

A Note on Equilibrium Turbulent Boundary Layer Wall Pressure Models

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ABSTRACT

Turbulent boundary layers are a significant source of vibration and noise for vehicles moving through a fluid medium. Describing the forcing function for this noise source is an active area of research. Empirical models are commonly used in system noise models. Two common models as discussed by Mellen [1,2] are separable and non-separable models. The separable models are in a class generally known as Corcos models [3]. The separable models postulate that the wall pressure space time statistics are a function of time times a function of downstream separation times a function of cross stream separation. The non-separable models postulate that the space time statistics are a function of time times a function of space with the downstream and cross stream separation being coupled. Two examples of the non-separable models are Chase [4,5] and Smol'yakov-Tkachenko [6].

These models were used to compare to existing data from the MIT Low-Noise/Low-Turbulence Wind Tunnel as reported by Martini [7]. Martini made an effort to make low wavenumber measurements using both microphone arrays and plate structures similar to Martin [8] and Jameson [9]. The performance of the separable and non-separable models are compared and contrasted.

INTRODUCTION

The turbulent boundary layer, TBL that forms around a vehicle as it moves through a fluid medium is a significant source of noise. For automobiles, this noise manifests itself at high speeds as wind noise. For aircraft at cruise, the turbulent boundary layer noise can be a

significant if not dominant contributor to the aircraft interior noise. For marine craft, the turbulent boundary layer noise can limit the performance of their passive sonar. As a result, the turbulent boundary layer wall pressure continues to be of interest.

It is necessary to have an accurate model of the TBL wall pressure fluctuations to predict the vehicles vibro-acoustic response. Cockburn and Robertson [10] predicted the response of launch vehicles to TBL excitation. Bremner and Wilby [11], Davis [12], and Dandaroy and Bhuiyan [13] all used empirical models to make aircraft interior noise predictions. Most of the TBL empirical models in common use are in the separable class. Davis [12] uses a model developed by Efimtsov [14] that is an extension to the separable class that takes the boundary layer thickness into account for the wall pressure statistics.

The TBL wall pressure literature has been reviewed by Bull [15] in 1996 and by Graham [16] in 1997. There continues to be active research on the boundary layer wall pressure.

Because of the complexity of the true situation and the difficulty of fully resolving the wavenumber frequency wall pressure spectrum, empirical models have been developed to make response predictions. The first empirical model was developed by Corcos [3]. His hypothesis was that the space time statistics were fully separable and could be described by a function of frequency times a function of downstream separation times a function of cross stream separation. These classes of separable models have been the most commonly adopted empirical models in the literature.

The first non-separable model postulated was one by Chase [5]. More recently Smol'yakov-Tkachenko [6] postulated another non-separable model. These models are starting to get more attention in the literature including work by Finnveden et al. [17] and Borisyuk and Grinchenko [18].

There have been several attempts to characterize the full wavenumber frequency spectrum of the TBL wall pressure. Extensive measurements have been done using both microphone arrays and mechanical structures as wavenumber filters. Blake and Chase [19], Farabee and Geib [20], Abraham and Keith [21], and Manoha [22,23] used one dimensional microphone arrays to measure the streamwise wavenumber pressure distributions. Arguillat et al. [24] used a line array that was rotated to characterize the full wavenumber frequency spectrum. Ko [25] used two 11 by 11 arrays to make wavenumber frequency measurements. Jameson [9] and Martin [8] used plates as filters to make wall pressure measurements off the convective peak. Martini [7] made plate and line array measurements in the same facility.

TBL WAVENUMBER FREQUENCY SPECTRA

The initial efforts to describe the TBL wall pressure statistics were conducted in the space time domain. These were carried out with either a single microphone or pairs of microphones and a tape recorder. The autocorrelation function and the cross correlation function were estimated from the measurements.

$$R(\xi_1, \xi_2, \tau) = \langle p(x, y, t)p(x + \xi_1, y + \xi_2, t + \tau) \rangle \quad (1)$$

It is difficult to resolve the complex structure of the TBL wall pressure in the space time domain as there is a rich scale of motion and a large difference in the energy between the gross and fine scales. Therefore, a wavenumber frequency domain approach was adopted to better capture the structure. The space time statistics are Fourier transformed to the wavenumber frequency domain.

$$\Phi_{pp}(k_x, k_y, \omega) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R(\xi_1, \xi_2, \tau) e^{-i(k_x \xi_1 + k_y \xi_2 + \omega \tau)} d\xi_1 d\xi_2 d\tau \quad (2)$$

This function describes the wavenumber distribution of the energy at each frequency. If the energy at all wavenumbers is summed, the result is the single point wall pressure spectrum. The single point wall pressure spectrum has been of considerable interest in its own right.

$$\Phi(\omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_{pp}(k_x, k_y, \omega) dk_x dk_y \quad (3)$$

Initially the models were used to discuss the ability to resolve the turbulence with a finite area microphone. Lueptow [26] used a wavenumber frequency approach to the measurement problem. Microphones are essentially wavenumber filters. He examined two types of sensing elements in his analysis. The first type has a uniform sensing area. The second is a displacement type transducer, such as a condenser microphone, where the edges of the microphone are insensitive to the response and the center is most sensitive.

The wavenumber filter shape for a microphone of radius, r_0 with uniform sensitivity is:

$$H(\mathbf{k}) = \frac{2J_1(kr_0)}{kr_0} \quad (4)$$

where

$$k = \sqrt{k_x^2 + k_y^2} \quad (5)$$

J_1 = Bessel function of order one

The wavenumber filter shape for a circular deflection microphone of radius r_0 is modeled by the first mode of vibration of a membrane and is given by:

$$H(\mathbf{k}) = \frac{a^2 J_0(kr_0)}{a^2 - (kr_0)^2} \quad (6)$$

where

$$a = 2.40482556043883$$

J_0 = Bessel function of order zero

The measured microphone responses are then the TBL wall pressure spectrum times the wavenumber filter shape integrated over all wavenumber.

$$\Phi_M(\omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_{pp}(k_x, k_y, \omega) |H(k_x, k_y)|^2 dk_x dk_y \quad (7)$$

In the next section we will use various empirical models to predict the response of microphones and microphone arrays and compare the predictions to measured results. The empirical models used in this study (Corcos, Chase type 1 and Smol'yakov-Tkachenko) were models summarized by Graham [16]. The Mellen elliptical model is from reference [1]. There was no effort expended to fit the models to the Martini [7] dataset. The details of these models are summarized in Appendix A.

EMPIRICAL TBL MODELS

The empirical models that we considered in this paper are the Corcos model, Mellen elliptical model, Chase type I model, the Smol'yakov-Tkachenko model. Following Graham [16] and Liu et al. [27], we work with the normalized wavenumber frequency spectra, $\tilde{\Phi}_{pp}(k_x, k_y, \omega)$.

$$\tilde{\Phi}_{pp}(k_x, k_y, \omega) = \frac{\omega^2}{U_c^2 \Phi(\omega)} \Phi_{pp}(k_x, k_y, \omega) \quad (8)$$

For this analysis, we used the Goody model [28] of the single point wall pressure spectrum, $\Phi(\omega)$. The first check is to require that when integrated over all wavenumbers, the normalized wavenumber frequency spectrum integrates as follows:

$$\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{\Phi}_{pp}(k_x, k_y, \omega) \left(\frac{U_c}{\omega} \right)^2 dk_x dk_y = 1 \quad (9)$$

The models are detailed in Appendix A and a surface and contour plot are shown for each model.

Mellen [1] pointed out that the separable Corcos model resulted in an anomalous behavior because of the overly restricted model. The features that he postulates as non-representative are the 'shoulders' on the wavenumber frequency spectrum. A surface plot of Mellen's elliptical and a Corcos model is given in Figure 1. The shoulders are readily apparent. The Corcos model is partially transparent for clarity in Figure 1.

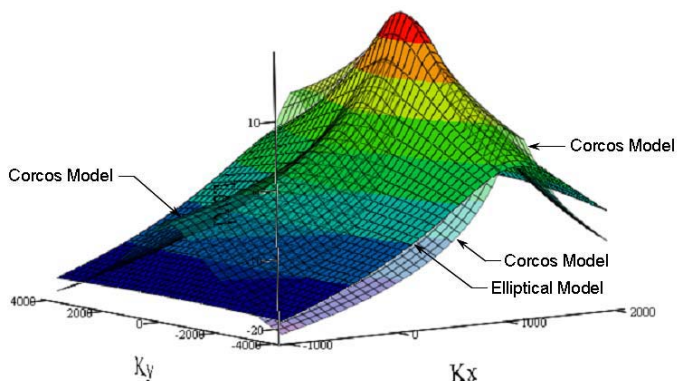


Figure 1. Surface plot for Mellen elliptical and Corcos models using Martini 15 m/s boundary layer at 3000 Hz.

SINGLE ELEMENT RESPONSE

The one inch B&K flush mounted microphone was used to compare to the wavenumber filtered single point wall pressure spectrum. It is sufficient to look at the single one inch microphone response to make a judgment on

the performance of the empirical TBL wall pressure models considered here. The measurements used for comparison purposes were carried out in the former MIT Low-Noise/Low-Turbulence Wind Tunnel in the MIT Acoustics and Vibration Laboratory.

The one inch microphone was similar to the one characterized in Farabee and Geib [20]. Both Martini [7] and Farabee and Geib [20] had special caps manufactured that allowed the one inch microphones to be flush mounted. Farabee and Geib [20] had the displacement shape measured and calculated the wavenumber filter shape from the measured displacement response. Current technology is available to measure the response using a laser vibrometer while driving the diaphragm electrically, but this was not available when these measurements were undertaken. For the purposes of this paper, the one inch B&K microphones were characterized by ideal displacement transducers where the sensitivity is not uniform across the surface.

The wind tunnel was an open circuit wind tunnel and the boundary layer was allowed to form naturally on the bottom wall of the wind tunnel. The wind tunnel details can be found in Martini [7] or Martin [8]. The test section was enclosed in a concrete blockhouse. The blockhouse walls were treated with layers of foam to make the space semi-anechoic. The tests described here were conducted at 15 m/s in the open jet configuration. The background noise level in the blockhouse was determined using B&K 4144 free standing microphones. The background noise was found to be comparable to those measured by Martin [8] for the same test configuration. The acoustic levels at the wavenumber filter location were more than 10 dB louder than the blockhouse background noise levels. The measurements used here and the boundary layer characteristics are summarized in Appendix A.

There is ongoing discussion in the literature about the effectiveness of pinhole microphone measurements. This discussion is summarized in Bull [15]. Lueptow [26] used the ratio of the transducer diameter, d , to the displacement thickness, δ^* as a measure of characteristic length scale. In this case, the pinhole measurement 3 dB down point is about 5000 Hz with $d/\delta^* = 0.145$. On the other hand, the one inch B&K flush mounted microphones had a $d/\delta^* = 3.44$ and exhibited significant wavenumber filtering. The wavenumber filter shape for the one inch B&K flush mounted microphone is shown in Figure 2.

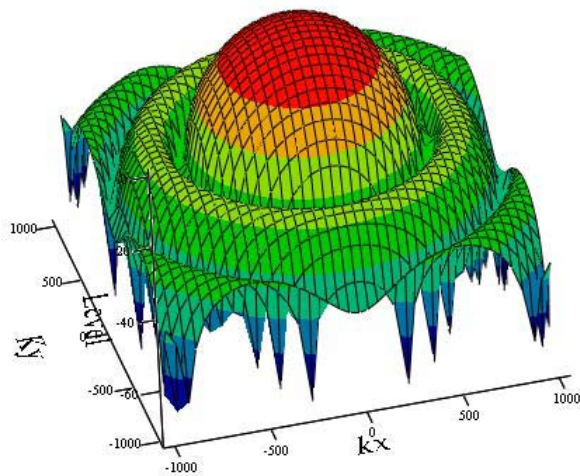


Figure 2. Wavenumber filter shape of a one inch B&K flush mounted microphone modeled as a 0.36 inch radius displacement transducer.

A pinhole microphone was used to characterize the single point wall pressure spectrum. The pinhole microphone was a 1/8 inch microphone with a 1/32 inch pinhole cap. The pinhole microphone was assumed to be uniformly responding. For analysis purposes, the Goody model [28] was selected as the baseline single point wall pressure spectrum. The Goody estimate and the Martini data are shown in Figure 3. The Martini pinhole data was not corrected for area averaging. It should be noted that the Goody estimate is somewhat higher than the Martini data. The source of this bias is unknown.

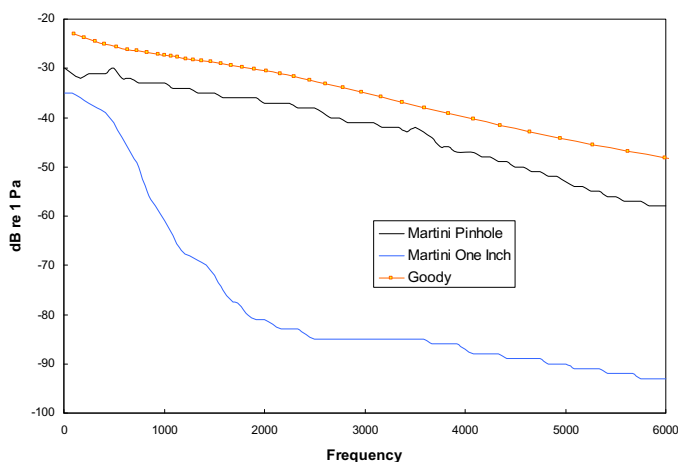


Figure 3. Comparison of the Martini measurement of the one inch B&K flush mounted microphone and the pinhole microphone to the Goody model estimate.

A bias can be seen in Figure 3 between the pinhole microphone measurement in the experiment and the Goody [28] estimate used in the analysis. To remove this bias, the following discussion will examine the

difference between the pinhole measurement and the one inch B&K flush mounted microphone measurement. Therefore, the computed model values are the difference between the one inch B&K flush mounted microphone and the pinhole microphone.

The results for the Chase and Corcos model are shown in Figure 4. The Corcos model under predicts the response difference for this simple measurement which is due to the shoulders of the Corcos model. The Corcos model is not compatible with the Martini data set. Above the 2000 Hz frequency, the microphone response is due to acoustic noise and it is no longer dominated by the TBL wall pressure. The measurement of the wind tunnel background noise is discussed in limited detail in Appendix B.

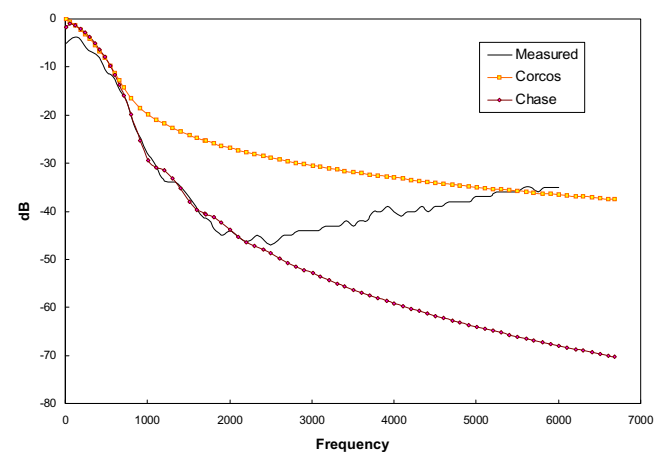


Figure 4. Difference between predicted one inch B&K flush mounted microphone and predicted pinhole microphone response for Chase and Corcos models compared to Martini measurements.

A surface plot of the Corcos model and the microphone wavenumber filter shape at 2000 Hz is shown in Figure 5. The response is dominated by the low wavenumber content of the TBL model. The microphone is a low pass filter in k_x and k_y . In the k_y dimension, it is centered right on the shoulder that Mellen postulates is an artifact of the model. The fact that the Corcos model significantly over predicts the response at this frequency supports Mellen's assertion. This over prediction results in the under prediction of the difference in response shown in Figure 4.