

The Application of the Vincent Circle to Vibro-Acoustic and Duct Acoustic Problems

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ABSTRACT

Over 30 years ago, A. H. Vincent of Westland Helicopters demonstrated that if a structure is excited harmonically, the response at another position (at a fixed frequency) will trace a circle in the complex plane as a result of a dynamic stiffness modification between two points. As either the real or imaginary part of an introduced dynamic stiffness is varied from minus infinity to plus infinity, the structural or acoustic response on any position will map a circle in the complex plane. This paper reviews the basis for this little known principle for vibro-acoustics problems and illustrates the viability for a cantilevered plate example. The applicability of the method is then considered for strictly acoustic systems like intake and exhaust systems. Specifically, it is shown that the response traces a circle in the complex plane if either the real or imaginary parts of the source or termination impedance are varied from minus to plus infinity.

INTRODUCTION

Noise can sometimes be mitigated by making a change to a structural energy path by adjusting mass, stiffness, damping, or some combination of the three. With the advent of high speed computing, numerical simulation is sometimes used to guide the design process. However, an over reliance on numerical simulation can mask important design insights that may be garnered easily using basic engineering principles. It is believed that basic principles like that discussed herein merit consideration.

The Vincent Circle principle can be applied to any linear structural and/or acoustic system excited harmonically. Tehrani et al [1] showed that the Vincent Circle Principle completely describes the response region when a structural modification is introduced between two points or between a single point and ground. That response might be structural or acoustic.

This paper will review the development of the Vincent Circle in light of the fact that it is unfamiliar to most in the NVH community. To the authors' knowledge, the usage of Vincent's Circle had previously been limited to vibration suppression problems [1-4]. This paper will demonstrate how the method can be extended to:

- Noise suppression whether the excitation is structure-borne or airborne.
- Multiple excitation problems.
- One dimensional structural and acoustic modifications.

THE VINCENT CIRCLE

Discovered by A. H. Vincent of Westland Helicopters in 1972 [2], it was apparently overlooked in the intervening years until a recent paper by Tehrani et al [1]. Vincent limited his scope to structures excited at a single point and assumed a stiffness modification between two positions on a structure. Tehrani et al [1] made an important contribution by discovering that the principle could be generalized to a dynamic stiffness modification in one dimension thereby incorporating mass and damping modifications.

The development of the method that follows is similar to that shown by Done and Hughes [3] over 30 years ago. Figure 1 shows a schematic of a structure with a modification (in this case a spring) between points r and s . The structure is excited at a point p and the response will be computed at a point q . Done and Hughes supposed that point q was on the structure and the response was a structural vibration. However, the derivation shown herein will assume a structural force at point p and an acoustic response at a point q . Whether the excitation or the response is structural or acoustic has little bearing on the applicability of the principle. It will be shown later that the modification need not be structural. Assuming acoustic plane waves, a modification to the admittance is equally valid. However,

the modification should be one-dimensional whether structural or acoustic.

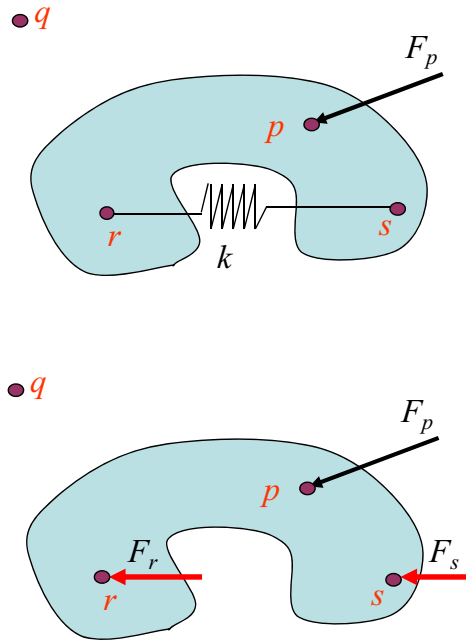


Figure 1 Schematic to illustrate the development of the Vincent Circle principle.

Assume that the spring is replaced by two forces F_r and F_s . In that case, the vibrational responses at points r and s , and the acoustic response at point q can be written in terms of the applied forces F_p , F_r , and F_s . Thus,

$$p_q = H_{qp} F_p + H_{qr} F_r + H_{qs} F_s \quad (1a)$$

$$x_r = H_{rp} F_p + H_{rr} F_r + H_{rs} F_s \quad (1b)$$

$$x_s = H_{sp} F_p + H_{sr} F_r + H_{ss} F_s \quad (1c)$$

where H_{ij} are the unmodified transfer functions between the vibration or acoustic responses at point i and the forces or inputs at point j .

Note that the forces F_r and F_s can be expressed in terms of the spring stiffness k and the displacement responses x_r and x_s as

$$F_r = k(x_s - x_r) = -F_s \quad (2)$$

and then substituted into Equation 1. This results in a set of 3 simultaneous equations with 3 unknown responses x_r , x_s , and p_q . Solving for the modified transfer function (p_q/F_p), the following expression is obtained

$$\frac{p_q}{F_p} = H_{qp} + \frac{k(H_{sp} - H_{rp})(H_{qr} - H_{qs})}{1 + k(H_{rr} + H_{ss} - H_{rs} - H_{sr})} \quad (3)$$

Tehrani et al [1] observed that Equation 3 is a particular case of the Moebius Transformation. It is recommended that the reader consult the aforementioned paper by Tehrani et al [1] for a more complete discussion of the Moebius Transformation.

What is most relevant to this discussion is that Equation 3 can be written in the form

$$a = b + \frac{Dc}{1 + Dd} \quad (4)$$

Where a , b , c , and d are complex numbers defined as

$$a = \frac{p_q}{F_p} \quad (5a)$$

$$b = H_{qp} \quad (5b)$$

$$c = (H_{sp} - H_{rp})(H_{qr} - H_{qs}) \quad (5c)$$

$$d = (H_{rr} + H_{ss} - H_{rs} - H_{sr}) \quad (5d)$$

And D is the modification (k in Equation 3). A characteristic of the Moebius Transformation is that the complex number a will trace a circle in the complex plane as a real or imaginary D varies from plus to minus infinity given that b , c , and d are complex numbers. For a real modification, Done and Hughes [3] showed that the radius ρ and center ξ of this circle can be expressed as

$$\rho = \left| \frac{c}{2 \operatorname{imag}(d)} \right| \quad (6)$$

and

$$\xi = b - \frac{jc\rho}{|c|} \quad (7)$$

respectively. If D is purely imaginary (i.e. a viscous damper modification), the radius ρ and center ξ of this circle can be expressed as

$$\rho = \left| \frac{c}{2 \operatorname{real}(d)} \right| \quad (8)$$

and

$$\xi = b + \frac{c\rho}{|c|} \quad (9)$$

respectively. As mentioned previously, Tehrani et al [1] noted that the modification D is a dynamic stiffness modification. Thus,

$$D = k - \omega^2 m + j\omega c_D \quad (10)$$

in which k , m and c_D are stiffness, mass and damping respectively.

The minimum value of a in Equation 4, which corresponds to the possible maximum suppression of the response, should be the point on the circle closest to the origin of the complex plane. Hence, the approach is ideal for selecting a passive control mechanism for minimizing noise at a particular frequency.

GENERALIZATION TO MULTIPLE INPUTS

The Vincent Circle can straightforwardly be extended to cases having multiple excitations in the following