

Synthesis of a Vehicle Suspension with Constrained Lateral Space using a Roll-plane Kineto-dynamic Model

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ABSTRACT

The larger chassis space requirements of hybrid vehicles necessitates considerations of the suspension synthesis with limited lateral space, which may involve complex compromises among performance measures related to vehicle ride and handling. This study investigates the influences of suspension linkage geometry on the kinematic and dynamic responses of the vehicle including the wheel load in order to facilitate synthesis of suspension with constrained lateral space. A kineto-dynamic half-car model is formulated incorporating double wishbone suspensions with tire compliance, although the results are limited to kinematic responses alone. An optimal synthesis of the suspension is presented to attain a compromise among the different kinematic performance measures with considerations of lateral space constraints. In the kineto-dynamic model, the struts comprising linear springs and viscous dampers are introduced as force elements. Kinematic formulations of the proposed model are derived using displacement matrix method. The kinematic responses, particularly the variations in the camber angles and the wheel track width are investigated under wheel vertical displacement, chassis roll, and simultaneous inputs of wheel center displacement and chassis roll. The results attained from a sensitivity analysis suggested that variations in the joint coordinates could yield reduction in the lateral space, while these would involve complex compromises among the kinematic responses of the suspension. A composite objective function of camber angle and track width measures under wheel vertical displacement and chassis roll excitations is subsequently formulated and solved with constraints on variations in the roll center height and the suspension lateral packaging space to seek optimal joint coordinates. The proposed synthesis with optimal joint coordinates could yield nearly 10% reductions in the lateral

packaging space, and camber angle and wheel track variations with only minimal increase in the peak roll camber.

INTRODUCTION

The current trends in hybrid vehicles developments impose considerable challenges related to the sprung mass and sub-frame space requirements and thus the chassis design [1]. The larger space requirements of the hybrid vehicles also necessitate considerations of the suspension synthesis with limited lateral space, which would most likely involve complex compromises among performance measures related to vehicle ride and handling. The suspension linkage kinematics and the joints coordinates are known to influence the roll and lateral dynamic responses of a vehicle most significantly under a steering input [2,3]. A recent study investigated the suspension synthesis using a kineto-dynamic quarter-car model, and showed that variations in suspension linkage geometry can yield substantial and conflicting changes in the ride response such as sprung mass acceleration, rattle space, and tire road-holding ability, thereby posing difficult design compromise challenges in identifying a synthesis to realize reduced lateral spacing [4]. Both the ride and handling dynamics of a vehicle are strongly coupled through its dynamic roll response, while there are no reported studies on the influences of linkage geometry on the roll dynamic response under vertical road excitations. The synthesis of a suspension with constrained lateral space thus necessitates investigations of the influences of joint coordinates on the kinematic and dynamic responses, and related measures including the variations in the wheel load.

Suspension kinematic analysis and related suspension design concepts have been studied extensively [5,6,7]. The kinematic analyses of independent suspension mechanisms have been reported using quarter-car models for McPherson

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[8,9,10], double wishbone [11], quadra-link [12] and multi-link [13,14] types of suspensions. These studies have generally evaluated variations in the camber and toe angles, and wheel track width under wheel displacement, while considering fixed chassis. A kinematic half-car model comprising double wishbone suspension has also been analyzed to study the influence of kinematics on the lateral acceleration and roll angle responses of a vehicle under steering inputs, while the contributions due to vertical displacement of the wheels was not considered [2].

The above studies were mostly limited to the analysis of the suspension mechanism with known coordinates, while different studies considered synthesis of suspension in order to achieve compromise among the kinematic performances of the suspension. Simionescu [15] identified optimal joint coordinates of a five link suspension subject to wheel vertical travel through minimization of an objective function comprising the differences between the lengths and the distances between the joint coordinates of the linkages. Although, the study showed reductions in the camber and toe angle variations during wheel vertical motion, the roll camber and roll steer effects were not considered. Habibi et al. [16] achieved reduction in the roll steer and roll camber responses of a McPherson suspension under chassis roll inputs by minimizing a performance index using genetic algorithm. It has been identified that the suspension synthesis to yield minimal variations in camber angle under wheel vertical displacement input (bump camber) and the minimal wheel camber angle under chassis roll (roll camber) involves design conflicts [5, 17]. A few studies have suggested that inclusion of an additional member between the chassis and the suspension [17] or kinematic interconnection between the left and right suspension [18] can yield reduction in both bump and roll camber variations, while these measures may add to the suspension weight and the cost. Suspension synthesis with desired camber and toe angle responses under wheel vertical travel have been achieved by axiomatic design methods [19], or nonlinear optimization methods [20], while these desired kinematic responses were determined based on the past experiences.

Despite the numerous reported studies, the identification of an optimal synthesis that can yield acceptable compromise among various kinematic performance measures continues to be a challenging task. This is partly attributed to conflicting bump and roll camber responses of the suspension. The synthesis and kinematic response analyses of a suspension may thus require consideration of both wheel vertical displacement and chassis roll inputs simultaneously. Moreover, synthesis of a suspension with constrained lateral space for hybrid vehicles may involve added design compromise among the kinematic responses.

This study presents a synthesis of the suspension with constrained lateral space and influences of the suspension

linkage geometry on the kinematic responses through analysis of a half-car kineto-dynamic model. While the results are limited to kinematic responses, the proposed half-car model could be conveniently applied to evaluate the dynamic responses. The kinematic responses, particularly the variations in the bump and roll camber angles and the wheel track of a double wishbone suspension mechanism are investigated under wheel vertical displacement, chassis roll and simultaneously applied vertical displacement and chassis roll inputs. A sensitivity analysis is performed to investigate the influence of each suspension joint coordinate on the kinematic responses, and an optimal synthesis is realized by minimization of an objective function comprising the camber angles and wheel track variations under wheel vertical displacement and chassis roll, with constraints on the lateral packaging space and roll center height.

HALF CAR ROLL-PLANE MODEL

Figure 1 illustrates the proposed 4-degrees of freedom (DOF) half-car kineto-dynamic model comprising the double wishbone type of suspension. The suspension in the planar model includes upper and lower control arms, and a strut consisting of a linear spring and a viscous damper. The control arms are connected to the chassis and the wheel knuckle through revolute joints, while the joints are considered to be frictionless. The control arms are modeled as massless elements, and the unsprung mass is assumed to be lumped at the center of gravity (*cg*) of the wheel assembly. For the kineto-dynamic analysis, the tire is modeled as a combination of a vertical linear spring and a viscous damper, while the lateral compliance of the tire is represented by a lateral linear stiffness. The chassis and suspension kinematics are formulated considering the chassis, suspension linkages and the wheel knuckle as rigid bodies, while the rigid body assumption was also applied to tire when kinematic analyses alone were concerned. The model is formulated assuming vertical (z_s) and roll (ϕ_s) displacements of the sprung mass, left and right wheels vertical displacements (z_{ul} and z_{ur}), as the generalized coordinates. The rotation of the chassis is assumed to occur about the kinematic roll center *G*, as shown in the **Fig. 1**.

<figure 1 here>

KINEMATICS OF CHASSIS: The chassis kinematics is formulated in order to evaluate the instantaneous positions of the suspension joints on the chassis under chassis rotation or vertical motions. The suspension kinematic responses are subsequently determined from the coordinates of the linkage joints. A fixed coordinate system is assumed with its origin located at the mass center (*cg*) of the vehicle sprung mass in its static equilibrium. The sprung mass vertical and rotational displacements are considered about the kinematic roll center *G* of the vehicle body. The initial (G_{y0} , G_{z0}) and

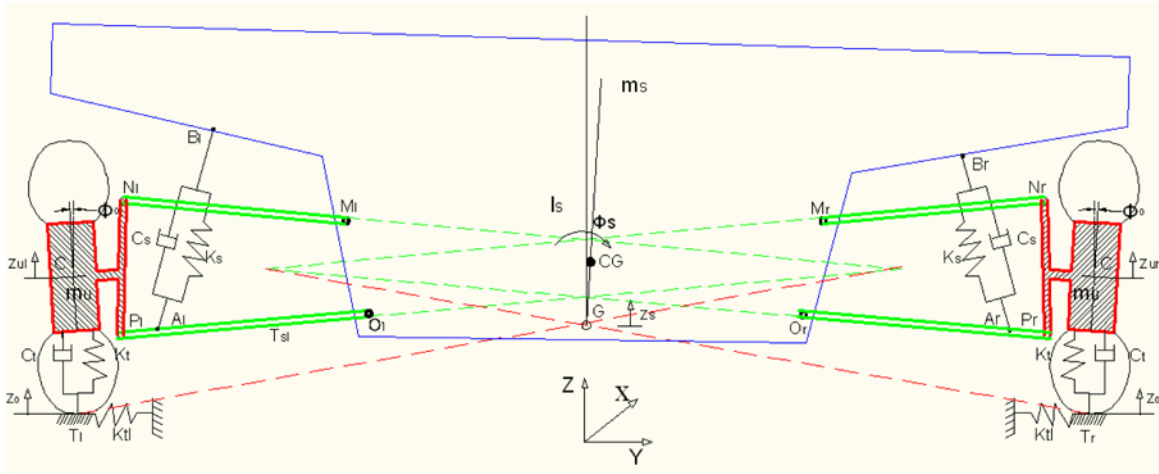


Figure 1. Half-car roll-plane kineto-dynamic model of a double wishbone suspension

instantaneous (G_y , G_z) coordinates of the roll center are related through the general displacement matrix D_c , formulated under a finite displacement of the chassis, given by [21]:

$$D_c = \begin{bmatrix} a_{11} & a_{12} & G_y - (a_{11}G_{y0} + a_{12}G_{z0}) \\ a_{21} & a_{22} & G_z - (a_{21}G_{y0} + a_{22}G_{z0}) \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where $a_{11} = a_{22} = \cos\phi_s$, $a_{12} = -a_{21} = \sin\phi_s$, and ϕ_s is the vehicle body rotation about the roll center.

The y - and z - coordinates of chassis-suspension joints, M_r , O_r , M_l , and O_l , shown in the Fig 1, can be determined using the displacement matrix D_c , such that: >

$$\begin{bmatrix} M_{ry} & O_{ry} & M_{ly} & O_{ly} \\ M_{rz} & O_{rz} & M_{lz} & O_{lz} \\ 1 & 1 & 1 & 1 \end{bmatrix} = D_c \begin{bmatrix} M_{ry0} & O_{ry0} & M_{ly0} & O_{ly0} \\ M_{rz0} & O_{rz0} & M_{lz0} & O_{lz0} \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad (2)$$

The subscripts ' r ' and ' l ' in Eq (2) refer to right or left suspension joints, respectively, while the subscripts ' y ' and ' z ' represent the lateral and vertical axes, respectively. The subscript ' 0 ' represents the initial coordinate of the joint. The expansion of the Eq (2) yields the expressions for the instantaneous coordinates of the suspension joints at the chassis, such that:

$$\begin{aligned} M_{jy} &= a_{11}(M_{jy0} - G_{y0}) + a_{12}(M_{jz0} - G_{z0}) + G_y \\ M_{jz} &= a_{21}(M_{jy0} - G_{y0}) + a_{22}(M_{jz0} - G_{z0}) + G_z \\ O_{jy} &= a_{11}(O_{jy0} - G_{y0}) + a_{12}(O_{jz0} - G_{z0}) + G_y \\ O_{jz} &= a_{21}(O_{jy0} - G_{y0}) + a_{22}(O_{jz0} - G_{z0}) + G_z \dots j = r, l \end{aligned} \quad (3)$$

where $G_y = G_{y0}$; and $G_z = G_{z0} + z_s$. The above equation can be solved to obtain instantaneous coordinates of the chassis-linkage joints for given chassis rotation ϕ_s about the roll center and/or a vertical displacement of the chassis, z_s .

KINEMATICS OF SUSPENSIONS: The kinematic analysis of the suspension links is performed to determine variations in the camber angles and wheel track width in terms of the generalized coordinates. For finite displacements of the right and left wheel knuckles in the given plane, the displacement matrices, D_{wr} and D_{wl} , of the right and left units, respectively, are formulated as:

$$D_{wj} = \begin{bmatrix} a_{11j} & a_{12j} & C_{jy} - (a_{11j}C_{jy0} + a_{12j}C_{jz0}) \\ a_{21j} & a_{22j} & C_{jz} - (a_{11j}C_{jy0} + a_{12j}C_{jz0}) \\ 0 & 0 & 1 \end{bmatrix} \dots j = r, l \quad (4)$$

where $a_{11j} = a_{22j} = \cos\phi_{uj}$, $-a_{12l} = a_{21l} = \sin\phi_{ul}$, and $a_{12r} = -a_{21r} = \sin\phi_{ur}$. In the above equation, ϕ_{ul} and ϕ_{ur} are the left and right knuckle rotations, respectively, about the x -axis. C_{jy} and C_{jz} refer to the lateral and vertical coordinates, respectively, of the j^{th} ($j=r, l$) wheel center. The instantaneous coordinates of the upper and lower ball joints (N_r , P_r , N_l and P_l) following the displacement can be expressed using the right and left wheel knuckle displacement matrices, D_{wr} and D_{wl} , as:

$$\begin{bmatrix} N_{jy} & P_{jy} \\ N_{jz} & P_{jz} \\ 1 & 1 \end{bmatrix} = D_{wj} \begin{bmatrix} N_{jy0} & P_{jy0} \\ N_{jz0} & P_{jz0} \\ 1 & 1 \end{bmatrix} \dots j = r, l \quad (5)$$

The expansion of Eq (5) yields eight equations in the joints coordinates of the left and right wheel knuckles. The above formulation, however, comprises a total of 12 unknown parameters corresponding to the wheel center displacements z_{ur} and z_{ul} , namely: the y and z coordinates of joints N_r , P_r , N_l and P_l ; the y coordinates of the wheel centers C_r and C_l ; and the camber angles ϕ_{ur} and ϕ_{ul} . Equation (5) is thus solved in conjunction with the constraint equations, which for a planar double wishbone suspension may be formulated considering the constant control arm lengths. The expanded form of Eq (5) together with the constraint equations thus yield 12 non-linear equations, given by:

$$\begin{aligned} N_{jy} &= a_{11j}(N_{jy0} - C_{jy0}) + a_{12j}(N_{jz0} - C_{jz0}) + C_{jy} \\ N_{jz} &= a_{21j}(N_{jy0} - C_{jy0}) + a_{22j}(N_{jz0} - C_{jz0}) + C_{jz0} + z_{uj} \\ P_{jy} &= a_{11j}(P_{jy0} - C_{jy0}) + a_{12j}(P_{jz0} - C_{jz0}) + C_{jy} \\ P_{jz} &= a_{21j}(P_{jy0} - C_{jy0}) + a_{22j}(P_{jz0} - C_{jz0}) + C_{jz0} + z_{uj} \\ (N_{jy} - M_{jy})^2 + (N_{jz} - M_{jz})^2 &= l_{MNj}^2 \\ (P_{jy} - O_{jy})^2 + (P_{jz} - O_{jz})^2 &= l_{OPj}^2 \dots j = r, l \end{aligned} \quad (6)$$

where l_{MNj} and $l_{OPj}(j=r, l)$ are the lengths of upper and lower control arms, respectively.

Equations (3) and (6) can be simultaneously solved to obtain kinematic responses of the suspension for given vertical displacements of left and/or right wheels, and/or vertical and roll displacements of the chassis.

KINEMATIC RESPONSES: The solution of Eqs (3) and (6) yield the camber angle responses (ϕ_{ur} and ϕ_{ul}) of the left and right wheels for given sprung mass and unsprung mass displacements either individually or simultaneously. The variations in the track width have been generally evaluated from the changes in the lateral coordinates of the wheel centres (C_r and C_l) [17], which may not truly define the track width due to lateral tire compliance and camber effects. In this study, the lateral displacement of the tire-ground contact points, T_r and T_l , as shown in Fig 1, during the wheel motions are considered as the variations in the wheel track width [5]. Thus, the wheel track variations yield wheel center lateral displacement coupled with the wheel camber angle variation. The rigid body assumption of the tire leads to expressions for the y - and z - coordinates of the tire-road contact points, T_r and T_l , as:

$$\begin{aligned} T_{jy} &= a_{11j}(T_{jy0} - C_{jy0}) + a_{12j}(T_{jz0} - C_{jz0}) + C_{jy} \\ T_{jz} &= a_{21j}(T_{jy0} - C_{jy0}) + a_{22j}(T_{jz0} - C_{jz0}) + C_{jz0} + z_{uj} \dots j = r, l \end{aligned} \quad (7)$$

The y - coordinates of the tire-ground contact points in Eq (7) determine the wheel track width variations, while the kinematic roll center of the vehicle in the roll plane is estimated using the instantaneous centers of rotations of the wheel knuckles [5,6] as illustrated in Fig. 1.

RESULTS AND DISCUSSIONS

The formulations derived in the previous section are solved using Newton-Raphson method to obtain kinematic responses of the half-car model under either vertical motions of the wheels or roll motion of the chassis or a combination of sprung and unsprung masses motions. A sensitivity analysis is then performed to evaluate the relative contributions of variations in different joint coordinates. A composite performance index of camber angles and track width measures under wheel vertical displacement and chassis roll excitations is subsequently formulated and solved with constraints imposed on the roll center height and the suspension lateral packaging space to seek optimal joint coordinates.

The analyses are performed using the known geometry of a double-wishbone suspension. The coordinates of right suspension linkage joints, shown in Fig. 1, are taken in mm as: $M_r(430, 818)$, $N_r(644, 852)$, $O_r(365, 360)$, $P_r(743, 347)$, $C_r(787, 452)$, $A_r(660, 349)$ and $B_r(615, 920)$, respectively [4]. The left suspension is considered to be symmetric to the right suspension about a vertical line through the mass center of the vehicle body. Initial camber angles of the wheels are assumed to be zero, since the analyses are concerned with the variations in the responses alone.

KINEMATIC ANALYSIS: The kinematic responses are evaluated in terms of: (i) variations in the roll center height, the wheel track width variations and the bump camber angles under vertical displacement inputs at the wheel centers with fixed chassis; (ii) roll camber response to chassis roll input; and (iii) variations in the bump/roll camber angles under simultaneously applied wheel centers displacements and the chassis roll. The wheel center displacement input with fixed chassis was defined as a 100 mm amplitude harmonic input at a frequency of 1 Hz, such that $z_{ur} = z_{ul} = 100\sin(2\pi t)$ mm and $z_s = 0$. This input was chosen to realize peak jounce and rebound wheel motions of 100mm. The analyses were performed by considering two different types of simultaneous inputs, namely (i) relatively large chassis roll coupled with smaller wheel bump motions ($\phi_s = 5\sin(2\pi t)$ degrees; $z_{ur} = z_{ul} = 50\sin(2\pi t)$ mm; and $z_s = 0$); and (ii) relatively large wheel