. [8]). It is questionable which test case is most significant; in practice, the actual configuration varies significantly depending on the application. Here it is assumed that a fully turbulent velocity profile is developed, given that the duct length from the inlet to the side branch is $\sim 60D$.

The interface between two connected circular tubes is not well-defined: a plane wave approaching from the side branch will see a gradual area change. To study the influence on the reactance, the following approach was used. For each case, a position where the reactance was zero was determined when performing the initial measurements without flow. This could be interpreted as the position where the influence of the side branch (including end correction) ends. The correction from the geometrical length corresponds well with the results summarized in Dalmont et al. [18]. The results with mean flow show the deviations from the no flow case.

The forward and backward travelling waves were determined via the two-microphone wave decomposition technique [19] using external sources [20]. The effect of uncorrelated disturbances, such as local turbulence, were minimized using the transfer functions between the input voltage to the loudspeakers and the microphone signals, rather than the microphone signals alone [21]. To cover the entire frequency range of interest, two different microphone separation distances were needed; hence three microphones were used at each port. Sufficient excitation over the entire frequency range was provided by an array of loudspeakers at each port. The separation distance between the loudspeakers in each array varied to avoid any wave interference. The mean flow was provided by a pressurized, anechoic

stagnation chamber. High-performance resistive silencers at each port reduced system reflections and flow noise generated by the valves. The flow was monitored with Prandtl tubes in Branch I and Branch II.

Bruel & Kjaer Type 4938 microphones and Nexus conditioning amplifiers were used together with an Agilent VXI data acquisition system (E1421B Mainframe with an E1432A data acquisition card) to acquire the data. An E1432A source card was used to generate the excitation signal, which was amplified with an NAD Model 319 stereo amplifier. Stepped-sine excitation was used, with the number of averages taken continuously updated with respect to the signal to noise ratio. The frequency range was 50 to 2000 Hz, covering most of the plane-wave range, with a frequency increment of 6.25 Hz.

ACOUSTIC IMPEDANCE OF A SIDE BRANCH

IMPEDANCE DEFINITION

A general definition of impedance is the ratio of some potential to flow quantity. The common choice in acoustics is the ratio between fluctuating pressure and acoustic volume velocity. An alternative formulation, suitable for ducts with mean flow, is the ratio between acoustic stagnation enthalpy and mass:

$$Z_h = \frac{h'}{m'} \quad (2)$$

where

$$h' = \frac{p'}{\rho_0} + T_0 s' + U_0 u' \quad (3)$$

$$m' = (\rho_0 u' + \rho' U_0) A. \quad (4)$$

The prime denote a fluctuating acoustic field quantity. Assuming an isentropic plane wave, Eqs. (2-4) yield:

$$Z_h = \frac{c_0}{\rho_0 A} \frac{(1+M)+R(1-M)}{(1+M)-R(1-M)} \quad (5)$$

Now, having access to the full scattering matrix for the three-port, we can study the orifice impedance for an incident wave from any port and with any flow configuration. In the present test setup, mean flow is provided from Branch I and exits through Branch II. Assuming anechoic terminations in the two other ports, and using the definition of the three-port in Figure 1 and the scattering matrix in Eq. (1), the normalized impedance at the orifice experienced by an incident travelling plane wave from the three ports is:

$$Z_{I-III} = \frac{h'_I}{m'_{III+}} = \frac{(1+M_I)+R_I(1-M_I)}{T_{I-III}} \quad (6)$$

$$Z_{II-III} = \frac{h'_{II}}{m'_{III+}} = \frac{(1-M_{II})+R_{II}(1+M_{II})}{T_{II-III}} \quad (7)$$

$$Z_{III} = \frac{h'_{III}}{m'_{III-}} = \frac{1+R_{III}}{1-R_{III}} \quad (8)$$

If one defines the acoustic impedance as the ratio of acoustic pressure and volume velocity, the Mach number compensation terms are not included in equations (6-7). That is, the relative frequency characteristics are the same but there is an off-set in amplitude.

GOVERNING PARAMETERS

The present work differs from most of the previous studies in that the orifice is not of finite thickness, but rather is a side branch. Nor is it a deep cavity, since the termination is open and considerably damped. Also, the actual dimensions of the orifice are larger here than commonly studied. However the basic physics of the flow-related effects apply, and some phenomena are perhaps even more apparent with this configuration.

Following the theory of vortex sound and Howe's energy corollary [1], the resistance of the orifice is controlled by the exchange of energy between the hydrodynamic and acoustic fields. At sufficiently large Reynolds numbers, vortices are shed periodically at the upstream leading edge [22]. These vortices grow and interact with the acoustic field (if present) while convected downstream over the orifice. This interaction results in the transfer of energy between the hydrodynamic and acoustic fields. This is usually expressed via the time averaged acoustic power:

$$\langle P \rangle = -\rho_0 \int_V \langle (\boldsymbol{\omega} \times \mathbf{u}) \cdot \mathbf{u}' \rangle dV, \quad (9)$$

where $\boldsymbol{\omega} = \nabla \times \mathbf{u}$ is the vorticity vector, $\mathbf{u}$ is the local velocity and V is the volume enclosing the vorticity. $\mathbf{u}'$ is the acoustic particle velocity, usually assumed to be uniform over the orifice area and harmonic. If the time averaged power is negative, the acoustic field is attenuated, while a positive value indicates sound generation. In the three-port format used here the calculation of power can be greatly simplified. The volume V is represented by the three well-defined port elements, at which one can compute the relative power directly from the scattering matrix. Hence, the relative powers from the three ports are: