

# Application of the Wave Based Technique to Predict the Engine Noise Radiation Under Anechoic Conditions

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## ABSTRACT

As an alternative to the element based methods, recently a wave based technique (WBT) has been developed. Since it is based on the indirect Trefftz approach, exact solutions of the governing differential equation are used to approximate the dynamic field variables. This paper discusses the extensions of the WBT for the analysis of engine noise radiation problems in 3 dimensions under anechoic conditions. Furthermore, necessary extensions of shape functions, numerical integration and a methodology to create a WBT radiation models are described. The performance of the method compared to a commercial BEM solution is demonstrated with a real engine example.

## INTRODUCTION

Efficient simulation of noise radiation of new engines and power units is highly requested for developing low noise engine designs. In recent years the boundary element method (BEM) has become a well established tool for the analysis of acoustic problems involving unbounded domains, such as sound radiation or sound scattering problems. The BEM is based on a boundary integral formulation, which relates the field variables in the solution domain to a set of variables defined on the problem boundary. In this way, only the boundary of the considered problem has to be discretized, which yields relatively small numerical models. However, due to the

global nature of the boundary integral formulation, the system matrices are fully populated with complex coefficients and are frequency dependent. Furthermore, since the dynamic boundary variables are expressed in terms of locally defined polynomial shape functions, a large number of elements are required in order to get reasonably prediction accuracy, especially for 3-dimensional problems at higher frequencies. As a result, an efficient use of the BEM is restricted to low-frequency applications.

As an alternative to the element based methods, recently a wave based technique (WBT) has been developed. Since it is based on the indirect Trefftz approach, exact solutions of the governing differential equation are used to approximate the dynamic field variables, instead of simple polynomial functions. As a result, a fine element discretization is no longer required. This results in very small numerical models which exhibit an enhanced computational efficiency as compared to the element based methods. To extend the method to radiation problems under anechoic conditions an artificial spherical truncation boundary is introduced. It divides the problem into a bounded and an unbounded domain. The dynamic field in the unbounded domain is expressed by radiation functions, consisting of linear combinations of spherical harmonics and Hankel functions, while the bounded domain uses the interior wave functions. In order to be able to model the fluid volume between the model body and the truncation sphere, new curved elements had to be introduced.

These additional elements are spherical faces and circular segments to describe the spherical coupling boundary.

This paper discusses the extensions of the WBT for the analysis of engine noise radiation problems in 3 dimensions under anechoic conditions. Furthermore, necessary extensions of shape functions, numerical integration and a methodology to create a WBT model in case of engine radiation problems are described. The performance of the method compared to a commercial BEM solution is demonstrated with a real engine example.

## PROBLEM DEFINITION

Consider an unbounded 3-dimensional acoustic problem as shown in Figure 1. A physical model boundary  $\partial\Omega$  is surrounded by a fluid domain, which is characterized by its speed of sound  $c$  and its ambient fluid density  $\rho$ . The time-harmonic pressure response is given by  $p(\mathbf{r}, t) = p(\mathbf{r}, t)e^{j\omega t}$  at position  $\mathbf{r}$ , where  $j$  represents the imaginary unit,  $t$  the time and  $\omega$  the circular frequency. Under the assumption that the system is linear, the fluid is inviscid and the process adiabatic, the steady-state pressure field is governed by the homogenous Helmholtz equation

$$(\Delta + k^2)p(\mathbf{r}) = 0, \quad (1)$$

where  $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$  represents the Laplace

operator and  $k = \frac{\omega}{c}$  the wave number.

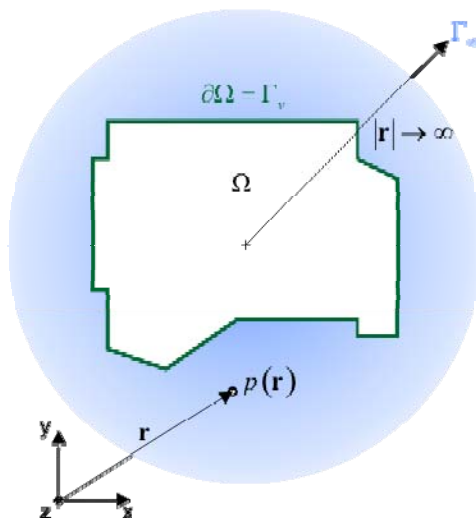


Figure 1: 3d unbounded acoustic problem definition

In case of the problem definition shown in Figure one, the boundary  $\partial\Omega$  consists only of one part  $\Gamma_v$ , where a normal velocity  $\bar{v}_n$  boundary condition is imposed:

$$v_n(\mathbf{r}) = \frac{j}{\rho\omega} \frac{\partial p(\mathbf{r})}{\partial n} = \bar{v}_n(\mathbf{r}), \quad \mathbf{r} \in \Gamma_v \quad (2)$$

Two other boundary conditions are possible, namely the pressure  $\bar{p}$  boundary condition

$$p(\mathbf{r}) = \bar{p}(\mathbf{r}), \quad \mathbf{r} \in \Gamma_p \quad (3)$$

and the normal impedance  $\bar{Z}$  boundary condition

$$p(\mathbf{r}) = \bar{Z}_n(\mathbf{r})v_n(\mathbf{r}) = \bar{Z}_n(\mathbf{r})\frac{j}{\rho\omega} \frac{\partial p(\mathbf{r})}{\partial n}, \quad \mathbf{r} \in \Gamma_z. \quad (4)$$

Furthermore, the Sommerfeld radiation condition for outgoing waves at infinity  $\Gamma_\infty$  is applied. This condition ensures that no acoustic energy is reflected at infinity [8] and is expressed as

$$\lim_{|\mathbf{r}| \rightarrow \infty} \left( |\mathbf{r}| \left( \frac{\partial p(\mathbf{r})}{\partial |\mathbf{r}|} + jkp(\mathbf{r}) \right) \right) = 0. \quad (5)$$

Solution of the Helmholtz equation (1) together with the boundary conditions (2)-(4) and (5) yields a unique dynamic acoustic pressure field  $p(\mathbf{r})$ .

## WAVE BASED TECHNIQUE

The wave based technique [1,3] is based on an indirect Trefftz approach [2] in that the dynamic acoustic pressure is expressed as a linear combination of wave functions, which exactly satisfy the homogenous Helmholtz equation (1). In contrast to the well known element based methods like FEM and BEM, which use polynomial shape functions, the wave functions do not satisfy the imposed boundary conditions. A weighted residual formulation is used to enforce the residuals at the boundaries to be zero in an integral sense.

A sufficient condition for the WBT approximations to converge towards the exact solution is convexity of the considered problem domain. Since in general acoustic problems are non-convex, the problem domain must be partitioned into convex sub-domains [1, 4,]. In order to tackle problems in unbounded domains, an additional partitioning must be performed. By introducing an artificial truncation boundary  $\Gamma_T$  as shown in Figure 2, the problem domain is divided into a bounded region  $\Omega_B$  and an unbounded region  $\Omega_U$ .

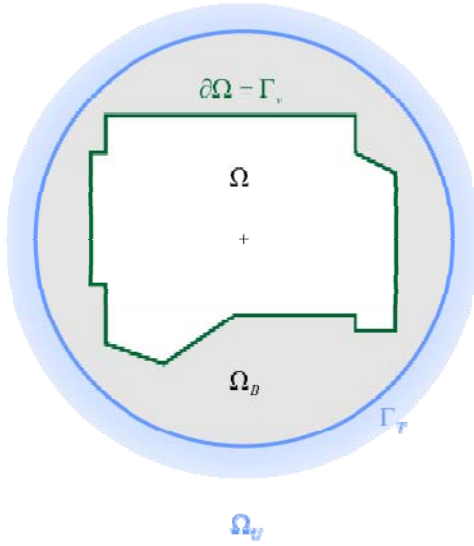


Figure 2: concept of the truncation boundary

### BOUNDED REGION

Within the bounded region  $\Omega_B$ , a linear combination of acoustic wave functions  $\Phi_B$  approximates the exact pressure solution

$$p(r) \approx \hat{p}(r) = \sum_{i=1}^{n_B} p_{B,i} \Phi_{B,i}(r), \quad (6)$$

where the unknown wave function contribution factors  $p_{B,i}$  are forming the  $n_B$  degrees of freedom. The acoustic wave functions  $\Phi_{B,i}(r)$  consist of the three following forms:

$$\Phi_{B,i} = \begin{cases} \Phi_{B,r}(x, y, z) = \cos(k_{xB,r}x) \cos(k_{yB,r}y) e^{-jk_{zB,r}z} \\ \Phi_{B,s}(x, y, z) = \cos(k_{xB,s}x) e^{-jk_{yB,s}y} \cos(k_{zB,s}z) \\ \Phi_{B,t}(x, y, z) = e^{-jk_{xB,t}x} \cos(k_{yB,t}y) \cos(k_{zB,t}z) \end{cases} \quad (7)$$

Since the only requirement for the selection of the wave numbers  $k_{xB,v}$ ,  $k_{yB,v}$  and  $k_{zB,v}$  is that  $k_{xB,v}^2 + k_{yB,v}^2 + k_{zB,v}^2 = k^2$  with  $v = r, s, t$  an infinite number of acoustic wave functions (7) can be defined for the approximation (6). It is proposed to select the following wave number components,

$$\begin{aligned} (k_{xB,r}, k_{yB,r}, k_{zB,r}) &= \left( \frac{b_1\pi}{L_x}, \frac{b_2\pi}{L_y}, \pm \sqrt{k^2 - \left(\frac{b_1\pi}{L_x}\right)^2 - \left(\frac{b_2\pi}{L_y}\right)^2} \right) \\ (k_{xB,s}, k_{yB,s}, k_{zB,s}) &= \left( \frac{b_3\pi}{L_x}, \pm \sqrt{k^2 - \left(\frac{b_3\pi}{L_x}\right)^2 - \left(\frac{b_4\pi}{L_z}\right)^2}, \frac{b_4\pi}{L_z} \right) \\ (k_{xB,t}, k_{yB,t}, k_{zB,t}) &= \left( \pm \sqrt{k^2 - \left(\frac{b_5\pi}{L_y}\right)^2 - \left(\frac{b_6\pi}{L_z}\right)^2}, \frac{b_5\pi}{L_y}, \frac{b_6\pi}{L_z} \right) \end{aligned} \quad (8)$$

with  $b_1, b_2, b_3, b_4, b_5, b_6 = 0, 1, 2, \dots, n_B$ .  $L_x$ ,  $L_y$  and  $L_z$  are the dimensions of the rectangular domain, enclosing the considered problem domain.

As mentioned in the beginning of this chapter, a non-convex problem domain has to be split into convex sub-domains. To couple two convex sub-domains at an interface  $\Gamma_i$ , two types of continuity conditions are considered in the wave based technique. One of these coupling methods is the pressure and velocity-coupling method.

#### • pressure and velocity-coupling

Both pressure and normal velocity continuity are imposed at the interface:

$$\begin{aligned} p_1(r_1) &= p_2(r_2), & \forall r_1, r_2 \in \Gamma_i \\ \frac{j}{\rho\omega} \frac{\partial p_1(r)}{\partial n_1} &= -\frac{j}{\rho\omega} \frac{\partial p_2(r)}{\partial n_2}, & \forall r_1, r_2 \in \Gamma_i \end{aligned} \quad (9)$$

The other coupling method is referred to as impedance- or equivalent normal velocity-coupling method.

#### • impedance-coupling

By taking a linear combination of the two conditions (9) and by the introduction of a pressure scaling factor  $\bar{Z}_{int}$  combined continuity conditions can be formulated at the interface:

$$\begin{aligned} \frac{j}{\rho\omega} \frac{\partial p_1(r)}{\partial n_1} - \frac{1}{\bar{Z}_{int}} p_1(r_1) &= -\frac{j}{\rho\omega} \frac{\partial p_2(r)}{\partial n_2} - \frac{1}{\bar{Z}_{int}} p_2(r_2) \\ \frac{j}{\rho\omega} \frac{\partial p_2(r)}{\partial n_2} - \frac{1}{\bar{Z}_{int}} p_2(r_2) &= -\frac{j}{\rho\omega} \frac{\partial p_1(r)}{\partial n_1} - \frac{1}{\bar{Z}_{int}} p_1(r_1) \end{aligned} \quad (10)$$

The expression equivalent normal velocity results from the fact that the relations in (10) actually express continuity of an equivalent acoustic velocity quantity [4]. The value  $\bar{Z}_{int}$  is a non-zero complex number and determines the relative importance of the velocity term as compared to the pressure. Notice that for the verification in the next chapter only the impedance-coupling is used.