

A Hybrid Wave Based - Modally Reduced Finite Element Method for the Efficient Analysis of Low- and Mid-frequency Car Cavity Acoustics

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ABSTRACT

This paper presents a newly developed hybrid simulation technique for uncoupled acoustic analysis of interior cavities. This method applies a Wave Based model for a large, geometrically simple portion of the acoustic cavity. The superficial details of the problem domain are modeled using a modally reduced finite element model. The resulting hybrid model benefits from the computational efficiency of the Wave Based Method, while retaining the Finite Element Method's ability to model the actual geometry of the problem in great detail. Application of this approach to the analysis of a moderately simplified acoustic car cavity shows the improved computational efficiency as compared to classical finite element procedures and illustrates the potential of the hybrid method as a powerful tool for the analysis of three-dimensional interior acoustic systems.

INTRODUCTION

The tightening of legal regulations on the human exposure to noise and vibrations and the ever-increasing customer demands for better comfort force product designers to take the dynamic and acoustic properties of their products into account throughout the entire design process. In the classical design process these properties are studied by means of extensive measurement campaigns carried out on physical prototypes. This approach is not only time-consuming but also very expensive, resulting in severe limitations on the number of design possibilities which can be fully explored. In order to reduce the number of physical prototypes and to shorten the product design cycle, a vast amount of effort is put into the development of Computer Aided Engineering (CAE) techniques. During the past decades,

these techniques have become invaluable tools in many design processes, resulting in the application of virtual prototypes for design optimization in every stage of the design process.

For low-frequency applications deterministic simulation techniques, such as the Finite Element Method [1] (FEM) and the Boundary Element Method [2] (BEM) are well established. At higher frequencies probabilistic prediction techniques, such as Statistical Energy Analysis (SEA) [3], are often used.

The element-based simulation techniques discretize the considered problem (or its boundary) into a finite number of elements. Within these elements, the dynamic response variables are described by an expansion of simple (polynomial) shape functions, which are not exact solutions of the governing differential equations. As a result, as frequency increases, the prediction accuracy of the element based models decreases, mainly due to two types of errors: interpolation and pollution errors [4]. Keeping these errors within acceptable bounds requires the use of extremely dense problem discretizations, especially at higher frequencies. This results in prohibitively large numerical models for real-life acoustic problems. As a result, the applicability of the element based techniques is limited to problems in the low-frequency range. One of the major advantages, however, of the element based methods is their ability to model any problem, regardless of its geometrical complexity.

The SEA techniques divide the entire system into a limited number of components. The mathematical expression of the power balance between the various subsystems provides the space- and frequency-

averaged dynamic behavior of each of the structural-acoustic system components. These methods are however based on the assumption that all of the subsystems have a high modal overlap in the frequency range of interest, which limits their use to high-frequency modeling.

In between the low-frequency application range of the deterministic element based techniques and the high-frequency application range of the statistical methods, there exists a wide frequency-gap, the so-called mid-frequency range, for which currently no adequate and mature prediction methods are available. One of the modeling techniques which is currently under investigation is the Wave Based Method (WBM) [5]. This technique belongs to the family of so-called Trefftz methods [6] in that it applies globally defined wave functions, which are exact solutions of the governing differential equations, instead of approximating shape functions to describe the dynamic response variables. As a result, the use of very fine domain discretizations at higher frequencies is no longer required. The size of the numerical models and the associated computational resources are substantially lower as compared to element based methods. Because of the enhanced convergence properties, the WBM has proven to be applicable for low- as well as mid-frequency steady-state acoustic problems [7]. A sufficient condition for convergence of the applied wave function expansions is the convexity of the considered problem domains. Non-convex domains have to be partitioned into a number of (convex) subdomains. As a result, in order to fully benefit from the method's computational efficiency, only problems of moderate geometrical complexity are considered.

In recent years, the combination of the geometrical flexibility of the FEM with the enhanced convergence properties of the WBM in a hybrid Wave Based-Finite Element (WB-FE) modeling technique has been successfully explored for two-dimensional (2D) [8] and three-dimensional (3D) [9] uncoupled acoustic and uncoupled structural problems [10]. Recently, this hybrid modeling approach has been successfully applied to the direct coupling between a structural FE model and an acoustic WB model [11]. As a result, this family of methods allows predictions to be made for uncoupled structural, acoustic or fully coupled structural-acoustic problems in the mid-frequency range. However, when direct physical coordinates are applied in the FE submodel, a large percentage (30 to 50%) of the execution time is used for the solution of the sparse system of FE equations. In previous work [12], the hybrid methodology for coupled structural-acoustic problems was refined by reducing the number of dynamic structural degrees of freedom (dofs) using modal reduction techniques. This results in smaller, diagonal system matrices describing the dynamic behavior of the structural part of the problem and in a significant reduction of the solution times for the system equations in the hybrid Wave Based-Modal Finite Element (WB-MFE) method with respect to the hybrid

WB-FE method, while retaining a similar level of accuracy. In a recent paper by the authors [13], this work was further extended to the existing 3D acoustic hybrid methodology. Instead of applying a set of uncoupled acoustic modes for the reduction of the acoustic FE part of the model, classical Component Mode Synthesis [14] techniques are applied to accurately capture the outer boundary layer of the cavity, while a large, inner portion of the cavity is modeled very efficiently using the WBM. In order to further optimize the computational efficiency, the hybrid Wave Based-Projected Modal Finite Element (WB-PMFE) method, an extension based on projection of the component modes on analytical functions is explored. In this paper, the applicability of the hybrid simulation technique for the mid-frequency acoustic analysis of an interior car cavity is assessed. Application of this approach to the analysis of a moderately simplified acoustic car cavity illustrates the potential of the hybrid method as a powerful tool for the analysis of three-dimensional interior acoustic systems.

The first section of the paper describes the mathematical formulation of a general uncoupled steady-state acoustic problem. Next, the hybrid coupling between an acoustic WB model and a direct acoustic FE model and the extension of this coupling to a hybrid coupling using modal reduction techniques for the FE submodel are discussed. A final section illustrates the improved computational efficiency of the method as compared to classical FE procedures through an application study of the interior acoustics of a rigid-walled acoustic car cavity.

PROBLEM DEFINITION

The steady-state dynamic behavior of a general 3D uncoupled acoustic system, as shown in figure 1, can be described by the acoustic pressure $p(\mathbf{r})$ at a position $\mathbf{r}(x, y, z)$.

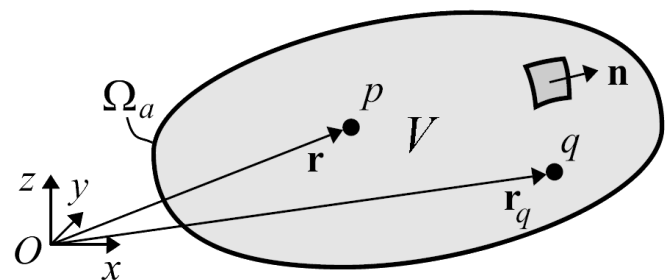


Figure 1: A 3D uncoupled acoustic problem

The acoustic cavity V is filled with air, with ambient density ρ_a and speed of sound c . An acoustic point source $\mathbf{q}$ at position $\mathbf{r}_q(x_q, y_q, z_q)$ within the cavity excites the fluid at circular frequency ω . Under the assumption that the fluid in the cavity exhibits linear, inviscid and adiabatic behavior, the Helmholtz equation

governs the steady-state acoustic pressure $p(\mathbf{r})$ inside the cavity [15]:

$$\Delta p(\mathbf{r}) + k_a^2 \cdot p(\mathbf{r}) = -j\rho_a \omega q \cdot \delta(\mathbf{r}, \mathbf{r}_q), \quad \mathbf{r} \in V \quad (1)$$

with $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ the Laplacian operator,

$k = \frac{\omega}{c}$ the acoustic wave number, $j = \sqrt{-1}$ the imaginary unit and δ a Dirac delta function.

Since the Helmholtz equation (1) is a second-order differential equation, one boundary condition needs to be specified at each point of the boundary in order to obtain a well-posed problem. The boundary Ω_a of the acoustic domain consists of three non-overlapping parts, $\Omega_a = \Omega_p \cup \Omega_v \cup \Omega_z$. On each of the parts of the boundary, Ω_p , Ω_v and Ω_z , respectively acoustic pressure, acoustic normal velocity or normal impedance boundary conditions are specified:

Dirichlet BCs on Ω_p : $p(\mathbf{r}) - \bar{p}(\mathbf{r}) = 0$

Neumann BCs on Ω_v : $\frac{j}{\rho_a \omega} \frac{\partial p(\mathbf{r})}{\partial \mathbf{n}} - \bar{v}_n(\mathbf{r}) = 0 \quad (2)$

Robin BCs on Ω_z : $\frac{j}{\rho_a \omega} \frac{\partial p(\mathbf{r})}{\partial \mathbf{n}} - \frac{p(\mathbf{r})}{\bar{Z}_n(\mathbf{r})} = 0$

with $\frac{\partial}{\partial \mathbf{n}}$ the normal derivative and $\bar{p}(\mathbf{r})$, $\bar{v}_n(\mathbf{r})$ and

$\bar{Z}_n(\mathbf{r})$ prescribed values for the acoustic pressure, normal velocity and normal impedance and the normal direction $\mathbf{n}$ pointing out of the acoustic domain.

HYBRID WB-FE METHOD

The hybrid WB-(PM)FE method combines an acoustic FE model of the geometrically complex parts of the problem with an acoustic WB model of a large acoustic cavity enclosed in the actual problem geometry. This section describes the basic concepts of the WBM and the FEM for uncoupled acoustic problems. Furthermore, the applied hybrid coupling methodology is described.

FEM FOR STEADY-STATE INTERIOR ACOUSTICS -

The FEM is a well-known simulation technique to model the steady-state dynamic behavior of acoustic cavities. The technique determines an approximate solution to the problem described by the Helmholtz equation (1) and the imposed acoustic boundary conditions (2) by applying an approximation of type (3) in terms of simple (polynomial) shape functions $N_i(\mathbf{r}')$. These functions are locally defined within a discretisation of the problem

domain into a large, but finite number of small, non-overlapping elements.

$$p(\mathbf{r}') \approx \hat{p}_{fe}(\mathbf{r}') = \sum_{n=1}^{n_k} N_n(\mathbf{r}') p_{fe,n} = \mathbf{N} \cdot \mathbf{p}_{fe} \quad (3)$$

The polynomial shape functions do not fulfill the Helmholtz equation and they may violate the imposed Neumann or Robin boundary conditions. The approximation errors are minimized in an integral sense by application of a Galerkin weighted residual formulation [1]. This results in a set of algebraic equations of the form:

$$(\mathbf{K} + j\omega\mathbf{C} - \omega^2\mathbf{M}) \cdot \mathbf{p}_{fe} = \mathbf{Z} \cdot \mathbf{p}_{fe} = \mathbf{f}_{fe} \quad (4)$$

with $\mathbf{Z}$ the acoustic dynamic stiffness matrix, $\mathbf{p}_{fe}$ the vector containing the unknown nodal acoustic pressures and $\mathbf{f}_{fe}$ the acoustic loading vector. Solution of (4) yields the acoustic pressure values $\mathbf{p}_{fe}$ in the nodes of the FE discretization. The matrix $\mathbf{Z}$ is large, symmetric and sparsely populated and can be decomposed into frequency independent submatrices $(\mathbf{K}, \mathbf{C}, \mathbf{M})$. These properties allow the use of very efficient solution algorithms to compute the unknown nodal displacements. A major advantage of the FEM is its versatility regarding geometrical complexity of the problem domain.

WBM FOR UNCOUPLED ACOUSTIC PROBLEMS -

The WBM, which is based on an indirect Trefftz approach, partitions the entire problem domain V into a small number of large, convex subdomains. Within these subdomains, the dynamic acoustic pressure $p(\mathbf{r})$ is written as a weighted sum $\hat{p}_{wb}(\mathbf{r})$ of wave functions, which exactly satisfy the Helmholtz equation, but which may violate the imposed boundary conditions:

$$\begin{aligned} p(\mathbf{r}) \approx \hat{p}_{wb}(\mathbf{r}) &= \sum_{a=1}^{n_a} \Phi_a(\mathbf{r}) \cdot p_{wb,a} + \hat{p}_q(\mathbf{r}) \\ &= \Phi \cdot \mathbf{p}_{wb} + \hat{p}_q(\mathbf{r}) \end{aligned} \quad (5)$$

with $\mathbf{p}_{wb}$ a $(n_a \times 1)$ vector of unknown wave function contributions $p_{wb,a}$ and Φ a $(1 \times n_a)$ vector collecting the wave functions $\Phi_a(\mathbf{r})$. $\hat{p}_q(\mathbf{r})$ is a particular solution of the inhomogeneous Helmholtz equation. For a detailed description of these functions used to describe the pressure in a bounded acoustic domain, the reader is referred to [5].

Since the pressure expansion (5) exactly satisfies the governing equation, the only error consists of the violation of the imposed boundary conditions. In order to

obtain a numerical model for the wave function contributions p_{wb} , this error is minimized by applying a weighted residual formulation, yielding a system of n_a equations in the n_a unknowns:

$$\mathbf{A}_{ww} \cdot \mathbf{p}_{wb} = \mathbf{f}_{wb} \quad (6)$$

Solution of this system for the unknown wave function contributions $\mathbf{p}_{wb}$ and substitution of these results in (5) yields an approximation $\hat{p}(\mathbf{r})$ for the acoustic pressure response. The acoustic system matrix $\mathbf{A}_{ww}$ is fully populated with complex and frequency dependent elements. The major advantage of the WBM is the substantially smaller number of dofs required in comparison to the FEM. This property combined with the enhanced convergence properties of the method, make the WBM a computationally more efficient simulation technique than the FEM and allow the WBM to be used for structural-acoustic analysis in the mid-frequency range. The requirement of convexity of the wave based subdomains imposes, however, a limitation to the practical applicability of the method for complex geometries.

HYBRID COUPLING STRATEGY - Table 1 summarizes the characteristic properties of the FEM and the WBM [16]. This table shows that both techniques are complementary. It forms the basis for the development of the hybrid modeling techniques.

	FEM	WBM
Model size	large	small
Convergence rate	medium	high
Frequency range	low	low, mid, (high)
Geometrical complexity	high	moderate

Table 1: Model properties of the FEM and WBM

In many practical examples (e.g., acoustic modeling of a car interior), the acoustic domain contains large homogeneous subcavities which can be modeled using the WBM, but the entire problem domain is too complex to be partitioned into a limited number of convex WBM subdomains. As a result, the WBM's computational efficiency is not fully exploited. A FE model is far more suited for this kind of domain description. A FE discretization of the entire problem domain is possible but needs to be rather fine in order to control the associated modeling errors which increase rapidly with increasing characteristic dimensions of the FE submodel. This results for practical applications in a very large number of acoustic FE dofs and unfeasibly long calculation times.

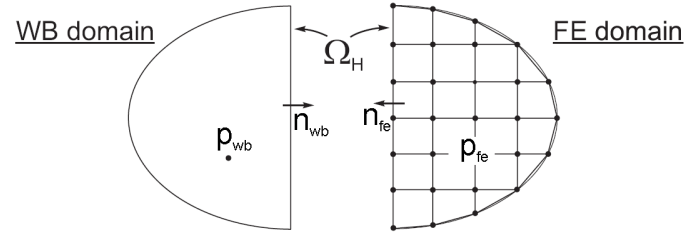


Figure 2: Hybrid WB-FE coupling approach

As proposed by Pluymers in [9], the acoustic pressure fields in the WB and FE submodels can be directly coupled in a hybrid WB-FE model by enforcing various types of coupling conditions along the hybrid interface Ω_H , see figure 2. In this work, the so-called *direct pressure and velocity* coupling strategy is adopted. That approach imposes the continuity of each of the two main acoustic variables, being the pressure and normal velocity, directly on the WB and FE submodel respectively.

Combination of the updated weighted residual formulations for both domains and application of a Galerkin approach yields the following matrix equation for the entire acoustic system in terms of the wave function contributions $\mathbf{p}_{wb}$ and the nodal acoustic pressure values $\mathbf{p}_{fe}$:

$$\begin{bmatrix} \mathbf{A}_{ww} + \mathbf{C}_{ww} & \mathbf{C}_{wf} \\ \mathbf{C}_{fw} & \mathbf{Z} \end{bmatrix} \begin{Bmatrix} \mathbf{p}_{wb} \\ \mathbf{p}_{fe} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_{wb} + \mathbf{f}_{fw} \\ \mathbf{f}_{fe} + \mathbf{f}_{wf} \end{Bmatrix} \quad (7)$$

With $\mathbf{Z}$, $\mathbf{A}_{ww}$, $\mathbf{f}_{fe}$ and $\mathbf{f}_{wb}$ the system matrices and loading vectors from the uncoupled acoustic FE and WB models. The WB-FE and FE-WB coupling matrices $\mathbf{C}_{wf}$ and $\mathbf{C}_{fw}$ describe the continuity conditions for the acoustic quantities and $\mathbf{C}_{ww}$ is the acoustic back-coupling matrix. The supplementary loading terms $\mathbf{f}_{wf}$ and $\mathbf{f}_{fw}$ result from the presence of an acoustic point source q in the WB submodel. When solving this system of equations, the rows and columns for the entire system of equations (7) cannot be reordered such that the sparse structure of the dynamic stiffness matrix $\mathbf{Z}$ can be exploited. To make optimal use of the properties of the submatrices Van Hal [16] and Pluymers [9] propose a three-step solution sequence. In a first step the uncoupled sparse FE model needs to be solved to eliminate the FE dofs from the model. Next, the solution of a condensed small and fully populated system delivers the wave function contributions. Finally the acoustic pressure in the FE submodel can be reconstructed by means of a matrix multiplication.