

Robust Observation of Tractor-trailer Vertical Forces Using Inverse Model and Exact Differentiator

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ABSTRACT

In this paper, we are interested in developing a robust tire-force estimator for heavy duty vehicles. We use a combined model of the articulated vehicle: a yaw plane model for the chassis motion and a vertical plane model for the axles. In the proposed method, we make use of the on-board available sensors to which low-cost sensors are added. In order to optimize the sensors configuration, a robust exact differentiator is used in order to obtain accelerations from the measured velocities. Once the differentiation is obtained, the model is inverted to determine the unknown input forces. The approach is validated by comparing the estimation results to those given by the software simulator *prosper*.

INTRODUCTION

The problem of the estimation of the contact forces for heavy duty vehicles is primordial for obtaining the necessary information on the interaction between the vehicle and the pavement. This information is particularly useful in order to avoid road and vehicle damage (tire, suspension, fragile payload) and to enhance security by preventing rollover of trucks.

The issue of estimating vehicle contact forces is considered in many previous works. In [9] an Extended Kalman Filter (EKF) is applied to estimate the longitudinal and lateral forces in a vehicle bicycle model, the forces are considered as additional states and their dynamics are modeled by the use of a shaping filter driven by white noise. The authors in [11] have applied a similar estimation algorithm to an off-highway mining truck. The main limitation of this approach is that

some assumptions are made on the behavior of the forces (slow variation) when tuning the shaping filter, this requires the exact knowledge of the nature of the forces and the way they vary. We can also cite [2], [8] who use sliding mode observers to estimate contact forces in heavy duty vehicles. First, the height of the wheel hub is estimated, then by measuring the road profile, the tire deflection is calculated by assuming the linear stiffness of the tire. An estimation of the vertical force is obtained by multiplying the tire deflection with its stiffness. The longitudinal and lateral forces are also derived using the Pacejka tire model. This method involves knowing accurately the vehicle and the pneumatic parameters and also to measure the road profile.

Another way to estimate the forces is to use strain gauges in the wheel hub. The solution provides precise measurement but it is expensive. Axle bar can also be instrumented by shear strain gauges. This last method is limited to straight constant speed maneuvers and does not suit to turning corners maneuvers with large lateral forces [4]. Furthermore, strain gauges in vehicles are complex to install and to calibrate.

The proposed method is to use a sliding mode observer [1] to evaluate the forces which are considered as unknown inputs of the systems. The advantage of this approach is to avoid the use of many parameters of the vehicle (usually unknown), to optimize the sensors configuration (using embedded sensors and adding low-cost sensors) and to present a good alternative to the use of strain gauges especially while turning corners (simple installation and calibration). This paper is divided into three main sections. In the first section, we describe the model and the validation by simulator software.

In the second section, we present the estimation method. In the third section, we present the simulation results. We finally conclude with some perspectives in the last section.

MODEL DESCRIPTION

To be able to develop observers and to validate our approach, we first introduce an observable model derived from the non linear yaw plane model described in [6]. We assume that only front tractor wheels are steerable. It is also assumed that the effect of aerodynamic forces is negligible. The model is represented in [Figure 1](#).

The behavior of the articulated vehicle is related to the forces applied by pavement on the tires. In the yaw plane model, two axles for the tractor are considered and the axle-group in the trailer are represented by unique equivalent axle. In order to describe the dynamics of the articulated vehicle, we represent separately the tractor and the trailer. The hitch force which is an internal force appears as an external force for both the tractor and the trailer. Moreover, we represent the hitch force by two components: one longitudinal component noted X_h and one lateral component noted Y_h .

Let us note F_{fx} and F_{fy} respectively the tractor front longitudinal and lateral tire forces, F_{rx} and F_{ry} are respectively the tractor rear longitudinal and lateral tire forces and F_{tx} and F_{ty} the total forces applied on trailer-axle. The equation of motion of the tractor in the yaw plane is given in the longitudinal axis by:

$$F_{fy} \sin(\delta) + F_{fx} \cos(\delta) + F_{rx} - X_h = M_1 \ddot{x} \quad (1)$$

where M_1 is the total mass of the tractor, δ is the steering angle and $\ddot{x}$ is the longitudinal tractor acceleration. For the lateral direction:

$$F_{fy} \cos(\delta) - F_{fx} \sin(\delta) + F_{ry} - Y_h = M_1 \ddot{y} \quad (2)$$

here $\ddot{y}$ corresponds to the tractor lateral acceleration. The yaw moment of the tractor at the hitch point:

$$(l_f + l_h)(F_{fy} \cos(\delta) - F_{fx} \sin(\delta)) - (l_r - l_h)F_{ry} = I_{z1} \ddot{\alpha}_1 \quad (3)$$

where l_f and l_r the distances between the centre of gravity of the tractor and respectively the front and the rear axles. l_h is the distance between the centre of gravity of the tractor and the hitch. $\ddot{\alpha}_1$ is the yaw acceleration of the tractor.

<figure 1 here>

Remark 1. the parameter I_{z1} is the moment of inertia of the tractor around a vertical axis crossing the hitch. It is obtained by the use of Huyghens theorem:

$$I_{z1} = I_{z1cog} + M_1 l_h^2 \quad (4)$$

where I_{z1cog} is the inertia moment according to the vertical axis crossing the centre of gravity of the tractor [7]. The trailer dynamics is governed by the following equations. In the longitudinal direction,

$$X_h + F_{tx} \cos(\theta) - F_{ty} \sin(\theta) = M_2 (\ddot{x}_t \cos(\theta) - \ddot{y}_t \sin(\theta)) \quad (5)$$

where M_2 is the total mass of the trailer. θ is the relative yaw angle between tractor and trailer. $\ddot{x}_t$ and $\ddot{y}_t$ are respectively the longitudinal and lateral accelerations of the trailer. In the lateral direction,

$$Y_h + F_{tx} \sin(\theta) + F_{ty} \cos(\theta) = M_2 (\ddot{x}_t \sin(\theta) + \ddot{y}_t \cos(\theta)) \quad (6)$$

and the yaw dynamics,

$$(Y_h \cos(\theta) - X_h \sin(\theta))l_{ft} - F_{ty}l_{rt} = I_{z2} \ddot{\alpha}_2 \quad (7)$$

here l_{ft} and l_{rt} correspond to the distance between the centre of gravity of the trailer and respectively the hitch and rear axle.

The state space vector consists of measured velocities: the trailer yaw rate $\dot{\alpha}_1$, the front and rear tractor wheel rotational velocities noted respectively ω_{fw} and ω_{rw} , the trailer yaw rate $\dot{\alpha}_2$ and the trailer wheel rotational velocity noted ω_{tw} . We select these variables as components of the state vector because of, first, they are measured variables and second, these variables are velocities so that their derivative correspond to accelerations needed for estimating the unknown input forces as shown further in this paper.

$$X_v = \begin{bmatrix} \dot{\alpha}_1 \\ \omega_{fw} \\ \omega_{rw} \\ \dot{\alpha}_2 \\ \omega_{tw} \end{bmatrix} \quad (8)$$

The output vector Y is composed of the longitudinal and lateral accelerations of the tractor and the longitudinal and

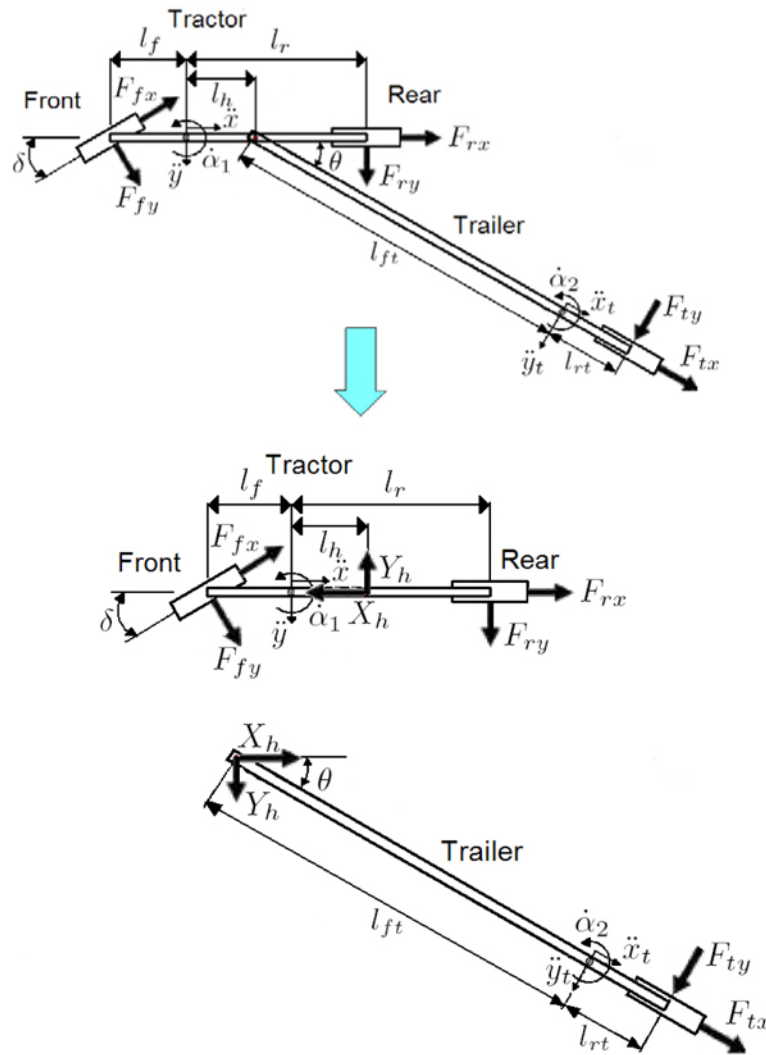


Figure 1. Extended semi-trailer bicycle model

lateral accelerations of the trailer considered in the tractor frame and noted respectively $\ddot{x}_t^*$ and $\ddot{y}_t^*$.

$$\ddot{y}_t^* = \ddot{x}_t \sin(\theta) - \ddot{y}_t \cos(\theta) \quad (11)$$

$$Y = \begin{bmatrix} \ddot{x} = \frac{1}{M_1} (F_{rx} + F_{fx} \cos(\delta) + F_{fy} \sin(\delta) - X_h) \\ \ddot{y} = \frac{1}{M_1} (F_{ry} - F_{fx} \sin(\delta) + F_{fy} \cos(\delta) - Y_h) \\ \ddot{x}_t^* = \frac{1}{M_2} (F_{tx} \cos(\theta) - F_{ty} \sin(\theta) + X_h) \\ \ddot{y}_t^* = \frac{1}{M_2} (F_{tx} \sin(\theta) - F_{ty} \cos(\theta) + Y_h) \end{bmatrix} \quad (9)$$

with

$$\ddot{x}_t^* = \ddot{x}_t \cos(\theta) - \ddot{y}_t \sin(\theta) \quad (10)$$

and

Equations of wheels rotation are also introduced, they are given by this set of equations for the front tractor axle wheel:

$$\begin{cases} I_{fw} \dot{\omega}_{fw} = T_{fw} + r_w F_{fx} \\ I_{rw} \dot{\omega}_{rw} = T_{rw} + r_w F_{rx} \\ I_{tw} \dot{\omega}_{tw} = T_{tw} + r_w F_{tx} \end{cases} \quad (12)$$

where I_{fw} , I_{rw} and I_{tw} respectively the rotational inertias for front and rear tractor wheels and trailer wheel. r_w is the wheel rolling radius supposed constant. T_{fw} , T_{rw} et T_{tw} are respectively the torques at the front and rear trailer axles and trailer equivalent axle.

The state-space model can be written in this form:

$$\begin{cases} \dot{X}_v = Bu + W_1(u)u_F \\ Y = W_2(u)u_F \end{cases} \quad (13)$$

where u is the known input vector composed of the tractor front and rear wheel torques respectively T_{fw} and T_{rw} , the trailer wheel torque T_{tw} , the steering angle δ and the relative yaw angle θ .

$$u = \begin{bmatrix} T_{fw} \\ T_{rw} \\ T_{tw} \\ \delta \\ \theta \end{bmatrix} \quad (14)$$

Remark 2. In practice, steering angle, yaw rate, wheel rotational velocity, the lateral acceleration and the engine torques are signals available on the CAN-bus of the vehicle. The braking torque can be deduced from the brakes pressure which is also available at the CAN-bus. In our application, we do not consider braking situations. The relative yaw angle θ between the tractor and the trailer can be obtained by integrating the difference between the measured tractor yaw rate $\dot{\alpha}_1$ and the measured trailer yaw rate $\dot{\alpha}_2$.

The unknown input forces u_F

$$u_F = \begin{bmatrix} F_{fx} \\ F_{rx} \\ F_{tx} \\ F_{fy} \\ F_{ry} \\ F_{ty} \\ X_h \\ Y_h \end{bmatrix} \quad (15)$$

The matrix $B \in R^{5 \times 5}$ is given by:

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \frac{1}{I_{fw}} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{I_{rw}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_{tw}} & 0 & 0 \end{bmatrix} \quad (16)$$

By considering $l_{fh} = l_f + l_h$ and $l_{rh} = l_r - l_h$. The matrix $W_1(u) \in R^{5 \times 8}$ is given by:

$$W_1(u) = \begin{bmatrix} \frac{l_{fh}}{I_{z1}} \sin(\delta) & 0 & 0 & \frac{l_{fh}}{I_{z1}} \cos(\delta) & -\frac{l_{rh}}{I_{z1}} & 0 & 0 & 0 \\ \frac{r_w}{I_{fw}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{r_w}{I_{rw}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{l_{rt}}{I_{z2}} & -\frac{l_{ft}}{I_{z2}} \sin(\theta) & \frac{l_{ft}}{I_{z2}} \cos(\theta) \\ 0 & 0 & \frac{r_w}{I_{tw}} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (17)$$

and $W_2(u) \in R^{4 \times 8}$ is given by:

$$W_2(u) = \begin{bmatrix} \frac{1}{M_1} \cos(\delta) & \frac{1}{M_1} & 0 & \frac{1}{M_1} \sin(\delta) & 0 & 0 & -\frac{1}{M_1} & 0 \\ -\frac{1}{M_1} \sin(\delta) & 0 & 0 & \frac{1}{M_1} \cos(\delta) & \frac{1}{M_1} & 0 & 0 & -\frac{1}{M_1} \\ 0 & 0 & \frac{1}{M_2} \cos(\theta) & 0 & 0 & -\frac{1}{M_2} \sin(\theta) & \frac{1}{M_2} & 0 \\ 0 & 0 & \frac{1}{M_2} \sin(\theta) & 0 & 0 & \frac{1}{M_2} \cos(\theta) & 0 & \frac{1}{M_2} \end{bmatrix} \quad (18)$$

An axle model is introduced to evaluate the vertical forces. The forces applied to the axle are shown in [Figure 2](#).

We can set the equations of motion of the free body in the vertical plane as:

$$F_{z,l} + F_{z,r} = F_{s,l} + F_{s,r} + M_{ax}g + M_{ax}a_{ax} \quad (19)$$

And for the equation of moments

$$(F_{z,l} - F_{z,r})\ell = (F_{s,l} - F_{s,r})\ell_s + F_y r_w + I_{ax} a_{\Phi} \quad (20)$$

with gravity acceleration $g = 9.81 \text{ m/s}^2$. $F_{z,l}$ and $F_{z,r}$ are respectively the vertical wheel forces at the left and at the right of the axle, $F_{s,l}$ and $F_{s,r}$ represent respectively the suspension forces at the left and the right of the axle, F_y is the resultant lateral force *i.e.*, the sum of the left $F_{y,l}$ and right $F_{y,r}$ lateral tyre-force applied to the axle, M_{ax} the mass of the axle, I_{ax} the moment of inertia of the axle around its roll axis, a_{ax} the axle-hop acceleration. a_{Φ} the axle-roll acceleration. ℓ is the distance between the application point of the tyre force and the centre of gravity of the axle. ℓ_s is the distance between the application point of the suspension force and the centre of gravity of the axle and finally r_w is the