

Influence of Gyroscopic Effect on Hypoid and Bevel Geared System Dynamics

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ABSTRACT

The noise and vibration response of hypoid or bevel geared rotor system, primarily excited by transmission error (TE), and mesh vector and stiffness variations, can be affected significantly by the coupling between the driveline rotor dynamics and gear vibratory response. This is because of the inherent design comprising of non-parallel rotational axes and time-varying as well as spatial-varying gear mesh characteristics. One of the important factors of the driveline system dynamics is the rotor gyroscopic effect that has not been studied extensively in traditional gear dynamics. To address this gap in the literature, this paper attempts to examine the influence of incorporating gyroscopic terms in the hypoid gear dynamic simulation. A multi-degrees-of-freedom, multi-body dynamic model is used as a generalized representation of a hypoid geared rotor system. The gyroscopic matrices are introduced to formulate both the linear time-invariant (LTI) and nonlinear time-varying (NLTV) dynamic equations. The system vibratory responses are solved in the frequency domain for LTI model and through time domain numerical integration for NLTV model. The results are compared to those calculations obtained from the traditional models without including the gyroscopic effect. The influence of gyroscopic behavior on the typical hypoid gear dynamic response is analyzed and the condition where the gyroscopic effect needs concern or can be neglected is revealed.

INTRODUCTION

Hypoid and spiral bevel gears are widely used in modern vehicles, but research into their dynamic behavior has not been as extensive and in-depth as parallel axis gears due to complicated tooth geometry and unique mesh characteristics. Because of the largely oblique mesh force vector and complex time-varying mesh

characteristics, hypoid and bevel gear vibratory response often significantly couples with the driveline large displacement rotational motion and rotor dynamics. Even though it is an important factor in the treatment of rotor dynamics [1-2], the gyroscopic effect is usually not included in the analysis of hypoid or bevel gear dynamics. The motivation of the present study presented in this paper is to improve on the previous approach and to help in understanding the influences of the gyroscopic effect on the gear mesh excited vibration, with emphasis on hypoid and bevel geared rotor systems.

Gyroscopic effect is often neglected in traditional gear dynamics to reduce the model complexity. Only a few previous investigations emphasized its role for some special cases. In one study, Choy and Ruan (1991) [3] incorporated the gyroscopic matrices in the analytical model of a gearbox with involute spur gears, and solved the linear dynamic response under typical operation conditions. Derouiche et al. (2004) [4] showed an appreciable influence of the gyroscopic behavior on the predicted noise response spectra of a geared transmission without housing. Parker and Lin (1999) [5-6] regarded the gyroscopic effect as a key factor in their analyses of planetary gear vibration. However, the inclusion or analysis of gyroscopic terms in the dynamics of non-parallel axis gears, namely hypoid and spiral bevel gears, has not been seen in the existing literature even though those geared systems are extensively applied and frequently experience undesirable noise and vibration problem.

In the present study, a generalized multi-degrees-of-freedom, multi-body dynamic model of a hypoid geared rotor system with gyroscopic terms is applied to address the gap discussed above. Using the proposed model, the gyroscopic matrices are introduced in the NLTV formulation. Under certain conditions, the equations of motion can be simplified into a LTI form that can be

solved directly in frequency domain. However, for more general cases, the raw NLTV equations of motion must be retained and analyzed through time domain numerical integration. In our work, the resulting system dynamic responses are compared to analytical solutions without the gyroscopic terms in order to quantify the influence of incorporating or neglecting the gyroscopic effect. Furthermore, the modal response is analyzed to gain a more in-depth understanding of how gyroscopic behavior affects dynamic response.

ANALYSIS MODEL

GEAR ROTOR SYSTEM DYNAMIC MODEL

A fourteen degrees-of-freedom (DOF) lumped parameter model is proposed to represent a multi-DOF, multi-body hypoid geared rotor system. The model consists of four inertia elements that include a hypoid gear pair comprising of a pinion and a gear, an engine or driver and a load as shown in Figure 1. The pinion and gear are modeled as rigid bodies supported on the flexible shaft-bearing structure and each body is represented using 6 independent coordinates. Between the pinion and the gear, there is a damped flexible connection representing the gear meshing relationship which is characterized by a lumped mesh stiffness and damping element. The engine/driver and load each has one DOF that is the rotation about the respective shaft axis as their other coordinates are normally decoupled from the gear dynamics by design. The shaft and bearing are also represented using a set of lumped stiffness and damping elements.

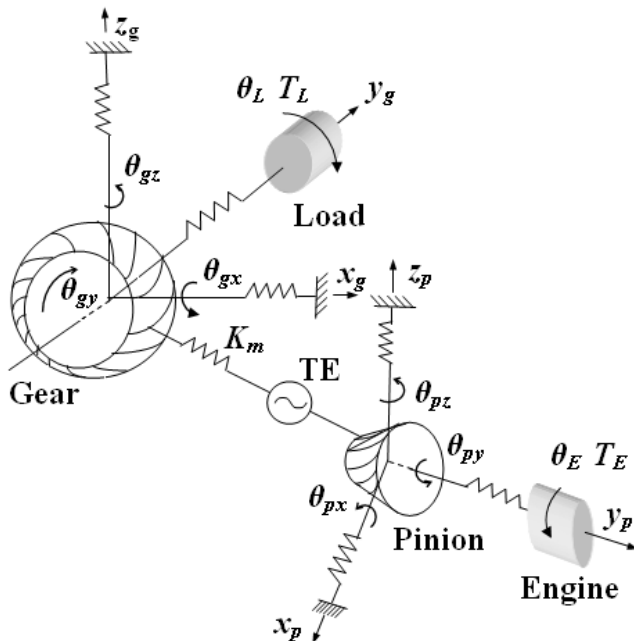


Figure 1 - Fourteen degrees-of-freedom model of a hypoid geared rotor system.

The pinion side coordinates (X_p , Y_p , Z_p , θ_{px} , θ_{py} , θ_{pz} , θ_E) are defined relative to a local Cartesian inertial

coordinate systems S_p whose origin is at the centroid of the pinion when the system is in static equilibrium. Similarly, the gear side coordinates (X_g , Y_g , Z_g , θ_{gx} , θ_{gy} , θ_{gz} , θ_L) are expressed in the gear local Cartesian inertial coordinate system S_g whose origin coincides with the static equilibrium position of the gear centroid. The angular coordinates θ_{py} and θ_E are rotation about the Y-axis of S_p , which coincides with the pinion shaft axis. Similarly, the angular coordinates θ_{gy} and θ_L are rotation about the gear shaft axis. These 4 rotational coordinates are large displacement type unlike the other angular coordinates in the model that are small vibratory quantities. However, when the system is under stationary operation condition and the assumption that the system vibration is uncoupled from the shaft torsional motion is satisfied, θ_{py} , θ_{gy} , θ_E and θ_L also can be small perturbations. Furthermore all the coordinate axes are assumed to coincide with the principal axes of inertia of each body in static equilibrium condition and no static or dynamic imbalance exists.

The detailed equations of motion of the 14-DOF hypoid geared rotor system dynamic model can be written as:

$$I_E \ddot{\theta}_E + c_{pyr} (\dot{\theta}_E - \dot{\theta}_{py}) + k_{pyr} (\theta_E - \theta_{py}) = T_E \quad (1)$$

$$m_p \ddot{x}_p + c_{pxt} \dot{x}_p + k_{pxt} x_p = -n_{px} F_m \quad (2)$$

$$m_p \ddot{y}_p + c_{pyt} \dot{y}_p + k_{pyt} y_p = -n_{py} F_m \quad (3)$$

$$m_p \ddot{z}_p + c_{pzt} \dot{z}_p + k_{pzt} z_p = -n_{pz} F_m \quad (4)$$

$$I_{px} \ddot{\theta}_{px} + c_{pxr} \dot{\theta}_{px} + k_{pxr} \theta_{px} - I_{py} \dot{\theta}_{pz} \dot{\theta}_{py} - I_{py} \theta_{pz} \ddot{\theta}_{py} = -\lambda_{px} F_m \quad (5)$$

$$I_{py} \ddot{\theta}_{py} + c_{pyr} (\dot{\theta}_{py} - \dot{\theta}_E) + k_{pyr} (\theta_{py} - \theta_E) = -\lambda_{py} F_m \quad (6)$$

$$I_{pz} \ddot{\theta}_{pz} + c_{pzt} \dot{\theta}_{pz} + k_{pzt} \theta_{pz} + I_{py} \dot{\theta}_{px} \dot{\theta}_{py} + I_{py} \theta_{px} \ddot{\theta}_{py} = -\lambda_{pz} F_m \quad (7)$$

$$m_g \ddot{x}_g + c_{gxt} \dot{x}_g + k_{gxt} x_g = n_{gx} F_m \quad (8)$$

$$m_g \ddot{y}_g + c_{gyt} \dot{y}_g + k_{gyt} y_g = n_{gy} F_m \quad (9)$$

$$m_g \ddot{z}_g + c_{gzt} \dot{z}_g + k_{gzt} z_g = n_{gz} F_m \quad (10)$$

$$I_{gx} \ddot{\theta}_{gx} + c_{gxr} \dot{\theta}_{gx} + k_{gxr} \theta_{gx} - I_{gy} \dot{\theta}_{gz} \dot{\theta}_{gy} - I_{gy} \theta_{gz} \ddot{\theta}_{gy} = \lambda_{gx} F_m \quad (11)$$

$$I_{gy} \ddot{\theta}_{gy} + c_{gyr} (\dot{\theta}_{gy} - \dot{\theta}_L) + k_{gyr} (\theta_{gy} - \theta_L) = \lambda_{gy} F_m \quad (12)$$

$$I_{gz} \ddot{\theta}_{gz} + c_{gzr} \dot{\theta}_{gz} + k_{gzr} \theta_{gz} + I_{gy} \dot{\theta}_{gx} \dot{\theta}_{gy} + I_{gy} \theta_{gx} \ddot{\theta}_{gy} = \lambda_{gz} F_m \quad (13)$$

$$I_L \ddot{\theta}_L + c_{gyr} (\dot{\theta}_L - \dot{\theta}_{gy}) + k_{gyr} (\theta_L - \theta_{gy}) = T_L \quad (14)$$

To be concise for the subsequent analysis, the above fourteen equations of motion can be expressed more compactly in matrix form as:

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} + [G]\{\dot{q}\} + [G_a]\{q\} = \{F\} \quad (15)$$

The generalized coordinates are given by:

$$\{q\} = \{\theta_E, q_p^T, q_g^T, \theta_L\}^T \quad (16)$$

$$\{q_l\} = \{x_l, y_l, z_l, \theta_{lx}, \theta_{ly}, \theta_{lz}\}^T \quad (\text{Subscript } l = p, g) \quad (17)$$

The lumped mass matrix is expressed as:

$$[M] = \text{diag}[I_E, M_{px}, M_{py}, M_{pz}, I_{px}, I_{py}, I_{pz}, M_{gx}, M_{gy}, M_{gz}, I_{gx}, I_{gy}, I_{gz}, I_L] \quad (18)$$

The matrix $[K]$ represents the shaft and bearing assembly stiffnesses lumped at the appropriate generalized coordinates. The damping matrix $[C]$ is normally assumed to be proportional viscous type. It is noted that in a more general case, other than the expression in Equation (1~14), any of the off-diagonal elements in $[K]$ and $[C]$ can be non-zero values to incorporate the stiffness and damping coupling between the different coordinates through the shaft-bearing structure.

As noted earlier, no previous analysis of hypoid gear pairs have incorporated the gyroscopic effect of the pinion and gear inertias the way they are included in Equation (15) as two additional items on the left hand side of the equation, namely $[G]\{\dot{q}\}$ and $[G_a]\{q\}$. The first gyroscopic matrix $[G]$ is associated with the rotational velocities of the pinion and gear, and can be explicitly expressed as,

$$[G] = \begin{bmatrix} I_{py} \dot{\theta}_{py} & -I_{py} \dot{\theta}_{py} & & \\ \uparrow (7,5) & \uparrow (5,7) & & \\ & & I_{gy} \dot{\theta}_{gy} & -I_{gy} \dot{\theta}_{gy} \\ & & \uparrow (13,11) & \uparrow (11,13) \end{bmatrix}_{14 \times 14} \quad (19)$$

where the number pairs in parenthesis, such as (5,7), refers to the row and column of the element in the matrix. All other elements not shown are zeros.

The second gyroscopic matrix $[G_a]$ is associated with the angular accelerations of the pinion and gear. The explicit form of $[G_a]$ is expressed as in following equation where again the number pairs in parenthesis indicates the row and column of the element in the matrix with all other elements not shown are zeros,

$$[G_a] = \begin{bmatrix} I_{py} \ddot{\theta}_{py} & -I_{py} \ddot{\theta}_{py} & & \\ \uparrow (7,5) & \uparrow (5,7) & & \\ & & I_{gy} \ddot{\theta}_{gy} & -I_{gy} \ddot{\theta}_{gy} \\ & & \uparrow (13,11) & \uparrow (11,13) \end{bmatrix}_{14 \times 14} \quad (20)$$

It is noted the above two gyroscopic terms make the system equations of motion nonlinear and intensify the coupling of the large displacement rolling motion with the small vibratory response. However, when the system is under stationary operation condition where the mean shaft speed (rolling velocity) is fixed with only very small perturbations, the gyroscopic terms can be linearized by neglecting the second order small quantities. In the linearized case, $[G]$ can be expressed as given below and $[G_a]$ is assumed zero,

$$[G] = \Omega \begin{bmatrix} I_{py} & -I_{py} & & \\ \uparrow (7,5) & \uparrow (5,7) & & \\ & & I_{gy} / R & -I_{gy} / R \\ & & \uparrow (13,11) & \uparrow (11,13) \end{bmatrix}_{14 \times 14} \quad (21)$$

where Ω is the mean shaft speed and R is the gear ratio,

$$R = -N_g / N_p \quad (22)$$

The negative sign in the above equation is due to the way the pinion and gear coordinate directions are defined. Also, note that N_p and N_g are the tooth numbers of the pinion and the gear, respectively.

The excitation force vector $\{F\}$ in Equation (15) is given by,

$$\{F\} = [T_E, h_p F_m, -h_g F_m, -T_L]^T \quad (23)$$

where T_E and T_L can be constant or time-varying torques exerted on the engine or driver and load elements as