

Comparative Linear Analysis of Alternative Layouts of Heavy Goods Vehicles

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ABSTRACT

This paper presents comparative analyses of steering wheel responses of various layouts for heavy goods vehicles, including rigid and articulated configurations. Newton and Lagrange techniques have been adopted to formulate and verify the generalized linear model for multi-axle rigid and articulated vehicles. The model is then used to simulate, analyze and compare steering angle response of a variety of commercially available vehicles. The study includes the analyses of steady state response and dynamic behavior for different layouts in terms of axle positions, number of axles and multiple steered axles.

INTRODUCTION

Many linear analyses of articulated vehicles have been produced and published in terms of vehicle response to steering wheel input [1], [2], [3], [5] and [8]. However, the literature is still missing a comprehensive dynamic response comparison of the advantages and disadvantages of the different vehicle layouts, in terms of number of axles and their position. Moreover, the linear analyses presented in previously published articles do not consider the dynamics induced by the relaxation length of the tires, which, according to the data related to common tires, can be very significant. The first aim of this paper is to produce a comparison based on analyses of different configurations of rigid and articulated vehicles. This comparison can be used to assess their handling in terms of steady-state and dynamic response, stability and overall safety.

The first part of the paper deals with the formulation of the generalized set of equations of motion. Newton force

and moment balance method [10], [12] has been employed to derive a generic Bicycle Model for both rigid and articulated layouts. The second approach is to develop the model equations by using Lagrangian formulation [4], [10]. Both of the above techniques are used in order to derive three degree of freedom (DOF) models (two in the case of rigid vehicles) i.e. lateral displacement, yaw motion of the tractor, relative angular motion between the tractor and the trailer. These models have been subjected to linearization for the frequency response analysis. Linearized models obtained from each of the two methods (Newton and Lagrange) reveal identical sets of equations.

Eight different layouts of heavy goods commercial vehicles, including rigid and articulated configurations, are addressed and analyzed from the viewpoint of their properties and purpose. The vehicle data used in the research refer to common production trucks and related tires. The procedure for the linearization of tire data is dealt with, as the variation of tire properties as functions of their vertical load is the most influential parameter.

The second part of the article presents results, comparisons and conclusions. The steady-state response plots showing vehicle performance in terms of gains (lateral velocity gain, yaw rate gain, etc...) vs. vehicle longitudinal speed are discussed. The implemented model is an extension of the Milliken model [10] based on stability derivatives. The final section of the paper presents the Bode diagrams of the response of the vehicles subjected to steering angle inputs. These diagrams are commonly used for the evaluation of vehicle response and the design of control systems for cars [14], [15]. Sensitivity analyses are carried out for the main parameters. In particular, the performance of the alternative vehicle layouts which are usually

characterized by similar values of Gross Vehicle Weight (GVW) is discussed and compared in detail.

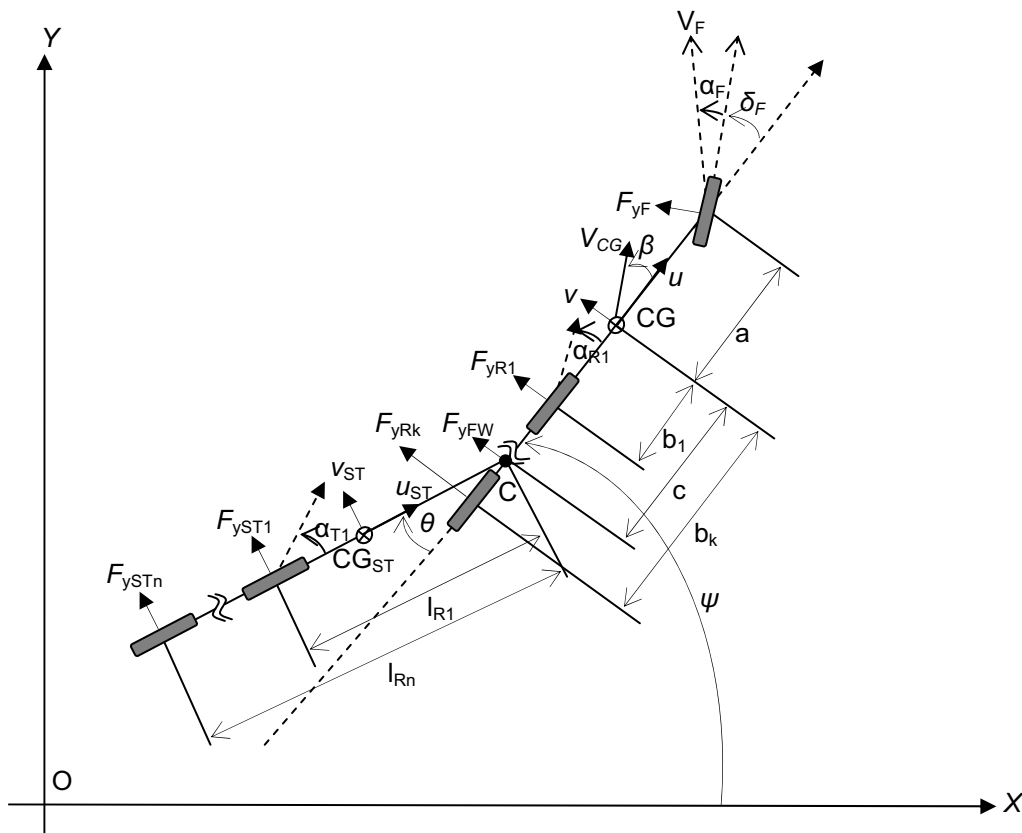


Figure 1: Schematic of the Bicycle Model for multi-axle articulated

ARTICULATED VEHICLE MODELS

The modeled system is depicted in Figure 1. This generic articulated vehicle model is an extension of Milliken's Bicycle Model for rigid vehicles [10] and also an improvement of Genta's articulated vehicle model [9] as it has an additional provision for a multi-axle tractor and trailer. To analyze dynamic and steady state responses of a motor vehicle, a three DOF model is adopted. In Figure 1, two planar rigid vehicles are hinged with each other with a cylindrical articulated link perpendicular to the road at point C. The longitudinal and lateral velocities of the tractor (in its reference system) are represented by u and v respectively and the yaw rate of the vehicle is denoted by $\dot{\psi}$. θ is the hitch/towing angle between tractor and trailer, and it is positive in clockwise direction. u_{ST} is the longitudinal velocity of the trailer/semi-trailer, v_{ST} refers to its lateral velocity (in the reference system of the trailer/semi-trailer). The absolute yaw rate of the towed vehicle is defined as $\dot{\psi} - \dot{\theta}$. The front and the rear axles of the truck/tractor are considered to be steered and the steering angle for the steered tires is denoted by δ . All

the angles defined in the figure are positive. All lateral forces F_y are defined in the reference system of the tractor and all lateral forces F_{yST} are defined in the reference system of the trailer/semi-trailer.

NEWTON FORCE AND MOMENT BALANCE METHOD

In this approach lateral force and moment balance equations for the two rigid bodies are written, and additional equations are defined to express the towed vehicle co-ordinates in terms of truck/tractor co-ordinates according to the constraint represented by the articulated link.

Equations (1) and (2) are tractor force and moment balances:

$$m_T(\dot{v} + u\dot{\psi}) = F_{yF} + F_{yR1} + \dots + F_{yRk} - F_{yFW} \quad (1)$$

$$J_{Tz}\ddot{\psi} = aF_{yF} - b_1F_{yR1} - \dots - b_kF_{yRk} + cF_{yFW} \quad (2)$$

Where k is the total number of rear axles of the truck/tractor.

The equations for trailer force and moment balance are below:

$$m_R (\dot{v}_{ST} + u_{ST} (\dot{\psi} - \dot{\theta})) = F_{yST1} + \dots + F_{ySTn} + F_{yFW} \quad (3)$$

$$J_{Rz} (\ddot{\psi} - \ddot{\theta}) = a_R F_{yFW} - (l_{R1} - a_R) F_{yST1} - \dots - (l_{Rn} - a_R) F_{ySTn} \quad (4)$$

Where n is the total number of axles of the semi-trailer.

Trailer longitudinal and lateral velocities may be expressed in terms of tractor coordinates (velocities experienced at CG_{ST} , the center of gravity of the trailer).

$$m_T (\dot{v} + u\dot{\psi}) + m_R (\dot{v} + u\dot{\psi}) \cos(\theta) - m_R (a_R + c \cos(\theta)) \ddot{\psi} + m_R a_R \ddot{\theta} - m_R (v\dot{\psi} + c\dot{\psi}^2) \sin(\theta) = F_{yF} + F_{yR1} + \dots + F_{yRk} + F_{yST1} + \dots + F_{ySTn} \quad (8)$$

$$(J_{Tz} + J_{Rz} + m_R a_R^2 + m_R c^2 \cos(\theta) + 2m_R a_R c) \ddot{\psi} - m_R (a_R + c) \dot{v} - (J_{Rz} + m_R a_R^2 + m_R a_R c) \ddot{\theta} - m_R (a_R + c \cos(\theta)) u \dot{\psi} - m_R c \sin(\theta) v \dot{\psi} - m_R c^2 \dot{\psi}^2 \sin(\theta) = a F_{yF} - b_1 F_{yR1} - b_2 F_{yR2} - \dots - b_k F_{yRk} - (c + l_{R1}) F_{yST1} - \dots - (c + l_{Rk}) F_{ySTk} \quad (9)$$

$$m_R a_R (v + u\dot{\psi}) - (J_{Rz} + m_R a_R^2 + m_R a_R c) \ddot{\psi} + (J_{Rz} + m_R a_R^2) \ddot{\theta} = l_{R1} F_{yST1} + l_{R2} F_{yST2} + \dots + l_{Rk} F_{ySTn} \quad (10)$$

LAGRANGE METHOD

In Lagrange method, equations of motion for the various degrees of freedom are obtained by using the total kinetic energy (i.e. translational and rotational) of the articulated vehicle [9]. The hitch constraint can be easily introduced by defining the coordinates of the towed vehicle in terms of towing vehicle (truck/tractor). $\dot{x}$ and $\dot{y}$ can be expressed in terms of vehicle coordinates through equation (51) in the Appendix and the velocities of CG_{ST} are functions of the velocities of CG and $\dot{\theta}$. (equation (52) in the Appendix).

The total kinetic energy of the system is then represented as:

$$T = \frac{1}{2} m_T V_{CG}^2 + \frac{1}{2} m_R V_{CG_{ST}}^2 + \frac{1}{2} J_{Tz} \dot{\psi}^2 + \frac{1}{2} J_{Rz} (\dot{\psi} - \dot{\theta})^2 \quad (11)$$

By substituting the expressions for the velocities from equations (51) and (52) into equation (11), the total kinetic energy of the system is:

$$T = \frac{1}{2} m (u^2 + v^2) + \frac{1}{2} J_1 (\dot{\psi})^2 + \frac{1}{2} J_3 \dot{\theta}^2 - J_2 (\dot{\psi}) \dot{\theta} + m_R v [c\dot{\psi} + a_R (\dot{\psi} - \dot{\theta}) \cos(\theta)] - m_R a_R u (\dot{\psi} - \dot{\theta}) \sin(\theta) \quad (12)$$

Equations (5) and (6) define the articulated link between tractor and trailer/semi-trailer:

$$u_{ST} = u \cos(\theta) - (v - c\dot{\psi}) \sin(\theta) \quad (5)$$

$$v_{ST} = u \sin(\theta) - (c\dot{\psi} - v) \cos(\theta) - a_R (\dot{\psi} - \dot{\theta}) \quad (6)$$

$$\dot{v}_{ST} = u \dot{\theta} \cos(\theta) + \dot{u} \sin(\theta) - [a_R + c \cos(\theta)] \ddot{\psi} + a_R \ddot{\theta} - \dot{v} \cos(\theta) - v \dot{\theta} \sin(\theta) + c \dot{\theta} \dot{\psi} \sin(\theta) \quad (7)$$

Incorporating equation (5) and (7) into (3) and combining equations (1),(2),(3) and (4) leads to obtain the three DOF model as the longitudinal dynamics are not considered (i.e. vehicle moving with constant longitudinal velocity) [9].

Where:

$$\begin{aligned} m &= m_T + m_R \\ J_1(\theta) &= J_{Tz} + J_{Rz} + m_R [a_R^2 + c^2 + 2a_R c \cos(\theta)] \\ J_2(\theta) &= J_{Rz} + m_R [a_R^2 + a_R c \cos(\theta)] \\ J_3 &= J_{Rz} + m_R a_R^2 \end{aligned} \quad (13)$$

The general equation of motion regarding the q_i degree of freedom obtained in terms of Lagrange equations is:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial F}{\partial \dot{q}_i} = Q_i \quad (14)$$

The total kinetic energy expression (12) is subjected to Lagrange equation (14) with reference to each of the $\{v, \psi, \theta\}$ co-ordinates of the articulated vehicle model shown above. Finally the three nonlinear equations of motion (Q_i are forces or moments) are obtained:

$$\begin{aligned} m (\dot{v} + u\dot{\psi}) - m_R [a_R + c \cos(\theta)] \ddot{\psi} + m_R a_R \ddot{\theta} \cos(\theta) - m_R a_R (\dot{\psi} + \dot{\theta})^2 \sin(\theta) &= Q_y \end{aligned} \quad (15)$$

$$J_1(\theta)\ddot{\psi} - J_2(\theta)\ddot{\theta} - m_R[a_R + c\cos(\theta)](\dot{v} + u\dot{\psi}) - m_R a_R (\dot{u} - v\dot{\psi})\sin(\theta) + m_R a_R c(\dot{\theta}^2 - 2\dot{\theta}\dot{\psi})\sin(\theta) = Q_\psi \quad (16)$$

$$J_3\ddot{\theta} - J_2(\theta)\ddot{\psi} + m_R a_R (\dot{v} + u\dot{\psi})\cos(\theta) - m_R a_R [\dot{u} - \dot{\psi}(v - c\dot{\psi})]\sin(\theta) = Q_\theta \quad (17)$$

Generalized forces and moments

Generalized forces and moments due to tire road interaction are computed using virtual work displacement method [9]. For this particular activity the contribution of aerodynamic and external perturbation forces are not taken into account. The generalized forces based on virtual works and moments regarding each of the three generalized co-ordinates are written below:

$$Q_y = F_{x_F} \sin(\delta_F) + F_{y_F} \cos(\delta_F) + \sum_{i=1}^k F_{y_{Ri}} + \sum_{i=1}^n [F_{x_{STi}} \sin(\theta) + F_{y_{STi}} \cos(\theta)] \quad (18)$$

$$Q_\psi = F_{x_F} a \sin(\delta_F) + F_{y_F} a \cos(\delta_F) - \sum_{i=1}^k b_i F_{y_{Ri}} - \sum_{i=1}^n F_{y_{STi}} [l_{Ri} + c \cos(\theta)] \quad (19)$$

$$Q_\theta = \sum_{i=1}^n l_{Ri} F_{y_{STi}} \quad (20)$$

By combining forces and moment equations from sets (15), (16), (17) and (18), (19), (20), the complete set of model equations is obtained:

$$m(\dot{v} + u\dot{\psi}) - m_R[a_R + c\cos(\theta)]\ddot{\psi} + m_R a_R \ddot{\theta} \cos(\theta) - m_R a_R (\dot{\psi} + \dot{\theta})^2 \sin(\theta) = F_{y_F} + F_{y_{R1}} + \dots + F_{y_{Rk}} + F_{y_{ST1}} + \dots + F_{y_{STn}} \quad (21)$$

$$J_1(\theta)\ddot{\psi} - J_2(\theta)\ddot{\theta} - m_R[a_R + c\cos(\theta)](\dot{v} + u\dot{\psi}) - m_R a_R (\dot{u} - v\dot{\psi})\sin(\theta) + m_R a_R c(\dot{\theta}^2 - 2\dot{\theta}\dot{\psi})\sin(\theta) = aF_{y_F} - b_1 F_{y_{R1}} - \dots - b_k F_{y_{Rk}} - (c + l_{R1})F_{y_{ST1}} - \dots - (c + l_{Rk})F_{y_{STk}} \quad (22)$$

$$J_3\ddot{\theta} - J_2(\theta)\ddot{\psi} + m_R a_R (\dot{v} + u\dot{\psi})\cos(\theta) - m_R a_R [\dot{u} - \dot{\psi}(v - c\dot{\psi})]\sin(\theta) = l_{R1} F_{y_{ST1}} + l_{R2} F_{y_{ST2}} + \dots + l_{Rk} F_{y_{STk}} \quad (23)$$

LINEARIZATION OF THE EQUATIONS OF MOTION

The non-linearities present in each of the two sets of three equations consist of trigonometric functions and products of coordinates (state variables). For normal handling conditions θ and δ_F are found to be small enough to perform the linearization of their trigonometric functions i.e. $\cos(\theta)=1$ and $\sin(\theta)=0$. Moreover, the longitudinal velocity of the vehicle is considered as a constant, i.e.

$$u = U \quad (24)$$

The linearized equations from either set of (8), (9), (10) or (21), (22), (23) are obtained:

$$m(\dot{v} + U\dot{\psi}) - m_R(a_R + c)\ddot{\psi} + m_R a_R \ddot{\theta} = F_{y_F} + F_{y_{R1}} + \dots + F_{y_{Rk}} + F_{y_{ST1}} + \dots + F_{y_{STk}} \quad (25)$$

$$J_1\ddot{\psi} - J_2\ddot{\theta} - m_R(a_R + c)(\dot{v} - U\dot{\psi}) = aF_{y_F} - b_1 F_{y_{R1}} - b_2 F_{y_{R2}} - \dots - b_k F_{y_{Rk}} - (c + l_{R1})F_{y_{ST1}} - \dots - (c + l_{Rn})F_{y_{STk}} \quad (26)$$

$$J_3\ddot{\theta} - J_2\ddot{\psi} + m_R a_R (v + U\dot{\psi}) = l_{R1} F_{y_{ST1}} + l_{R2} F_{y_{ST2}} + \dots + l_{Rk} F_{y_{STn}} \quad (27)$$

where

$$\begin{aligned} m &= m_T + m_R \\ J_1 &= J_{Tz} + J_{Rz} + m_R(a_R^2 + c^2 + 2a_R c) \\ J_2 &= J_{Rz} + m_R(a_R^2 + a_R c) \\ J_3 &= J_{Rz} + m_R a_R^2 \end{aligned} \quad (28)$$

Sideslip Angles

From the geometry of the Bicycle Model presented in Figure 1, sideslip angles are defined as below:

$$\alpha_F = \tan^{-1}\left(\frac{v + \dot{\psi}a}{U}\right) - \delta_F \quad (29)$$

$$\alpha_{Ri} = \tan^{-1}\left(\frac{v - \dot{\psi}b_i}{U}\right) - \delta_{Ri} \quad \text{where } i=1,2,\dots,k \quad (30)$$

$$\alpha_{Ti} = \tan^{-1}\left[\frac{U\sin(\theta) + v\cos(\theta) - c\dot{\psi}\cos(\theta) - l_{Ri}(\dot{\psi} - \dot{\theta})}{U\cos(\theta) + v\sin(\theta) - c\dot{\psi}\sin(\theta)}\right] \quad (31)$$

where $i=1,2,\dots,n$

The former equations can be linearized as:

$$\alpha_F = \frac{v + \dot{\psi}a}{U} - \delta_F \quad (32)$$

$$\alpha_{Ti} = \theta + \frac{1}{U} [v - (c + l_{Ri})\dot{\psi} + l_{Ri}\dot{\theta}] \quad \text{where } i=1,2,\dots,n \quad (34)$$

$$\alpha_{Ri} = \frac{v - \dot{\psi}b_i}{U} - \delta_{Ri} \quad \text{where } i=1,2,\dots,k \quad (33)$$

Final Equations

By adopting a linear tire model the final linear set of equations of motions is achieved:

$$m(\dot{v} + U\dot{\psi}) - m_R(a_R + c)\ddot{\psi} + m_R a_R \ddot{\theta} = \left(\frac{v + \dot{\psi}a}{U} - \delta_F \right) C_{Stiff_F} + \frac{v - \dot{\psi}b_1}{U} C_{Stiff_{R1}} + \dots + \frac{v - \dot{\psi}b_k}{U} C_{Stiff_{Rk}} \\ \left(\frac{v - c\dot{\psi} - l_{R1}(\dot{\psi} - \dot{\theta})}{U} + \theta \right) C_{Stiff_{ST1}} + \dots + \left(\frac{v - c\dot{\psi} - l_{Rn}(\dot{\psi} - \dot{\theta})}{U} + \theta \right) C_{Stiff_{STn}} \quad (35)$$

$$J_1\ddot{\psi} - J_2\ddot{\theta} - m_R(a_R + c)(\dot{v} + U\dot{\psi}) = \left(\frac{v + \dot{\psi}a}{U} - \delta_F \right) a C_{Stiff_F} - \frac{v - \dot{\psi}b_1}{U} b_1 C_{Stiff_{R1}} \dots \\ - \frac{v - \dot{\psi}b_k}{U} b_k C_{Stiff_{Rk}} - \left(\frac{v - c\dot{\psi} - l_{R1}(\dot{\psi} - \dot{\theta})}{U} + \theta \right) (l_{R1} + c) C_{Stiff_{ST1}} \dots - \left(\frac{v - c\dot{\psi} - l_{Rn}(\dot{\psi} - \dot{\theta})}{U} + \theta \right) (l_{Rn} + c) C_{Stiff_{STn}} \quad (36)$$

$$J_3\ddot{\theta} - J_2\ddot{\psi} + m_R a_R (\dot{v} + U\dot{\psi}) = \left(\frac{v - c\dot{\psi} - l_{R1}(\dot{\psi} - \dot{\theta})}{U} + \theta \right) l_{R1} C_{Stiff_{ST1}} \dots + \left(\frac{v - c\dot{\psi} - l_{Rn}(\dot{\psi} - \dot{\theta})}{U} + \theta \right) l_{Rn} C_{Stiff_{STn}} \quad (37)$$

Where C_{Stiff} is the equivalent cornering stiffness for each axle.

$$J_1\ddot{\psi} - J_2\ddot{\theta} - m_R(a_R + c)\dot{v} = Q_{\dot{\psi}v}v + Q_{\dot{\psi}\dot{\psi}}\dot{\psi} + Q_{\dot{\psi}\dot{\theta}}\dot{\theta} \\ + Q_{\dot{\psi}\theta}\theta + Q_{\dot{\psi}\delta_F}\delta_F + m_R(a_R + c)U\dot{\psi} \quad (39)$$

MODEL IMPLEMENTATION

The model presented in the previous section is implemented in state space form using the stability derivative notation [10].

“The derivatives are slope of force/moment curves against motion variable β , ψ and δ . They relate the forces and moments to small excursions of these variables.” [10].

$$m\dot{v} - m_R(a_R + c)\ddot{\psi} + m_R a_R \ddot{\theta} - \\ = Q_{y_v}v + Q_{y_\psi}\dot{\psi} + Q_{y_\theta}\dot{\theta} + Q_{y_\delta}\delta_F - mU\dot{\psi} \quad (38)$$

$$J_3\ddot{\theta} - J_2\ddot{\psi} + m_R a_R \dot{v} = Q_{\dot{\theta}v}v + \\ Q_{\dot{\theta}\dot{\psi}}\dot{\psi} + Q_{\dot{\theta}\dot{\theta}}\dot{\theta} + Q_{\dot{\theta}\delta_F}\delta_F - a_R m_R U\dot{\psi} \quad (40)$$

The stability derivatives Q_s are mentioned in the Appendix. Model equations (38), (39) and (40) may be represented in matrix form:

$$\begin{bmatrix} m & -m_R(a_R + c) & m_R a_R & 0 \\ -m_R(a_R + c) & J_1 & -J_2 & 0 \\ m_R a_R & -J_2 & J_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{v} \\ \ddot{\psi} \\ \ddot{\theta} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} Q_{y_v} & Q_{y_\psi} - mU & Q_{y_\theta} & Q_{y_\delta} \\ Q_{\dot{\psi}v} & Q_{\dot{\psi}\dot{\psi}} + m_R(a_R + c)U & Q_{\dot{\psi}\dot{\theta}} & Q_{\dot{\psi}\delta_F} \\ Q_{\dot{\theta}v} & Q_{\dot{\theta}\dot{\psi}} - m_R a_R U & Q_{\dot{\theta}\dot{\theta}} & Q_{\dot{\theta}\delta_F} \\ Q_{\dot{\theta}v} & Q_{\dot{\theta}\dot{\psi}} & Q_{\dot{\theta}\dot{\theta}} & Q_{\dot{\theta}\delta_F} \end{bmatrix} \begin{bmatrix} v \\ \dot{\psi} \\ \dot{\theta} \\ \theta \end{bmatrix} + \begin{bmatrix} Q_{y_{\delta_F}} \\ Q_{\dot{\psi}\delta_F} \\ Q_{\dot{\theta}\delta_F} \\ Q_{\dot{\theta}\delta_F} \end{bmatrix} \delta_F \quad (41)$$