

Analytical Stress Intensity Factor Solutions for Spot Welds Joining Sheets of Different Materials and Thicknesses

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ABSTRACT

In this paper, analytical stress intensity factor solutions for spot welds with ideal geometry in lap-shear specimens of different materials and thicknesses are presented as functions of the applied load, the elastic material property parameters, and the geometric parameters of the weld and specimen. The analytical stress intensity factor solutions are selectively validated by the results of a three-dimensional finite element analysis for a dissimilar spot weld with ideal geometry in a lap-shear specimen. Finally, selected stress intensity factor solutions at the critical locations of spot welds in lap-shear specimens of dissimilar magnesium, aluminum and steel sheets with equal and different thicknesses are presented in the normalized forms as functions of the ratio of the specimen width to weld diameter.

INTRODUCTION

Resistance spot welding is widely used to join sheet metals in the automotive industry. These resistance spot welds are subjected to complex multiaxial loads under service conditions. The fatigue lives of resistance spot welds in various types of specimens have been investigated by many researchers, for example, see Zhang [1]. Since resistance spot welds provide natural cracks or notches along the nugget circumferences, the stress intensity factor solutions at the critical locations of the main and kinked crack fronts for the welds have been developed and adopted in order to investigate the fatigue behaviors of spot welds in various types of specimens [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25].

It should be noted that the analytical stress intensity factor solutions developed in Lin et al. [23] and Lin and Pan [25]

are for spot welds joining sheets of identical material and equal thickness. Zhang [16] developed a general procedure to obtain the stress intensity factor and J integral solutions for spot welds joining sheets of different materials and thicknesses. In Zhang [16], the stress intensity factor and J integral solutions are expressed as functions of the structural stresses. In this paper, the analytical stress intensity factor solutions for spot welds are presented as functions of the applied load, the elastic material property parameters, and the geometric parameters of the weld and specimen based on the available structural stress solutions developed by Lin and Pan [24]. The analytical stress intensity factor solutions are selectively validated by the results of a three-dimensional finite element analysis for a dissimilar spot weld with ideal geometry in a lap-shear specimen. Finally, selected stress intensity factor solutions at the critical locations of spot welds in lap-shear specimens of dissimilar magnesium, aluminum and steel sheets with equal and different thicknesses are presented in the normalized forms as functions of the ratio of the specimen width to weld diameter.

ANALYTICAL STRESS INTENSITY FACTOR SOLUTIONS

Fig. 1(a) shows a schematic plot of a lap-shear specimen with a spot weld idealized as a circular cylinder. The upper and lower sheets used to make the specimen have the nominal thicknesses t_u and t_l , respectively. In this paper, a subscript or superscript u represents the upper sheet and a subscript or superscript l represents the lower sheet. As shown in Fig. 1(a), the lap-shear specimen has the width $2b$ and the nugget diameter $2a$. Here, L represents the length of the upper and lower sheets of the specimen, and V represents the overlap



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length. In Fig. 1(a), a set of doublers is used to align the loads to avoid the initial realignment of the specimen under lap-shear loading conditions. The critical locations of the spot weld are marked as A, B, C and D in Fig. 1(a).

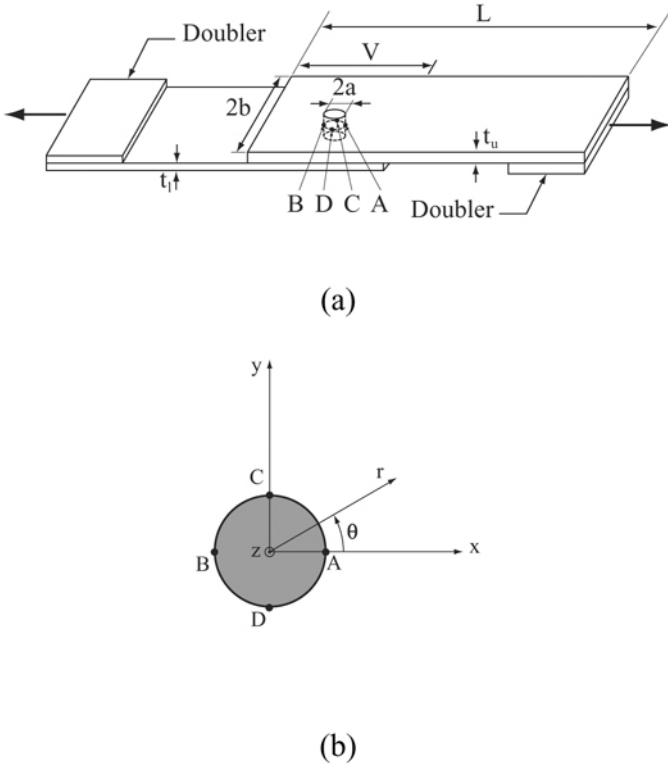


Fig. 1. Schematic plots of (a) a lap-shear specimen with a spot weld idealized as a circular cylinder, (b) a top view of the weld nugget in the upper sheet of the specimen.

Fig. 1(b) shows a schematic plot of a top view of the weld nugget in the upper sheet of the specimen with the cylindrical and Cartesian coordinate systems centered at the center of the upper half of the weld nugget. The critical locations of points A, B, C and D are 0° , 180° , 90° and 270° with respect to the x axis, respectively. The orientation angle θ represents the angular location along the nugget circumference of the spot weld.

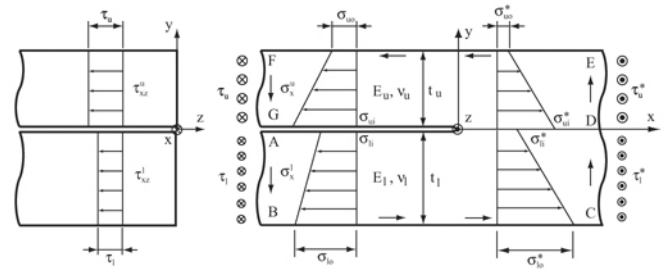


Fig. 2. The front and side views of the strip model of Radaj and Zhang [8,9,11].

Fig. 2 shows the front and side views of the strip model of Radaj and Zhang [8,9,11] with linearly distributed structural stresses based on the classical Kirchhoff plate theory. In this figure, The normal stresses σ_{ui} , σ_{uo} , σ_{li} and σ_{lo} represent the normal stresses for lines $\overline{FG}$ and $\overline{AB}$, and the normal stresses σ_{ui}^* , σ_{uo}^* , σ_{li}^* and σ_{lo}^* represent the normal stresses for lines $\overline{DE}$ and $\overline{CD}$ at the inner (i) and outer (o) surfaces of the upper (u) and lower (l) strips, respectively. The shear stresses τ_u and τ_l represent the shear stresses for lines $\overline{FG}$ and $\overline{AB}$, and the shear stresses τ_u^* and τ_l^* represent the shear stresses for lines $\overline{DE}$ and $\overline{CD}$ of the upper (u) and lower (l) strips, respectively.

The asymptotic in-plane stress field around an interface crack tip is an oscillatory field that can be characterized by a complex stress intensity factor $\mathbf{K} = K_1 + iK_2$ ($i = \sqrt{-1}$) (Rice and Sih [26]). The stresses σ_y and τ_{xy} at a small distance r ahead of the interface crack tip are characterized by $\mathbf{K}$ as

$$\sigma_y + i\tau_{xy} = \frac{K_1 + iK_2}{\sqrt{2\pi r}} \left(\frac{r}{t} \right)^{i\varepsilon} \quad (1)$$

Here, the bimaterial constant ε is defined as

$$\varepsilon = \frac{1}{2\pi} \ln \frac{\kappa_u / G_u + 1 / G_l}{\kappa_l / G_l + 1 / G_u} \quad (2)$$

where G_u and G_l are the shear moduli for the upper and lower sheet materials, respectively. Here, κ_u and κ_l are defined as

$$\kappa_u = 3 - 4\nu_u \quad (3)$$

and

$$\kappa_l = 3 - 4\nu_l \quad (4)$$

under plane strain conditions.

In Eq. (1), t represents a characteristic length (Rice [27]; Suo and Hutchinson [28]; Zhang [16]). In this investigation, t is taken as the smaller value of the upper and lower sheet thicknesses as in Suo and Hutchinson [28] and Zhang [16]. Based on the in-plane part of the J integral solution for the strip model and the analytical solutions for interface cracks of Suo and Hutchinson [28], Zhang [16] derived the stress intensity factor solutions K_1 and K_2 (as defined in Eq. (1)) for spot welds joining sheets of different materials and thicknesses in terms of the structural stresses σ_{ui} , σ_{uo} , σ_{li} and σ_{lo} (as defined in Fig. 2) as

$$K_1 = \frac{\cosh(\pi\varepsilon)\sqrt{t_u}}{2\sqrt{3(1+\eta)(1+4\eta\delta+6\eta\delta^2+3\eta\delta^3)(1+\tan^2\omega)}} \times \left\{ \left[\frac{(1+4\eta\delta+9\eta\delta^2+6\eta\delta^3)\tan\omega}{\sqrt{1+2\eta\delta(2+3\delta+2\delta^2)+\eta^2\delta^4}} - \sqrt{3} \right] \sigma_{ui} - \left[\frac{(1+4\eta\delta+3\eta\delta^2)\tan\omega}{\sqrt{1+2\eta\delta(2+3\delta+2\delta^2)+\eta^2\delta^4}} + \sqrt{3} \right] \sigma_{uo} + \eta \left[\frac{\delta(1-2\eta\delta-3\eta\delta^2)\tan\omega}{\sqrt{1+2\eta\delta(2+3\delta+2\delta^2)+\eta^2\delta^4}} + \sqrt{3}(2+\delta) \right] \sigma_{li} - \eta \left[\frac{(1+4\eta\delta+3\eta\delta^2)\tan\omega}{\sqrt{1+2\eta\delta(2+3\delta+2\delta^2)+\eta^2\delta^4}} + \sqrt{3} \right] \sigma_{lo} \right\} \quad (5)$$

$$K_2 = \frac{\cosh(\pi\varepsilon)\sqrt{t_u}}{2\sqrt{3(1+\eta)(1+4\eta\delta+6\eta\delta^2+3\eta\delta^3)(1+\tan^2\omega)}} \times \left\{ \left[\frac{(1+4\eta\delta+9\eta\delta^2+6\eta\delta^3)}{\sqrt{1+2\eta\delta(2+3\delta+2\delta^2)+\eta^2\delta^4}} + \sqrt{3}\tan\omega \right] \sigma_{ui} - \left[\frac{(1+4\eta\delta+3\eta\delta^2)}{\sqrt{1+2\eta\delta(2+3\delta+2\delta^2)+\eta^2\delta^4}} - \sqrt{3}\tan\omega \right] \sigma_{uo} + \eta \left[\frac{\delta(1-2\eta\delta-3\eta\delta^2)}{\sqrt{1+2\eta\delta(2+3\delta+2\delta^2)+\eta^2\delta^4}} - \sqrt{3}(2+\delta)\tan\omega \right] \sigma_{li} - \eta\delta \left[\frac{(1+4\eta\delta+3\eta\delta^2)}{\sqrt{1+2\eta\delta(2+3\delta+2\delta^2)+\eta^2\delta^4}} - \sqrt{3}\tan\omega \right] \sigma_{lo} \right\} \quad (6)$$

The thickness and modulus ratios δ and η are defined as

$$\delta = t_u/t_l \quad (7)$$

and

$$\eta = E'_u/E'_l \quad (8)$$

where

$$E'_u = E_u/(1-\nu_u^2) \quad (9)$$

and

$$E'_l = E_l/(1-\nu_l^2) \quad (10)$$

Here, E_u and E_l are the Young's moduli for the upper and lower sheet materials, respectively. Note that the expressions shown in Eqs. (5) and (6) are for spot welds joining sheets with $\delta \leq 1$. For spot welds joining sheets with $\delta > 1$, one should rotate the strip model by an angle of 180° to represent the same physical system but with $\delta < 1$. The values of the angular quantity ω in Eqs. (5) and (6) can be found in Suo and Hutchinson [28] and Zhang [16]. The K_3 solution can be obtained as (Tran and Pan [29])

