

Time-Dependent Reliability Estimation for Dynamic Systems Using a Random Process Approach

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ABSTRACT

Reliability is an important engineering requirement for consistently delivering acceptable product performance through time. As time progresses, the product may fail due to time-dependent operating conditions and material properties, component degradation, etc. The reliability degradation with time may increase the lifecycle cost due to potential warranty costs, repairs and loss of market share. Reliability is the probability that the system will perform its intended function successfully for a specified time interval. In this work, we consider the first-passage reliability which accounts for the first time failure of non-repairable systems. Methods are available in the literature, which provide an upper bound to the true reliability which may overestimate the true value considerably. This paper proposes a methodology to calculate the cumulative probability of failure (probability of first passage or upcrossing) of a dynamic system, driven by an ergodic input random process. Time series modeling is used to characterize the input random process based on data from a “short” time period (e.g. seconds) from only one sample function of the random process. Sample functions of the output random process are calculated for the same “short” time, assuming that it is impractical to perform the calculation for a “long” duration (e.g. hours). Our proposed methodology calculates the cumulative probability of failure or equivalently, the time-dependent reliability, at a “long” time using an accurate “extrapolation” procedure of the failure rate. A representative example of a quarter car model on a stochastic road demonstrates the improved accuracy of the proposed method compared with available methods.

1. INTRODUCTION

In today's technological world, we demand continued functionality of a wide array of complex equipment and processes. We expect our cars and appliances for example, to

be functional and reliable. In reality, as time goes on, degradation in the components (e.g. material properties) and stochastic operating conditions (loading, environmental conditions, etc.), cause these products to deliver inconsistent performance and sometimes to even fail prematurely. As a result, we may have high warranty cost, customer annoyance and lower resale value, or even a catastrophic injury or loss of life. It is important therefore, to maintain good reliability over a long time period in order to maximize the product value over its lifecycle.

In product design, it is desirable to consider the attributes of survivability, quality, reliability, maintainability, and safety and reduce for example, scrap, rework, and maintenance time. This can be achieved by selecting the dimensions and materials so that their possible degradation does not affect the performance of the system [1,2,3]. The performance measures (e.g. system responses) should continue to satisfy the design specifications over the planned lifetime of the system. For many dynamic systems, an analytical evaluation of the system performance (e.g. stress level) can be practically performed over a “short” period of time (e.g. seconds), say in seconds, as compared to the lifecycle which may be in years. This research presents an analytical methodology to estimate the system reliability or equivalently, the cumulative probability of failure [3], over a “long” time using performance evaluations over a “short” time period. The approach can be used to reduce the lifecycle cost [4,5,6,7] or to predict the reliability in real-time, similarly to the work in [8]. It can also be used to set up a schedule for preventive condition-based-maintenance.

Reliability is defined as the probability that the system will perform its intended function successfully for a specified interval of time, under stated operating and environmental conditions. It is therefore, related to product functionality over time which is determined by the so-called “hard” and

“soft” failures [1, 2]. In a hard failure the system loses functionality due to a complete breakdown of one or more of its components, while in a soft failure the system is functional but one or more performance measures are out of conformance. The reliability associated with the hard failure is important for non-repairable systems where the replacement or repair of a failed component is not possible. In this case, the failed system is removed permanently from the population. In contrast, repairable systems [9] consist of multiple components which can be repaired or replaced if failed keeping therefore, the original system in the population. In this research, we use time-dependent reliability concepts associated with the so-called first-passage of non-repairable systems.

Time-dependent reliability has been widely addressed in the literature. The time-dependent phenomena can be due to degradation, random input loading and/or uncertainty in the design parameters in a controlled time-varying environment. In the past, many problems related to degradation have been treated as monotonic where the performance function degrades with time. For example, Andrieu-Renaud et al. [10] proposed the PHI2 method which is a time-variant, first-order reliability method (FORM), based on an outcrossing rate [11] which can estimate reliability of monotonic systems using degrading characteristics of the system components. However, most dynamic systems are non-monotonic. For a non-monotonic random process, the sample functions may increase and decrease with time. In general, reliability is strongly influenced by degradation over time which must be accounted for [12].

Consider a system with multiple responses which must conform to specification limits set by the designer. For demonstration purposes, degradation in time changes the statistics of the inputs (e.g. operating loads, material properties, etc.) and the resulting statistics of the responses. Figure 1 shows a probabilistic view of this process over time.

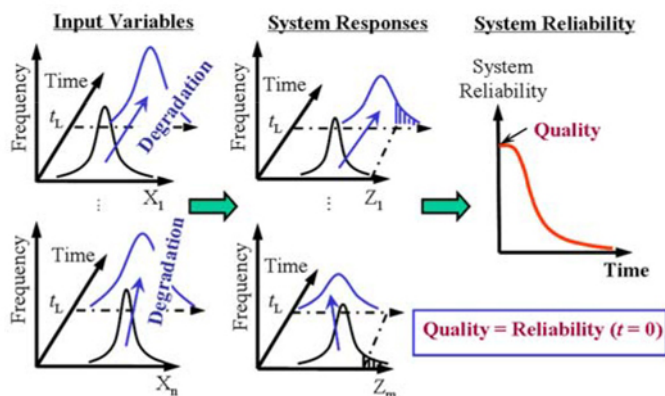


Figure 1. Uncertainty propagation for time-dependent system reliability (adopted from [2]).

The degradations of the components are shown in terms of input time-variant random variables with given probability density functions which change over time. The system response distributions also change over time due to input changes. Each response is a function of all inputs. The system reliability is shown on the right panel of Figure 1. It is built over time by calculating the system time-dependent reliability or probability of conformance. Note that the reliability at the initial time $t = 0$ is usually referred as quality.

Figure 2 shows our notation for the input and output of a system. The input random vector $\mathbf{X} = [\mathbf{X}_v, \mathbf{X}_p(t)]$ consists of a vector of random variables $\mathbf{X}_v$ and a vector of random processes $\mathbf{X}_p(t)$. An output random process $S(\mathbf{d}, \mathbf{X}, t)$ is obtained when $\mathbf{X}$ is passed through the system transfer function (TF). The vector of constant input parameters is denoted by $\mathbf{d}$.

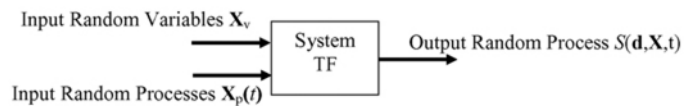


Figure 2. Schematic of input-output relations

A random input variable is quantified using its probability density distribution (PDF). If the PDF is not available, experimental data can be used to estimate it. A random input process must be however, characterized under certain assumptions (e.g. ergodicity and/or stationarity). It is often desirable to do so using experimental data which is usually available for a practically “short” period of time t_s (e.g. a few seconds of a finite-element analysis to calculate a stress level) compared to a “long” time T where $t_s \ll T$, the analysis must be carried out for (e.g. few hours of operation). The ergodicity assumption is common in order to avoid the need of collecting data for many sample functions of a random process in order to estimate its statistics. In this case, the statistics of the ergodic random process are determined from the temporal statistics of a single sample function.

In this paper, a time-series modeling approach is used to characterize an input random process using “short” time experimental data. Weak ergodicity is assumed in order to calculate ensemble averages (mean and autocorrelation function) using the temporal averages of only one sample function. Because ergodicity implies stationarity, a weakly ergodic process is also weakly stationary allowing us therefore to use the temporal averages over a “short” time duration to estimate the temporal averages (and because of stationarity, ensemble averages) of a “long” time duration. We propose in this paper, a methodology to calculate the cumulative probability of failure (probability of first passage - upcrossing-) of a dynamic system driven by an ergodic input random process. The input random process is

characterized by available experimental data during a “short” time period t_s from only one sample function. Sample functions of the output random process are calculated for the same “short” time, assuming that it is impractical to perform the calculation for a “long” duration T . Our proposed methodology calculates the cumulative probability of failure (or equivalently, time-dependent reliability) at a “long” time T , using an “extrapolation” procedure of the failure rate. A representative example of a quarter car model on a stochastic road is used to demonstrate the accuracy of the proposed method.

Section 2 provides the fundamentals of time-dependent reliability after a brief introduction of the classical time-independent structural reliability methods. The proposed methodology to calculate the cumulative probability of failure using Monte Carlo Simulation (MCS) is presented in section 3. Section 4 gives an overview of the time-series approach to characterize a random process and section 5 presents a quarter car example to demonstrate the potential of the proposed method. Summary and conclusions are presented in section 6.

2. RELIABILITY ANALYSIS

Reliability analysis estimates the probability of failure due to variability and/or uncertainty. It requires a probabilistic quantification of the variation/uncertainty and a definition of failure. The calculation of the probability of failure is used to express the constraints of a Reliability-Based Design Optimization (RBDO) problem probabilistically [13]. It can be time-independent or time-dependent.

In time-independent (or time-invariant) analysis, the uncertainty is quantified using random variables. A time-independent limit state function is used, where the vector $\mathbf{X}$ represents the input random variables with a joint probability density function $f_{\mathbf{X}}(\mathbf{x})$. The probability of failure is given by

$$P_f = P(g(\mathbf{X}) < 0) = \int \cdots \int_{g(\mathbf{x}) < 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \quad (1)$$

Because the multi-dimensional integral of Eq. (1) is difficult to evaluate, various analytical and simulation-based methods have been developed. The analytical methods include the well known first-order and second-order reliability methods (FORM [13, 14], SORM [15,16,17]), and multi-point approximation methods [18]. For system reliability analysis involving multiple limit states, the joint failure probability must be accounted for. Due to the difficulty in determining the joint failure probability of more than two failure modes except through MCS, approximations using first-order and second-order bounds have been developed [19, 20].

Simulation-based methods are also used for component or system reliability analysis. Among them, the MCS method is very simple and accurate. Because its computational cost can be prohibitively high, more efficient simulation-based techniques have been proposed. From the MCS-based methods seeking to improve computational efficiency, adaptive importance sampling (AIS) techniques [21] and multi-modal importance sampling methods have been proposed [22]. They use an importance sampling density function, which is gradually refined to reflect the increasing state of knowledge of the failure domain. A combination of analytical and simulation-based methods can also be used [23].

2.1. TIME-DEPENDENT RELIABILITY ESTIMATION

Let $S(\mathbf{d}, \mathbf{X}, t)$ be a random process and $S_{\text{threshold}}$ be an allowable threshold. A limit state is defined as

$$g(\mathbf{d}, \mathbf{X}, t) = S_{\text{threshold}} - S(\mathbf{d}, \mathbf{X}, t) \quad (2)$$

where $g(\mathbf{d}, \mathbf{X}, t) < 0$ indicates failure. Denoting the time interval of interest by $t_L = [t_{\min}, t_{\max}]$, the probability of failure within this time interval is defined as

$$P_f^c(t_{\min}, t_{\max}) = P(\exists t \in [t_{\min}, t_{\max}], g(\mathbf{d}, \mathbf{X}, t) < 0) \quad (3)$$

where failure occurs if the limit state function becomes negative at least at one instance within the time interval. This is known as *cumulative* probability of failure [3, 24, 25]. The definition of Eq. (3) is different from the *instantaneous* probability of failure

$$p_f^i(t) = P(g(\mathbf{d}, \mathbf{X}, t) < 0) \quad (4)$$

which is calculated in a quasi-steady sense, by fixing time t and replacing the random process by a random variable. As a result, it does not account for possible failures before time t . The conceptual difference between the cumulative and the instantaneous probabilities of failure is very important.

There is a special class of problems where $g(\mathbf{d}, \mathbf{X}, t)$ is monotonically decreasing (e.g. degradation phenomena) or increasing in $[t_{\min}, t_{\max}]$, i.e.,

$$\begin{aligned} \forall t_1, t_2 \in [t_{\min}, t_{\max}], t_1 < t_2 \Rightarrow g(\mathbf{d}, \mathbf{X} = \mathbf{X}_0, t_1) \geq \dots \\ \geq g(\mathbf{d}, \mathbf{X} = \mathbf{X}_0, t_2) \text{ (decreasing)} \end{aligned} \quad (5a)$$

or

$$\begin{aligned} \forall t_1, t_2 \in [t_{\min}, t_{\max}], t_1 < t_2 \Rightarrow g(\mathbf{d}, \mathbf{X} = \mathbf{X}_0, t_1) \leq \dots \\ \leq g(\mathbf{d}, \mathbf{X} = \mathbf{X}_0, t_2) \text{ (increasing)} \end{aligned} \quad (5b)$$

for every realization $\mathbf{X}_0$ of $\mathbf{X}$. This is true if $\frac{\partial g(\mathbf{X}, t)}{\partial t} \geq 0$ or ≤ 0 for all sample realizations of $\mathbf{X}$. It has been mentioned in [10, 24] that for monotonically decreasing systems,

$$\forall t \in [t_{\min}, t_{\max}], p_f^c(t_{\min}, t) = p_f^i(t). \quad (6)$$

For many time-dependent engineering problems, the monotonicity property of Eq. (5) does not hold and a different approach must be followed. For that, the commonly used outcrossing rate of a random process provides an upper bound on the time-dependent (cumulative) probability of failure. The outcrossing rate is defined [26] as

$$v^+(t) = \lim_{\Delta\tau \rightarrow 0, \Delta\tau > 0} \frac{P[g(\mathbf{d}, \mathbf{X}, t) > 0 \cap g(\mathbf{d}, \mathbf{X}, t + \Delta\tau) \leq 0]}{\Delta\tau}. \quad (7)$$

Ditlevsen and Madsen [13] calculated the mean number of outcrossings as

$$m_1 = E[N^+(t_{\min}, T)] = \int_{t_{\min}}^T v^+(t) dt \quad (8)$$

where $N^+(t_{\min}, t)$ is the number of outcrossings in the time interval $[t_{\min}, t]$, and Shinozuka [27] showed that the cumulative probability of failure is bounded as

$$\max_{t_{\min} \leq t \leq t_{\max}} p_f^i(t) \leq p_f^c(t_{\min}, t_{\max}) \leq p_f^i(t_{\min}) + m_1. \quad (9)$$

The computation of the outcrossing rate has been extensively studied [17, 28, 29] under the assumption of the outcrossings being statistically independent and Poisson distributed.

Methods such as the PHI2 and the AsM asymptotic method [30] compute an upper bound of the cumulative probability of failure of Eq. (3). Hagen and Tvedt [26] suggested a parallel system reliability formulation to compute the outcrossing rate. It uses two successive time-invariant analyses based on FORM, and the binomial cumulative distribution to calculate the probability of the joint event in Eq. (7).

In 1968, Rice [31] introduced an outcrossing rate approach to calculate the time-variant probability that a load exceeds a given threshold, known as the barrier crossing problem. Rice's method assumes that the system is linear resulting in a stationary output Gaussian process under the additional assumptions of Poisson distributed and statistically independent outcrossings and Gaussian input random process. The assumption of Poisson distributed outcrossings is commonly used in the literature [27, 28, 29, 32, 33, 34].

When the failure events are assumed statistically independent and Poisson distributed, an upper bound is calculated for the cumulative probability of failure. If we assume that 1) the probability of having two or more outcrossings in $[t, t+dt]$ is negligible compared to the probability of having exactly one outcrossing, 2) dt is sufficiently small, and 3) the outcrossings in $[t, t+dt]$ are statistically independent from previous outcrossings in $[0, t]$, then the number of outcrossings $N^+(t_{\min}, t)$ is a Poisson process and the probability that $N^+(t_{\min}, t)$ is equal to n is given by

$$P(N^+(t_{\min}, t) = n) = \frac{m_1^n}{n!} e^{-m_1}. \quad (10)$$

The probability of no failure in $[t_{\min}, t]$ is therefore, provided by

$$P(N^+(t_{\min}, t) = 0) = e^{-m_1} \quad (11)$$

and the probability of at least one failure in $[t_{\min}, t]$ is equal to

$$P(N^+(t_{\min}, t) > 0) = 1 - e^{-m_1}. \quad (12)$$

Based on the definition of P_f^c in Eq. (3) and Eq. (12), we get,

$$p_f^c(t_{\min}, t) = 1 - e^{-m_1}. \quad (13)$$

Eq. (13) assumes that $p_f^i(t_{\min}) = 0$ which may not be always true. It can be modified to include the initial condition $p_f^i(t_{\min})$ as [39],

$$p_f^c(t_{\min}, t) = 1 - (1 - p_f^i(t_{\min}))e^{-m_1} \quad (14)$$

Eq. (14) provides an upper bound to the first-passage cumulative probability of failure because of the assumption of statistically independent outcrossings. It estimates p_f^c relatively well only if $m_1 \ll 1$ which is true for a very small probability of failure [33]. However, it grossly overestimates the true p_f^c if the latter is larger than 0.1 to 0.3 depending on the problem at hand. The PHI2 method also provides an upper bound to p_f^c . This upper bound is even wider than the upper bound from Eq. (14) because the PHI2 method simply calculates $P[g(\mathbf{d}, \mathbf{X}, t) > 0 \cap g(\mathbf{d}, \mathbf{X}, t + dt) \leq 0]$ without accounting for upcrossings at earlier times (no “time memory”). Appendix 1 discusses the PHI2 approach in more detail.

For a stationary process, the outcrossing rate of Eq. (7) is constant; i.e., $\nu^+(t) = \nu_m$, and Eq. (8) becomes,

$$m_1 = \nu_m(t - t_{\min}) \quad (15)$$

It should be noted that if either ν_m or t is large, the value of m_1 from Eq. (15) is large, resulting in an upper bound of p_f^c provided by Eq. (14), which is close to 1. This upper bound usually deviates significantly from the actual value of p_f^c . If the threshold level is low, the outcrossing rate is large, and the outcrossings are strongly correlated violating the assumption of statistical independence. It is therefore, concluded that Eq. (14) is accurate only when the outcrossing rate is low which occurs when the threshold level is high.

In this paper, we propose a methodology to estimate p_f^c at a “long” time T using failure rate data over a very short time t_s . The failure rate data in $[0, t_s]$ is utilized in order to avoid the assumption of statistical independence at the initial time when the outcrossings are heavily correlated. The assumption of statistical independence of outcrossings is used later in the life cycle of the system when the failure rate declines significantly so that the outcrossings can be treated as

statistically independent. The proposed methodology is discussed over the next few sections.

3. CUMULATIVE PROPABILITY OF FAILURE

3.1. FAILURE RATE FOR FIRST-PASSAGE (NON-REPAIRABLE SYSTEMS)

The probability that an item will fail at time T_f where $t < T_f \leq t + dt$, given that the item is functioning at time t , is given by

$$P(t < T_f \leq t + dt | T_f > t) = \frac{P(t < T_f \leq t + \Delta t)}{P(T_f > t)} = \frac{p_f^c(t + dt) - p_f^c(t)}{R(t)} \quad (16)$$

where $R(t) = P(T_f > t)$ denotes the reliability at time t . The failure rate function is defined as [35],

$$\lambda(t) = \lim_{dt \rightarrow 0} \frac{P(t < T_f \leq t + dt | T_f > t)}{dt} = \frac{f(t)}{R(t)} \quad (17)$$

where $f(t)$ is the probability density function of the time to failure (first-passage failure) provided by

$$f(t) = \frac{d}{dt} p_f^c(t) \quad (18)$$

The failure rate function $\lambda(t)$ is also called hazard rate. It provides the average number of systems that fail per unit time, given that these systems survived up to time t . Note that the reliability is also equal to

$$R(t) = 1 - p_f^c(t) \quad (19)$$

Based on Eqs (17),(18),(19), the first-passage probability of failure (cumulative probability of failure) $p_f^c(t)$ is given in terms of the failure rate $\lambda(t)$ as

$$p_f^c(t) = 1 - [1 - p_f^i(0)] \exp^{-\int_0^t \lambda(\xi) d\xi} \quad (20)$$

Because $e^{-x} \approx 1 - x$ for small values of x , Eq. (20) can become,

$$p_f^c(t) = 1 - [1 - p_f^i(0)] \cdot (1 - \int_0^t \lambda(\xi) d\xi)$$

or

$$p_f^c(t) = p_f^i(0) + \int_0^t \lambda(\xi) d\xi \quad (21)$$

if $\int_0^t \lambda(\xi) d\xi$ is very small.

Because it is very challenging to “analytically” calculate $\lambda(t)$, a simulation approach such as MCS, can be used. However, MCS can become inefficient if the time t is too long or the number of sample functions is large. In this research, MCS efficiency is significantly improved by only calculating the failure rate over a “short” time t_s that represents a small fraction of the system lifecycle. Using this information, the failure rate is then “estimated” over a “long” time T . Instead of MCS, a “composite limit state” [3, 24, 25] approach can also be used to calculate $\lambda(t)$.

To use MCS, the time t is discretized using a time step Δt , so that $t = i \cdot \Delta t$ where $i = 0, 1, 2, \dots, I$ is the step number. The failure rate λ_i at time increment i is then computed as,

$$\lambda_i = \frac{\# \text{ of first time failures at the } i(\text{th}) \text{ step conditioned on system survival at } i-1 \text{ step}}{(\# \text{ of systems that did not fail at step } i-1) \cdot \Delta t} \quad (22)$$

or

$$\lambda_i = \frac{\sum_{n=1}^N p(i)}{N_s(i-1) * \Delta t}, \quad (23)$$

where the counter $p(i)$ is incremented by one if for the n^{th} ($n = 1, 2, \dots, N$) sample function, the event $\{g(k) > 0 \text{ for } k = 1, 2, \dots, i-1 \text{ and } g(i) \leq 0\}$ is true. This event captures the first failure as indicated by the limit state function g . Figure 3 shows this schematically. In Eq. (23), $N_s(i-1)$ denotes the number of sample functions (out of N) for which there is no outcrossing for all time steps $k = 1, 2, \dots, i-1$.

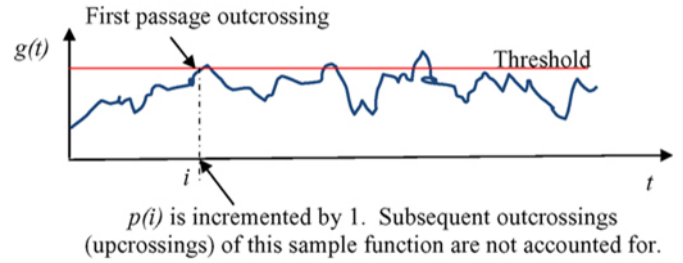


Figure 3. Schematic of first-passage failure

It should be noted that the failure rate $\lambda(t)$ of Eq. (17) is different from the outcrossing rate $\nu^+(t)$ of Eq. (7). The latter has “no memory” of past events. In contrast, the failure rate $\lambda(t)$ counts only the first failure implying that the state has been safe at all previous instances. Both the PHI2 method and the Poisson-based approach of Eq. (14) use the outcrossing rate $\nu^+(t)$. However, Eq. (20) provides the exact first-passage P_f^c considering the contribution of all past events. Figure 4 shows the “memory-less” outcrossings used in the PHI2 method and the Poisson-based approach.

Using MCS, the outcrossing rate ν_i at time increment i (discretized $\nu^+(t)$ in time) is calculated as

$$\nu_i = \frac{\# \text{ of failures at } i \text{ when safe at } i-1 \text{ step regardless of past failure events}}{(\text{total } \# \text{ of samples}) \cdot \Delta t} \quad (24)$$

or,

$$\nu_i = \frac{P(g(i-1) > 0 \cap g(i) \leq 0))}{\Delta t} = \frac{\sum_{n=1}^N p_2(i)}{N * \Delta t}. \quad (25)$$

The counter $p_2(i)$ is incremented by one if for the n^{th} ($n = 1, 2, \dots, N$) sample function, the event $\{g(i-1) > 0 \text{ and } g(i) \leq 0\}$ is true. The counter accounts for all upcrossings of the n^{th} sample function in the time period of interest. Figure 4 shows this schematically.