

A Fuzzy Based Stability Index Using a Right Sigmoid Membership Function

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ABSTRACT

The increasing use and implementation of yaw and roll stability control in heavy trucks has contributed to an increased level of safety for truck drivers and other motorists. It has been shown that the combination of the stability control systems with a predictive model-based stability index can dramatically improve the truck stability and hence road safety. In this respect the authors introduced a new Total Safety Margin (TSM) using a fuzzy logic-based stability index. That methodology utilized a smoothed step and provided acceptable results; however, continuing development has shown that a right sigmoid membership function distribution would provide more complete coverage of the fuzzy space. Compared to the more common triangular membership function which is discontinuous when the membership grade equals one, sigmoid functions facilitate obtaining smooth, continuously differentiable surfaces of a fuzzy model. This is advantageous since continuous surfaces presumably yield smooth control. This was simulated using a 3 degree-of-freedom, single track vehicle model in Matlab and validated using TruckSim 7, the simulation platform of choice. This allowed the new membership functions for Lateral Acceleration, Lateral Velocity, Yaw Rate, and Roll Angle to contribute to the updated Total Safety Margin (TSM).

1. INTRODUCTION

There have been many notable research publications describing the efforts to apply various membership functions to both automotive and nonautomotive fuzzy logic-based predictive methodologies. For example, the work by Jose Naranjo, et al [1] on lateral vehicle control involving fuzzy logic techniques utilized modified

trapezoidal membership functions, one each for angular error and lateral error. Defuzzification was by Sugeno's singletons, and their ultimate goal was autonomous vehicles. The work by Hessburg and Tomizuka [2] also used a fuzzy rule-based controller as a means for lateral control but triangular membership functions were used for fuzzification of the input variables. Defuzzification was by the weighted average method. Their ultimate goal was for use in an automated highway system. Boada, Boada, and Diaz [3] discussed a fuzzy logic-based yaw moment controller to improve vehicle handling and stability. Their controller used triangular membership functions with five fuzzy sets for the input and seven for the output. Defuzzification was via the center of area method. Abbasi and Abouec [4] used an Adaptive Neuro-Fuzzy Inference System (ANFIS) to investigate the stock price trend of the Iran Khodro Company at the Tehran Stock Exchange. A variety of membership functions including triangular, trapezoidal, and Gaussian combination were used. Defuzzification was by the weighted average method. Khanafer, et al [5] discussed a rule based collision avoidance system which focused on scene interpretation of a situation and its optimization. The rule-based fuzzy system used trapezoidal membership functions to define the longitudinal and lateral directions as well as the vehicle velocity. A sigmoidal function was used to define the steering path of the vehicle. Davoudi, et al [6] investigated a fuzzy based vehicle dynamic stability control. Their goal was to detect vehicle instability and the turnover threshold. Triangular membership functions were used for the position and velocity of the vehicle CG. The center of area (centroid) method was used for defuzzification.

The goal here is to update the Total Safety Margin (TSM), a fuzzy logic-based stability index first presented

in [7] using the weighted average defuzzification method. As defined in that paper, the fuzzy logic-based Total Safety Margin (TSM) was an attempt to define the instability level of a vehicle as a single, non-dimensional number. The TSM presented in that paper was based on a single vehicle speed and utilized a smooth step for both the steering input and the fuzzy logic membership functions. Furthermore, the weighting factors used in that study were determined arbitrarily based on the vehicle type being considered. The weighting factors are now calculated based on vehicle response data and are no longer assigned a priori, as they were in [7]. The calculation is based on a comparison between results from a ramp-steer maneuver run at 80 kph and a lifted wheel study. See Appendix B, Figure B.12. Furthermore, the steering input has also been updated. It is still a ramp steer input to the same maximum steering wheel angle as in [7], but it has been slowed down such that it now occurs over a period of one second. Two additional longitudinal vehicle velocities have been added to the study. This was done to provide a more complete coverage of the working vehicle speed range as well as to gain a better understanding of the vehicle dynamics behavior. Most importantly, and the focus of this paper, is the use of new right sigmoid membership functions to calculate the membership grades of the individual safety margins used in the Total Safety Margin.

2. MATHEMATICAL MODELING

The simulated benchmark vehicle was a two axle truck and was chosen based on available data and vehicle parameters. The truck was modeled using a 3 DOF Bicycle Model in planar motion including Yaw Rate, Lateral Velocity, and Roll. The linear tire assumption was used [8-14]. The front tire steer angle was calculated using a smooth ramp in MATLAB®.

The equations of motion were derived using the Lagrangian method. These are given by Equations 1-4:

$$F_{yf,r} = C_{\alpha f,r} \alpha_{f,r} \quad (1)$$

$$\begin{bmatrix} m & 0 & 0 & mh \\ 0 & I_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ mh & 0 & C_\phi & I_x + mh^2 \end{bmatrix} \begin{Bmatrix} \dot{v} \\ \dot{r} \\ \dot{\phi} \\ \ddot{\phi} \end{Bmatrix} = \begin{bmatrix} \frac{-(C_{\alpha f} + C_{\alpha r})}{u} & -mu + \frac{(bC_{\alpha r} - aC_{\alpha f})}{u} & 0 & 0 \\ \frac{bC_{\alpha r} - aC_{\alpha f}}{u} & \frac{-(a^2C_{\alpha f} + b^2C_{\alpha r})}{u} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -muh & mgh - K_\phi & 0 \end{bmatrix} \begin{Bmatrix} v \\ r \\ \phi \\ \dot{\phi} \end{Bmatrix} + \begin{Bmatrix} C_{\alpha f} \\ aC_{\alpha f} \\ 0 \\ 0 \end{Bmatrix} \delta_f \quad (2)$$

$$\alpha_f = \delta_f - \frac{v + ar}{u} \quad (3)$$

$$\alpha_r = -\left(\frac{v - br}{u} \right) \quad (4)$$

The ramp-steer maneuver (See Appendix B, Figure B.1) was modeled in both MATLAB®/Simulink® and TruckSim 7, with the unladed vehicle parameters listed in Appendix A.

The first vehicle mass was 13172.6 lb which is the baseline conventional van in the Mechanical Simulation Corporation program, TruckSim 7®. See Appendix A. The maximum vehicle mass simulated was 26000 lb, the assumed gross vehicle weight for a vehicle of this type. The masses in between these two limits were chosen to best represent various loaded cases (i.e. various total vehicle masses) throughout the assumed mass range. See Appendix B, Figure B.3.

3. SIMULATION MANEUVER ASSUMPTIONS

The list of vehicle stability control maneuvers is vast and includes simple step-steer inputs, accident avoidance maneuvers and decreasing radius turns, J-turn, and Fishhook, with the double lane change probably being the most common of the accident avoidance maneuvers. The fuzzy logic-based Total Safety Margin developed here is based on the following assumptions:

- The road is a dry, level, high-mu surface.
- The vehicle is symmetric left-to-right.
- No other traffic is present.
- The vehicle is traveling in the center of the road.
- Steering wheel angle (SWA) remains at a constant, non-zero value from the instant the ramp-steer input was completed for the duration of the maneuver.

4. APPLICATION OF FUZZY LOGIC

Fuzzy logic is a way of representing real world models in a format that is linguistic in nature. A fuzzy system can help model or control a system when we do not have a math model of how the system's output depends on its input. The fuzzy system uses commonsense rules in place of the math model, building a bridge from the input space to the output space [15-18]. In the case of vehicle control, if a driver is traveling down a road and realizes he is going too fast for the upcoming turn he would presumably react to slow the vehicle down. The goal is to apply fuzzy logic in an intuitive fashion to the vehicle responses being measured. The process used to develop the fuzzy logic-based stability index (TSM) will then be discussed.

4.1 MEMBERSHIP FUNCTION SELECTION

One of the more difficult areas of fuzzy logic application is the choice of the membership function. In the most basic terms [18], a membership function assigns to each element x of a given variable a certain value from the interval $[0, 1]$:

$$\mu_A(x): X \rightarrow [0,1], \forall x \in X \quad (5)$$

In other words, the variable $\mu_A(x)$ is a measure of the grade or degree of membership of a given variable over a specified range of x . The assignment of each element x of a given variable over the interval depends on the type of membership function used. For example, if a right sigmoid membership function is used then when the variable in question, Lateral Acceleration for example, is equal to zero, the membership is equal to zero. When the variable Lateral Acceleration is at its maximum, the membership is equal to one. See Figures 1 - 4.

Membership functions can have many different forms, including triangular, trapezoidal, Gaussian, and left and right Sigmoids [1-6, 15-18]. Since this paper is interested in determining the instability levels of a vehicle, the membership function needed to be non-repeating over its defined interval on the positive x -axis beginning at $x = 0$. Specifically, any membership function that is symmetrical with respect to the x -axis would in fact yield the same membership value for two response values. For example, if a symmetrical membership function such as a triangular, trapezoidal, or Gaussian were used for Lateral Acceleration, then depending on the specific shape of the membership function, a membership value of $\mu_A(x) = 0.5$ could be possible for Lateral Accelerations of 0.2g and 0.8g. This would imply that the vehicle was just as unstable at 0.2g as it was at 0.8g. Since this would not make sense, an asymmetrical membership function had to be used.

The membership function also could not possess any abrupt changes in slope as is the case with triangular and trapezoidal membership functions. The main disadvantage of polygonal membership functions are that they are not continuously differentiable [18]. Therefore a right sigmoid membership function was chosen because:

- (1) They facilitate smooth, continuously differentiable surfaces of the fuzzy model.
- (2) They facilitate theoretical analysis of fuzzy systems as they are continuously differentiable and infinitely differentiable; i.e. they have derivatives of any grade [18].

A right sigmoid membership function was created for each of the four vehicle responses being studied in this paper: Lateral Acceleration, Lateral Velocity, Yaw Rate, and Roll Angle. They are shown in Figures 1 – 4.

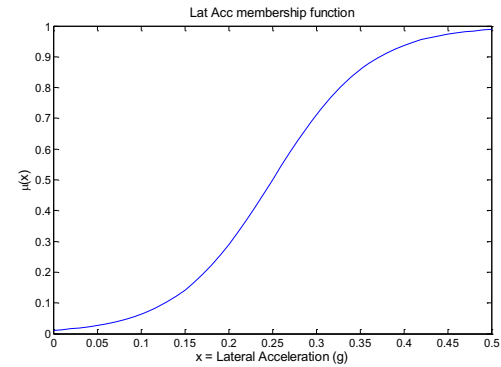


Figure 1: Right Sigmoid membership functions for Lateral Acceleration

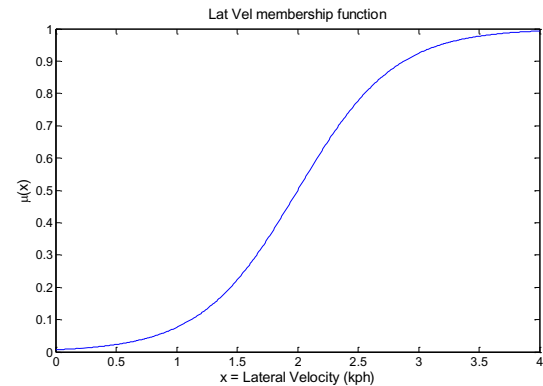


Figure 2: Right Sigmoid membership functions for Lateral Velocity

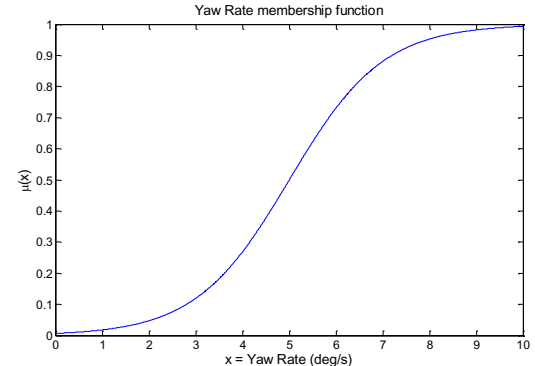


Figure 3: Right Sigmoid membership functions for Yaw Rate

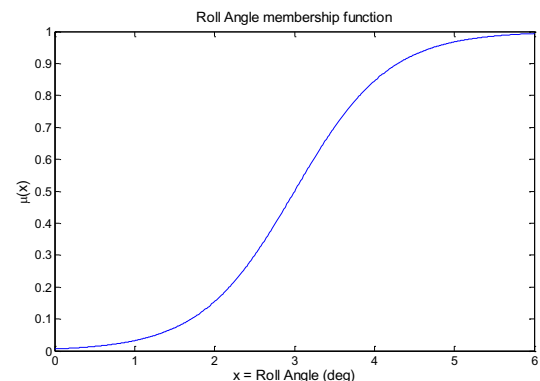


Figure 4: Right Sigmoid membership functions for Roll Angle

The equation for the right sigmoid is [18]:

$$\mu(x) = \frac{1}{1 + \exp[-a(x-b)]} \quad (6)$$

where:

$\mu(x)$ = Fuzzified value. Determines the degree of set membership. In this case this value is the calculated safety margin l_m , v_m , r_m , and p_m (Lateral Acceleration, Lateral Velocity, Yaw Rate, and Roll Angle, respectively) and is obtained by setting x equal to the maximum value of Lateral Acceleration, Lateral Velocity, Yaw Rate, and Roll Angle.

a = Slope factor. Increasing this value makes the sigmoid more vertical so with large values of a the sigmoid can approach a step function. The slope factor a can be calculated by estimating the value of $x_{0.99}$, i.e. the least value of the variable x which can be admitted with almost complete certainty to belong to a fuzzy set represented by the sigmoid function.

b = Determines the characteristic coordinate of the point k , at which the grade of membership in the fuzzy set amounts to 0.5, i.e. at $x = b$, $\mu(x) = 0.5$.

x = The x-axis of the membership function. In this case this variable represents the vehicle response ranges of Lateral Acceleration, Lateral Velocity, Yaw Rate, and Roll Angle.

The membership $\mu(x)$ represents the vehicle instability level for each variable: Lateral Acceleration, Lateral Velocity, Yaw Rate, and Roll Angle. These are referred to as the safety margins and represented as Lateral Acceleration (l_m), Lateral Velocity (v_m), Yaw Rate (r_m), and Roll Angle (p_m). When these vehicle responses were all zero, the vehicle was stable and therefore the memberships were all zero. When these vehicle responses were at their maximum and therefore represented an unstable situation for the given region, the memberships were all equal to one. Based on this convention the following fuzzy rules were established:

If membership $\mu(x)$ of output = 0 then Safe
If membership $\mu(x)$ of output = 1 then Danger

4.2 WHEEL LIFT STUDY

The next step was to determine the proper vehicle response ranges for these membership functions. Since the vehicle's instability level was of interest, it was necessary to determine the test vehicle's range of dynamic capabilities in the maneuver chosen. TruckSim

7 was used for this testing since real vehicle data was not available.

The vehicle was loaded to the same test loads as in the multimass study (See Appendix B) but this time each vehicle mass was run through the same ramp-steer maneuver (Figure B.1) until the inside tire lifted off the ground within the first seven seconds but touched back down by 10 seconds. The seven second time was chosen since in the multimass studies the vehicle had attained steady state values within four seconds and the total simulation time was seven seconds. Touchdown by 10 seconds was chosen since the goal was not to test for vehicle rollover, but rather when the vehicle would just lift its inside tires. Wheel lift was checked for during the TruckSim animation and confirmed on the normal load tire plots available within TruckSim. These speeds at which the inside wheel briefly lifted and touched back down indicated a maximum safe cornering speed. The TruckSim simulation was run long enough to ensure that the vehicle didn't rollover at any of these speeds.

The results of these tests are shown in Figures 5 – 9.

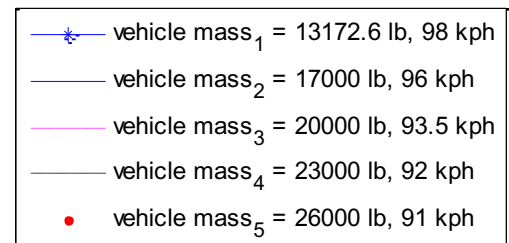


Figure 5: Legend for TruckSim Wheel Lift Study

Speed Comparison Chart	
98 kph =	60.89 mph
96 kph =	59.65 mph
93.5 kph =	58.10 mph
92 kph =	57.17 mph
91 kph =	56.54 mph

Figure 6: Speed Comparison Chart for TruckSim 7 Wheel Lift Speeds

Note that there is a 7 kph (4.35 mph) spread of speed between the lightest and heaviest loaded vehicles for limit cornering speeds in this test.

The Static Stability Factor, or SSF, is a common measure of static stability [19-20]:

$$SSF = t / 2h \quad (7)$$

where:

t = vehicle track width
 h = vehicle CG height

Using the rear track width of the indicated test vehicle, the TTR for the unladen vehicle in this study is:

$$SSF = \frac{t}{2h} = \frac{1.829}{2 \cdot 1.173} = 0.7796 \approx 0.78 \tag{8}$$

$$SSF = 0.78$$

Based on the results of [19-20], an SSF for a pick-up trucks, SUVs, and vans have SSF values in the range of 1.0 to 1.3. Thus, and SSF of 0.78 for a vehicle of the type simulated here is reasonable and indicates a higher propensity of static rollover than a typical passenger vehicle. It should be noted that including the tire and suspension stiffnesses result in a lower stability factor than the SSF calculation.

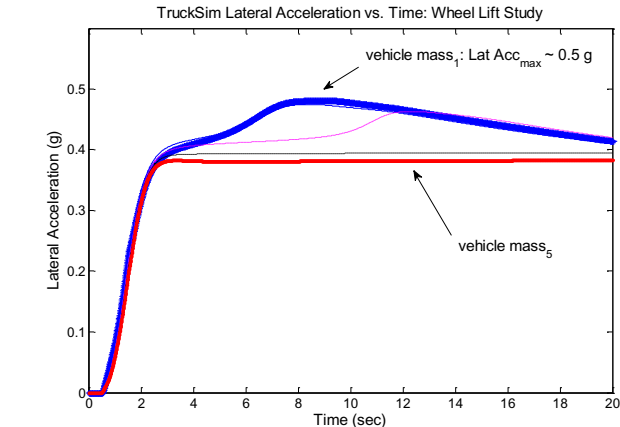


Figure 7: TruckSim Wheel Lift Study: Lateral Acceleration vs. Time

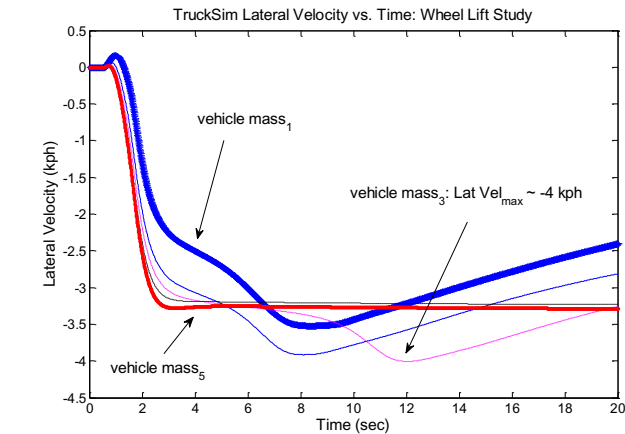


Figure 8: TruckSim Wheel Lift Study: Lateral Velocity vs. Time

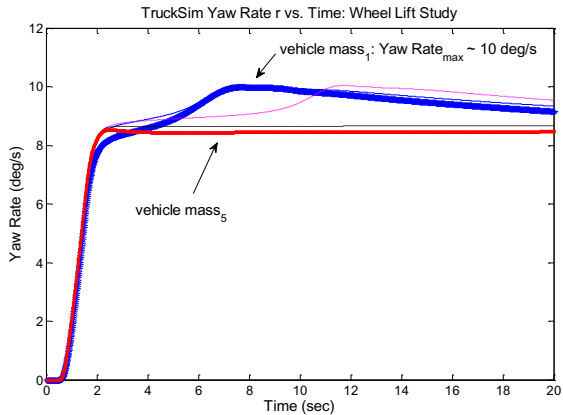


Figure 9: TruckSim Wheel Lift Study: Yaw Rate vs. Time

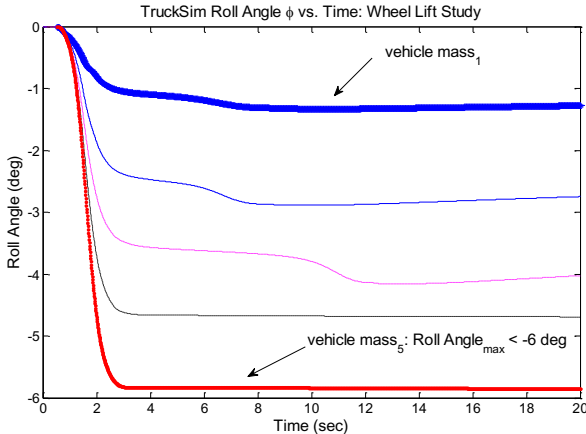


Figure 10: TruckSim Wheel Lift Study: Roll Angle vs. Time

Looking at these results it can be seen upper limits for each of the vehicle responses are:

Wheel Lift Study		
Lateral Accel _{max}	0.5	g
Lateral Velocity _{max}	4	kph
Yaw Rate _{max}	10	deg/s
Roll Angle _{max}	6	deg

Figure 11: Wheel Lift Study Vehicle Response Thresholds

These limits were chosen as the upper limits on the intervals for the membership functions. See Figures 1-4. Note that the magnitudes of the simulated values are shown and were used in the membership function calculations.

4.3 WEIGHTING FACTOR DETERMINATION

Defuzzification is the process by which a single numerical output value is determined from the summation of the grades of membership as determined by the membership functions. This operation is accomplished by a defuzzification mechanism such as [1-6, 15-18]: